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DIRICHLET PROBLEM FOR ONE CLASS OF NONUNIFORMLY DEGENERATE SECOND ORDER ELLIPTIC EQUATIONS

Abstract

In the article one class of nonuniformly degenerate second order elliptic equations is considered. The unique weak solvability of the first boundary value problem for these equations in anisotropic weighted Sobolev spaces is proved.

Introduction. Let the bounded domain D with the boundary ∂D be located in n-dimensional Euclidean space $\mathbf{E}_n$ of the points $x=(x_1,...,x_n)$, $n\geq 2$, $O\in D$.

In D consider the elliptic operator

$$\mathcal{L}=\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial}{\partial x_j}\right)+\sum_{i=1}^{n}b_i(x)\frac{\partial}{\partial x_i}+c(x)$$

and assume that the coefficients $a_{ij}(x)$ are measurable functions in D, $a_{ij}(x)=a_{ji}(x)$, $i,j=1,...,n$. Moreover for all $\xi\in\mathbf{E}_n$, $x\in D$

$$\mu\sum_{i=1}^{n}\lambda_i(x)\xi_i^2\leq\sum_{i,j=1}^{n}a_{ij}(x)\xi_i\xi_j\leq\mu^{-1}\sum_{i=1}^{n}\lambda_i(x)\xi_i^2. \tag{1}$$

Here $\mu\in(0,1]$ is a constant,

$$\lambda_i(x)=\left(|x|_\alpha\right)^{\alpha_i},\ |x|_\alpha=\sum_{i=1}^{n}|x_i|^{\frac{2}{2+\alpha_i}},\ \ \alpha_i\geq 0;\ i=1,...,n. \tag{2}$$

We'll assume that if $\alpha^-=\min\{\alpha_1,...,\alpha_n\}$, then

$$\alpha^-<2. \tag{3}$$

We'll formulate the restriction on the functions $b_i(x)\,(i=1,...,n)$ and $c(x)$ later. In addition it's assumed that for $x\in D$

$$c(x)\leq 0. \tag{4}$$

The aim of the present paper is the proof of unique weak solvability of the first boundary value problem

$$\mathcal{L}u=f+\sum_{i=1}^{n}\frac{\partial f_i}{\partial x_i},\ \ u|_{\partial D}=0 \tag{5}$$

for any $f(x)\in L_2(D)$, $\dfrac{f_i(x)}{\sqrt{\lambda_i(x)}}\in L_2(D)$; $i=1,...,n$.

Note that for second order uniformly elliptic equations of divergent structure we can find the proof of unique weak solvability of Dirichlet problem in papers [1-2].

The corresponding results for elliptic equations with uniform degeneration are obtained in [3-4]. For equations with weak (logarithmic) nonuniform degeneration we note paper [5]. In [6] the first boundary value problem is considered on the assumption that the minor coefficients of the operator $\mathcal{L}$ are equal to zero. Note that for the second order elliptic equations of nondivergent structure with non-uniform power degeneration the analogous results are obtained in [7].

Let's agree to some designations and definitions. Denote by $W^1_{p,\alpha}(D)$ a Banach space of the functions $u(x)$ given on D with the finite norm

$$\|u\|_{W^1_{p,\alpha}(D)} = \left[\int_D \left(|u|^p + \sum_{i=1}^n (\lambda_i(x))^{p/2} \left|\frac{\partial u}{\partial x_i}\right|^p\right) dx\right]^{1/p}, \quad (1 < p < \infty)$$

and by $\overset{\circ}{W}{}^1_{p,\alpha}(D)$ - the subspace $W^1_{p,\alpha}(D)$, dense set in which is a set of all functions from $C_0^\infty(D)$.

The function $u(x) \in \overset{\circ}{W}{}^1_{2,\alpha}(D)$ is called a weak solution of the Dirichlet problem (5), if for any $v(x) \in \overset{\circ}{W}{}^1_{2,\alpha}(D)$ the integral identity

$$\int_D \sum_{i,j=1}^n a_{ij}(x) \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_j} dx - \int_D \sum_{i=1}^n b_i(x) \frac{\partial u}{\partial x_i} v dx - \int_D c(x) uv dx =$$

$$= \int_D \left[\sum_{i=1}^n f_i \frac{\partial v}{\partial x_i} - fv\right] dx \tag{6}$$

is satisfied.

