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THE ESTIMATION AND SELECTION OF PARAMETERS IN PATTERN RECOGNITION PROBLEMS WITH THE HELP OF CONSTRUCTION OF A CLASS OF DECISIVE FUNCTIONS BASED ON CALCULATION OF ENTROPY

Abstract

By solving the problems on object recognition on manyspectrul videoinformation, the success depends on right definition of significance of each symptom and construction of decision rule that is invariant to admissible transformations. In the paper a method to construct decision rule on the basis of entropy calculation is considered.

In spite of significant experience in the interpretational processing of the aerocosmic information, today the one-valued procedures of transition from totality of the remote measured quantities to determination of types and states of objects of earth surface haven't been worked out. The absence of such procedures is related with variety of types and states of investigation objects, changeability, dependence of results of measurements on conditions of observation and atmosphere state, multipurpose character of the processing problems, and also insufficiency of systematic investigations in the sphere of physic-mathematical descriptions of earth surface on the basis of date of the remote measurements [1].

In conformity to problems of studying the earth from cosmos the investigation of possibilities and development of methods of description of types and states of objects of the earth surface in their dynamics is the main problem of modern aerocosmic investigations.

The important step in these investigations is the solution of objects recognition problem of the earth surface, in particularly the problem of choice of base of classification.

The base may be the object with known properties or country area determined by the geographical coordinates, then to which the dates about they texture and spectrum of the reflected and own radiation are assigned.

Such classical methods of analysis of images as: the method of training sample (recognition with an apriori information) and the method of cluster analysis (recognition without an apriori information) [2] correspond to the solution of this problem.

The method of training sampling requires the deep apriori attainments of the investigation object, that is related with great volume of surface investigations which have own sufficiently difficult problems. The methods of training sample is also constrained from these defects, but it permits in a great degree to formalize the classification process releasing it from necessity of coordination with the surface dates in certain stage. Giving preference to the case of classification on basis of the distantly measure quantities and correspondingly to the classification methods one can determine the following stages of objects recognition of surface by aerocosmic photos:

1. the determination of texture of some area of surface on the basis of multispectral videoinformation formed by principal-component method and formed by textured characteristic of array of initial dates (the base of classification are investigated objects composing the set of training sample);
2. the determination of spectro-energy characteristics on the basis of multispecral videoinformation for fragments of training sample, get in array of initial dates and formation by spectral- textured characteristics of array of intermediate dates;

3. the establishment of connection between the investigated country areas get in array of the initial dates, and statistic characteristics of the surface objects situated in them.

In the given paper the problem of formalization of the initial dates, otherwise the formalization of an apriori information about the investigated objects among the class of training sample and their centers on the basis of application of transformation by principal-components [2, 3] is considered.

The main calculating difficulty of the indicated above principal components is calculation of covariance matrix, matrices of eigen values and eigen vectors. The main preference is that with the help of this method it is possible to press the volume the information without loss of useful factor in this information.

Hence it follows the tendency of development of investigations as far as possible the decrease of calculation difficulty of the given method and increase of its effectiveness. Today in this direction the methods forming different classes of resolving rules and in particular the methods based on comparison of such statistic characteristic as entropy have got a development.

Principal- component method is based on orthogonal (unitary) transformation of coordinates of the initial space of signs describing the investigated objects by polyzonal aerocosmic information, the aim of which is the obtaining of the optimal system of coordinates.

The measure of "importance" or "weight" of each coordinate axis can be dispersion of projections to this axis of vectors representing the objects. Then for a "good" system of coordinates the large values of weights will be only in little number of parameters and for a "bad" system of coordinates the weights of all parameters will be approximately the same. In this case, if for the measure of uniformity of distribution of weights we will take the entropy, then the optimal system of coordinates will be that to which corresponds the minimal entropy. Hence the recognized, "unknown" object is categorized by this system and its entropy is calculated and one can expect that if the unknown object belongs to the class X then its entropy will be small and comparable with the training sampling which is an image of this class. And if the recognized object belongs to the class Y then obviously its entropy will be significantly lager than the mean entropy of the training sample of classes of the investigated objects by their characteristic parameters.

In the present paper the method of construction of decisive rule on the base of calculation of the is considered, with the help of which construction of the initial dates on the training sampling and selection of parameters permitting to separate the class of objects effectively, is possible.