Everywhere further the notation $C(\cdots)$ means that the positive constant C depends only on the content of brackets.

1^0. Friedrichs type inequality.

Lemma 1. *Let the functions $\lambda_i(x)$, $i = 1,\dots,n$ be determined by the equalities (2) and the condition (3) be satisfied. Then for any function $u(x) \in \overset{\circ}{W}{}^1_{2,\alpha}(D)$ the estimate*

$$\int_D u^2 dx \le C_1 \int_D \sum_{i=1}^n \lambda_i(x) \left(\frac{\partial u}{\partial x_i}\right)^2 dx \tag{7}$$

holds, where $C_1 = \frac{2+\alpha^-}{2-\alpha^-} 4^{\frac{\alpha^-}{2+\alpha^-}} \cdot d^{\frac{4}{2+\alpha^-}}$, $d = diam D$.

Proof. It's sufficient to prove (7) for smooth functions. Let $u(x) \in C_0^\infty(D)$. Since the domain D is bounded, then there exists a point $x^0 \in E_n$ such hat the cube $Q = \{x : x_i^0 < x_i < x_i^0 + d,\ i = 1,\dots,n$ contains $\overline{D}$. We continue the function $u(x)$ by zero in $Q \setminus D$ and denote the obtained extension again by $u(x)$. It's clear that $u(x) \in C_0^\infty(Q)$.

Without loss of generality we can assume that $\alpha^- = \alpha_1$. Denote $(x_2,\dots,x_n)$ by x'. We have for $x_1 \in (x_1^0, x_1^0 + d)$

$$u(x_1, x') = \int_{x_1^0}^{x_1} \frac{\partial u}{\partial x_1}(t, x') dt.$$

Using the Cauchy- Bunyakowsky inequality we get

$$u^2(x_1, x') \le \int_{x_1^0}^{x_1} \frac{dt}{\lambda_1(t, x')} \int_{x_1^0}^{x_1} \lambda(t, x') \left(\frac{\partial u}{\partial x_1}(t, x')\right)^2 dt. \tag{8}$$

But on the other hand

$$J=\int_{x_1^0}^{x_1}\frac{dt}{\lambda_1(t,x')}\le\int_{x_1^0}^{x_1^0+d}\frac{dt}{\lambda_1(t,x')}=\int_{x_1^0}^{x_1^0+d}\frac{dt}{\left(|t|^{\bar{\alpha}_1}+\sum_{i=2}^{n}|x_i|^{\bar{\alpha}_i}\right)^{\alpha_1}}\le$$

$$\le\int_{x_1^0}^{x_1^0+d}\frac{dt}{|t|^{\bar{\alpha}_1\alpha_1}}=\int_{x_1^0}^{x_1^0+d}\frac{dt}{|t|^{\frac{2\alpha_1}{2+\alpha_1}}},$$

where $\bar{\alpha}_i=\dfrac{2}{2+\alpha_i}$, $i=1,\dots,n$.

Without loss of generality we assume that $x_1^0\in\left[-\dfrac{d}{2},0\right]$. Then

$$J\le\frac{2+\alpha_1}{2-\alpha_1}\left(\left|x_1^0\right|^{\frac{2-\alpha_1}{2+\alpha_1}}+\left(d-\left|x_1^0\right|\right)^{\frac{2-\alpha_1}{2+\alpha_1}}\right).$$

Since the function $\varphi(z)=z^{\frac{2-\alpha_1}{2+\alpha_1}}+(d-z)^{\frac{2-\alpha_1}{2+\alpha_1}}$ doesn't decrease for $z\in\left[0,\dfrac{d}{2}\right]$ then

$$J\le\frac{2+\alpha_1}{2-\alpha_1}2\left(\frac{d}{2}\right)^{\frac{2-\alpha_1}{2+\alpha_1}}. \tag{9}$$

From (8)-(9) we obtain

$$u^2(x_1,x')\le\frac{2+\alpha_1}{2-\alpha_1}\cdot2\left(\frac{d}{2}\right)^{\frac{2-\alpha_1}{2+\alpha_1}}\int_{x_1^0}^{x_1^0+d}\lambda_1(t,x')\left(\frac{\partial u}{\partial x_1}(t,x')\right)^2dt.$$

Integrating the last inequality over the cube Q we conclude

$$\int_Q u^2dx\le\frac{2+\alpha_1}{2-\alpha_1}4^{\frac{\alpha_1}{2+\alpha_1}}\cdot d^{\frac{4}{2+\alpha_1}}\int_Q\lambda_1(x)\left(\frac{\partial u}{\partial x_1}\right)^2dx.$$

Now it's sufficient to consider that $u(x)=0$ in $Q\setminus D$ and the required estimate (7) is proved.

2^0. Embedding theorem.

We'll use the following classical embedding theorem [8]

Theorem 1. *For any function $u(x)\in\overset{\circ}{W}{}_p^1(D)$ and arbitrary $p\in(1,n)$ the estimate*

$$\|u\|_{L_{\frac{pn}{n-p}}(D)}\le C_2(n,p,d)\|\nabla u\|_{L_p(D)} \tag{10}$$

is valid. Here $W_p^1(D)$ is the space $W_{p,\alpha}^1(D)$ in the case when α coincides with zero-vector, and

$$\|\nabla u\|_{L_p(D)}=\left(\int_D\sum_{i=1}^{n}\left|\frac{\partial u}{\partial x_i}\right|^p dx\right)^{1/p}.$$

Denote $\max\{\alpha_1,\dots,\alpha_n\}$ by α^*.

Theorem 2. *Let the functions $\lambda_i(x)$, $i=1,\dots,n$ be determined by the equality (2). If*

$$\alpha^{+} < \frac{4-2p}{3p-2}, \tag{11}$$

then for any p, $\frac{2n}{n+2} < p < 2$ *and any function* $u(x) \in \overset{\circ}{W}{}^{1}_{2,\alpha}(D)$ *estimate*

$$\|u\|_{L_{\frac{np}{n-p}}(D)} \le C_3 \left(\int_D \sum_{i=1}^{n} \lambda_i(x) \left(\frac{\partial u}{\partial x_i} \right)^2 dx \right)^{1/2} \tag{12}$$

is valid. Here

$$C_3 = C_2 \left(2d^{n-1}\right)^{\frac{2-p}{2p}} \left(\sum_{i=1}^{n} \gamma_i^{\frac{p-2}{p}} \left(\frac{d}{2}\right)^{\frac{\gamma_i(2-p)}{p}} \right)^{1/2}, \quad \gamma_i = \frac{4-2p-\alpha_i(3p-2)}{(2+\alpha_i)(2-p)}, \; i=1,\dots,n.$$

Proof. It's sufficient to consider the case $u(x) \in C_0^{\infty}(D)$. According to the previous theorem the estimate (10) holds. We apply the Hölder inequality with the exponents $q = \frac{2}{p}$ and $q' = \frac{2}{2-p}$. We get

$$\|\nabla u\|_{L_p(D)} = \left(\int_D \sum_{i=1}^{n} \left| \frac{\partial u}{\partial x_i} \right|^p dx \right)^{1/p} \le \sum_{i=1}^{n} \left(\int_D (\lambda_i(x))^{-\frac{p}{2}} (\lambda_i(x))^{\frac{p}{2}} \left| \frac{\partial u}{\partial x_i} \right|^p dx \right)^{1/p} \le$$

$$\le \sum_{i=1}^{n} \left(\int_D (\lambda_i(x))^{-\frac{p}{2-p}} dx \right)^{\frac{2-p}{2p}} \left(\int_D \lambda_i(x) \left(\frac{\partial u}{\partial x_i} \right)^2 dx \right)^{1/2} \le$$

$$\le \left(\sum_{i=1}^{n} \left(\int_D (\lambda_i(x))^{-\frac{p}{2-p}} dx \right)^{\frac{2-p}{p}} \right)^{1/2} \left(\int_D \sum_{i=1}^{n} \lambda_i(x) \left(\frac{\partial u}{\partial x_i} \right)^2 dx \right)^{1/2}. \tag{13}$$