Mathematical statement of the above mentioned problem is considered.

Let $x^{(\alpha)}$ be a normed vector of the object $a, \alpha - 1,...,v$ in n dimensional space of parameters, i.e. $\left(x^{(\alpha)} \cdot x^{(\alpha)}\right) = 1$. Let $\{e_i\}$, $i = 1,2,...,n$ be an initial rectangle system of co-ordinates in which the set of objects and $(e_i \times e_k) = \delta_{ik}$ are given. Then

$$x_i^{(\alpha)} = \left(x^{(\alpha)} \times e_i\right),$$
$$x^{(\alpha)} = \sum_{i=1}^{n} x_i^{\alpha} \times e_i \,. \tag{1}$$

If we'll introduce another rectangle system of coordinates $\{\xi_j\}$ related to $\{e_i\}$ with orthogonal transformation T_{ij}

$$T_{ij} = \left(T'\right)_{ij} = \left(T^{-1}\right)_{ij}. \tag{2}$$

components of vector $x^{(\alpha)}$ in a new system of coordinates $\{\xi_j\}$ will be:

$$c_j^{(\alpha)} = \sum_{i=1}^{n} x_i^{(\alpha)} \times T_{ij} = \left(x^{(\alpha)} \times \xi_j\right), \tag{3}$$

where

$$x^{(\alpha)} = \sum_{j=1}^{n} c_j^{(\alpha)} \times \xi_j .$$

"Weight" of the axis ξ_j for the set $\{x^{(\alpha)}\}$ one can define as:

$$\rho(\xi_j) = \frac{1}{V} \sum_{\alpha=1}^{V} \left[c_j^{(\alpha)}\right]^2 ,$$

where

$$\sum \rho\{\xi_j\} = 1 . \tag{4}$$

"Uniformness" of the distribution of weights in the system of coordinates is measured with entropy

$$S(\{\xi_j\}) = \sum_{j=1}^{n} \rho(\xi_j) \log \rho\left(\xi_j^{(n)}\right). \tag{5}$$

Then the decisive rule has the form

$$S\{g_\alpha\} = \min S\{(\xi_j)\}. \tag{6}$$

In a practice the above described statement of the problem is fulfilled by the following way.

The correlation matrix of the form:

$$G_{ij} = \frac{1}{V} \sum_{\alpha=1}^{V} x_i^{(\alpha)} \times x_j^{(\alpha)} \tag{7}$$

is calculated, then eigen vectors g_k of the matrix G_{ij} are calculated as:

$$\sum_{j=1}^{n} G_{ij} g_j^{(k)} = \lambda^{(k)} \times g_i^{(k)} , \tag{8}$$

where $g_i^{(k)}$ are components of vector g_k in the direction e_j. If we expand the eigen vectors in decreasing order

$$\lambda^{(1)} \geq \lambda^{(2)} \geq \ldots \geq \lambda^{(n)}, \tag{9}$$

then g_1 is the most "important" parameter, g_2 is the second in the degree of importance and so on.

If we expand $x^{(\alpha)}$ in series similarly to (3) but with the number of terms in expansion less than n, for example $m(m < n)$, then the optimal system of coordinates minimizes the average error

$$\sum_{k=1}^{m} \rho(g_k) = \max_{\{\xi_j\}} \sum \rho(\xi_j) \tag{10}$$

for each integer $m < n$.

So we obtained the decisive rule (6) on the base of calculation of the entropy which defines the optimal system of coordinates $\{g_k\}$ which one can represent a set of eigen vectors of the correlation matrix (7). Then subject to the inequality (9) and selecting the optimal coordinate axis as far as importance of "weight", using the rule (10) one can reduce the initial n dimensional space of signs to the space of less dimension $m(m < n)$. The fundamental and main property of recognition by the decisive rules (6) and (10) is in the transformation reducing to the optimal system of coordinates of less dimensional initial one saves the properties of the investigated objects by which one can correctly identify and recognize them.

The last is one of the important problem on recognition of the earth surface objects.

Thus on calculation of entropy we'll obtain the class of decisive functions which unlike the linear transformation of principle-components introduces a more difficult function from components of initial vectors which in its turn raises the effectiveness of solution of recognition problem.

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