On the other hand for $i = 1,\dots,n$

$$J_i = \int_D (\lambda_i(x))^{-\frac{p}{2-p}} dx \le d^{n-1} \int_{x_i^0}^{x_i^0+d} |t|^{-\frac{2\alpha_i p}{(2+\alpha_i)(2-p)}} dt.$$

From the condition (11) it follows that for $i = 1,\dots,n$

$$\frac{2\alpha_i p}{(2+\alpha_i)(2-p)} < 1.$$

Without loss of generality we can assume that $x_i^0 \in \left[-\frac{d}{2}, 0\right]$. Therefore operating just as in the proof of lemma 1 we obtain

$$J_i \le 2d^{n-1} \gamma_i^{-1} \left(\frac{d}{2}\right)^{\gamma_i}.$$

Thus

$$\left(\sum_{i=1}^{n} J_i^{\frac{2-p}{p}} \right)^{1/2} \le \left(2d^{n-1}\right)^{\frac{2-p}{2p}} \left(\sum_{i=1}^{n} \gamma_i^{\frac{p-2}{p}} \left(\frac{d}{2}\right)^{\frac{\gamma_i(2-p)}{p}} \right)^{1/2}.$$

Using the last estimate in (13) we obtain the required inequality (12).
The theorem is proved.

3^0. Weak solvability of the Dirichlet problem.

We'll assume that with respect minor coefficients of the operator $\mathcal{L}$, the conditions

$$\omega=\sum_{i=1}^{n}\left\|\frac{b_i}{\sqrt{\lambda_i}}\right\|_{L_r(D)}<\frac{\mu}{C_3}, \tag{14}$$

$$c(x)\in L_r(D), \tag{15}$$

where $r>n$, and the constant C_3 (just as the constant C_2 in its representation) is calculated at $p=\frac{2rn}{(n-1)r+n}$, are satisfied.

Besides we'll assume that

$$\alpha^+<\frac{2(r-n)}{(n-1)r+n}. \tag{16}$$

Theorem 3. *Let in the domain D the coefficients of the operator $\mathcal{L}$ satisfying the conditions (1), (4), (14)-(16) be determined. Then the first boundary value problem (5) is uniquely weak solvable in the space $\overset{\circ}{W}{}^1_{2,\alpha}(D)$ for any $f(x)\in L_2(D)$ and $\frac{f_i(x)}{\sqrt{\lambda_i(x)}}\in L_2(D)$; $i=1,\dots,n$.*

Proof. Consider for $u(x),v(x)\in \overset{\circ}{W}{}^1_{2,\alpha}(D)$ the bilinear form

$$B(u,v)=\int_D\left[\sum_{i,j=1}^{n}a_{ij}(x)\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j}-\sum_{i=1}^{n}b_i(x)\frac{\partial u}{\partial x_i}v-c(x)uv\right]dx.$$

We'll show at first that this form is bounded. We have

$$|B(u,v)|\le\int_D\sqrt{\sum_{i,j=1}^{n}a_{ij}(x)\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}\sum_{i,j=1}^{n}a_{ij}(x)\frac{\partial v}{\partial x_i}\frac{\partial v}{\partial x_j}}dx+$$

$$+\int_D\sqrt{\sum_{i=1}^{n}\frac{b_i^2(x)}{\lambda_i(x)}v^2\sum_{i=1}^{n}\lambda_i(x)\left(\frac{\partial u}{\partial x_i}\right)^2}dx+\sqrt{\int_D c^2(x)v^2dx\int_D u^2dx}=k_1+k_2+k_3. \tag{17}$$

Using the Hölder inequality we obtain

$$k_1\le\sqrt{\int_D\sum_{i,j=1}^{n}a_{ij}(x)\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}dx\int_D\sum_{i,j=1}^{n}a_{ij}(x)\frac{\partial v}{\partial x_i}\frac{\partial v}{\partial x_j}dx}\le$$

$$\le\mu^{-1}\sqrt{\int_D\sum_{i=1}^{n}\lambda_i(x)\left(\frac{\partial u}{\partial x_i}\right)^2dx\int_D\sum_{i=1}^{n}\lambda_i(x)\left(\frac{\partial v}{\partial x_i}\right)^2dx}\le\mu^{-1}\|u\|_{W^1_{2,\alpha}(D)}\|v\|_{W^1_{2,\alpha}(D)}; \tag{18}$$

and further

$$k_2\le\sqrt{\int_D\sum_{i=1}^{n}\frac{b_i^2x}{\lambda_i(x)}v^2dx\int_D\sum_{i=1}^{n}\lambda_i(x)\left(\frac{\partial u}{\partial x_i}\right)^2dx}\le\sum_{i=1}^{n}\sqrt{\int_D\frac{b_i^2(x)}{\lambda_i(x)}v^2dx}\,\|u\|_{W^1_{2,\alpha}(D)}. \tag{19}$$

On the other hand for $i=1,\dots,n$

$$\left(\int_D\frac{b_i^2(x)}{\lambda_i(x)}v^2dx\right)^{1/2}\le\left(\int_D|v|^{2q}dx\right)^{1/2q}\left(\int_D\frac{|b_i(x)|^{2q'}}{(\lambda_i(x))^{q'}}dx\right)^{1/2q'}, \tag{20}$$

where $q>1, q'=\frac{q}{q-1}$.

Assume $2q=\frac{np}{n-p}$, where $p\in\left(\frac{2n}{n+2},2\right)$ will be chosen later. Then $q'=\frac{np}{(n+2)p-2n}$.

Since the function $\varphi(p)=\frac{np}{(n+2)p-2n}$ decreases by p, then $q'>\frac{n}{2}$ for $p\in\left(\frac{2n}{n+2},2\right)$.

Let's fix $2q'=r$. Then $q=\frac{r}{r-2}$ and from the equality $\frac{2r}{r-2}=\frac{np}{n-p}$ we find $p=\frac{2rn}{(n+2)r-2n}$. Now using embedding theorem 2 from (20) we obtain for $i=1,\dots,n$

$$\left(\int_D \frac{b_i^2(x)}{\lambda_i(x)}v^2dx\right)^{1/2}\le C_3\sqrt{\int_D\sum_{i=1}^n\lambda_i(x)\left(\frac{\partial v}{\partial x_i}\right)^2dx}\left\|\frac{b_i}{\lambda_i}\right\|_{L_r(D)}\le$$

$$\le C_3\|v\|_{W^1_{2,\alpha}(D)}\cdot\left\|\frac{b_i}{\sqrt{\lambda_i}}\right\|_{L_r(D)}.$$

Allowing for the last estimation in (19) we conclude

$$k_2\le C_3\omega\|u\|_{W^1_{2,\alpha}(D)}\|v\|_{W^1_{2,\alpha}(D)}. \tag{21}$$

We analogously obtain

$$k_3\le\left(\int_D c^2(x)v^2dx\right)^{1/2}\|u\|_{W^1_{2,\alpha}(D)}\le\left(\int_D|v|^{\frac{2r}{r-2}}dx\right)^{\frac{r-2}{2r}}\|c\|_{L_r(D)}\|u\|_{W^1_{2,\alpha}(D)}\le$$

$$\le C_3\|c\|_{L_r(D)}\|u\|_{W^1_{2,\alpha}(D)}\|v\|_{W^1_{2,\alpha}(D)}. \tag{22}$$

Using now (18), (21) and (22) in (17) we arrive at the conclusion that

$$|B(u,v)|\le C_4\|u\|_{W^1_{2,\alpha}(D)}\|v\|_{W^1_{2,\alpha}(D)},$$

where $C_4=\mu^{-1}+C_3\omega+C_3\|c\|_{L_r(D)}$.

Thus the boundedness of the bilinear form $B(u,v)$ is proved. Now we show its coerciveness. We have for $u(x)\in W^1_{2,\alpha}(D)$ according to the conditions (1) and (4)

$$B(u,u)\ge\mu\int_D\sum_{i=1}^n\lambda_i(x)\left(\frac{\partial u}{\partial x_i}\right)^2dx-\left|\int_D\sum_{i=1}^n b_i(x)u\frac{\partial u}{\partial x_i}dx\right|. \tag{23}$$

But on the other hand

$$\left|\int_D\sum_{i=1}^n b_i(x)u\frac{\partial u}{\partial x_i}\right|\le\sum_{i=1}^n\sqrt{\int_D\frac{b_i^2(x)}{\lambda_i(x)}u^2dx}\times$$

$$\times\sqrt{\int_D\sum_{i=1}^n\lambda_i(x)\left(\frac{\partial u}{\partial x_i}\right)^2dx}\le C_3\int_D\sum_{i=1}^n\lambda_i(x)\left(\frac{\partial u}{\partial x_i}\right)^2dx\sum_{i=1}^n\left\|\frac{b_i}{\lambda_i}\right\|_{L_r(D)}=C_3\omega\int_D\sum_{i=1}^n\lambda_i(x)\left(\frac{\partial u}{\partial x_i}\right)^2dx.$$

Using the last estimate in (23) we have

$$B(u,u) \geq C_5 \int_D \sum_{i=1}^{n} \lambda_i(x) \left(\frac{\partial u}{\partial x_i} \right)^2 dx, \tag{24}$$

where $C_5 = \mu - \omega C_3 > 0$.

Now it's sufficient to apply lemma 1, and from (24) it follows

$$B(u,u) \geq \frac{C_5}{2} \int_D \sum_{i=1}^{n} \lambda_i(x) \left(\frac{\partial u}{\partial x_i} \right)^2 dx + \frac{C_5}{2C_1} \int_D u^2 dx \geq C_6 \|u\|^2_{W^1_{2,\alpha}(D)}, \tag{25}$$

where $C_6 = \min\left\{ \frac{C_5}{2}, \frac{C_5}{2C_1} \right\}$. Thus the bilinear form $B(u,v)$ is coercive.

Now note that for any $\frac{f_i(x)}{\sqrt{\lambda_i(x)}} \in L_2(D)$, $(i = 1,...,n)$ and $f(x) \in L_2(D)$ the integral

$$\int_D \left[\sum_{i=1}^{n} f_i \frac{\partial v}{\partial x_i} - fv \right] dx$$

represents a linear bounded functional on $W^1_{2,\alpha}(D)$. Using the Lax-Milgram theorem [8] we arrive at the required statement.

The theorem is proved.

Theorem 4. *If the conditions of previous theorem are satisfied, then for the weak solution $u(x)$ of the first boundary value problem (5) the estimate*

$$\|u\|_{W^1_{2,\alpha}(D)} \leq C_7 \left(\|f\|_{L_2(D)} + \sum_{i=1}^{n} \left\| \frac{f_i}{\sqrt{\lambda_i}} \right\|_{L_2(D)} \right), \tag{26}$$

where the positive constant C_7 depends on n, d, r, μ, ω and the vector α, is valid.

Proof. Let $u(x)$ be a weak solution of the Dirichlet problem (5). Assume in the integral identity (6), $v = u$. Then according to (25) we obtain

$$C_6 \|u\|^2_{W^1_{2,\alpha}(D)} \leq \frac{1}{2} \|u\|_{W^1_{2,\alpha}(D)} \sum_{i=1}^{n} \left\| \frac{f_i}{\sqrt{\lambda_i}} \right\|_{L_2(D)} + \frac{1}{2} \|u\|_{W^1_{2,\alpha}(D)} \|f\|_{L_2(D)}. \tag{27}$$

Without loss of generality we can assume that $\|u\|_{W^1_{2,\alpha}(D)} > 0$ (otherwise the estimation (26) is obvious). Now the required estimate (26) with $C_7 = \frac{1}{2C_6}$ follows from (27).

The theorem is proved.

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