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MECHANICS

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THE MOTION OF THE SPHERICAL INCLUSION WITH THE OSCILLATING MASS IN THE ELASTIC MEDIUM

Abstract

The investigation of the interaction of constructions with environment has a great practical meaning. One can cite such examples as: the surface structures subjected to wind loads, shock blast wave; sea structures subjected to surface waves "acoustic waves", the structures subjected to the seismic actions.

In the paper [1,2] the motion of the spherical and cylindrical inclusions with the spring- loaded mass in acoustic medium has been investigated. The problem of non-stationary wave actions to the cylindrical cover containing the spring-loaded masses was considered in the work [3]. By solving the problem Fourier integral transformation of the coordinates on axis of cylinder and Laplace are used. The inversion integrals are represented with the help of Gauss-Lager quadratic formula and expansion in terms of Intraspherical functions. But final calculations were realized for the steel cover immersed to the water and doesn't contain a discrete system of masses.

The considered problems have been solved in acoustic statement.

In the present paper the motion of the spherical inclusion with the spring-loaded mass in the elastic medium after passage of wave, is investigated.

In the case of axial symmetry the vector components of the replacements u and v are represented by the wave potentials φ and ψ by the following way

$$u=\frac{\partial\varphi}{\partial r}-\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right),$$
$$v=\frac{1}{r}\frac{\partial\varphi}{\partial\theta}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial\theta}\right). \tag{1}$$

Satisfying the wave equations

$$\Delta\varphi=\frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2};\quad \Delta\psi=\frac{1}{b^2}\frac{\partial^2\psi}{\partial t^2}, \tag{2}$$

where a, b are velocities of propagation of the longitudinal and transverse waves, correspondingly and they are expressed by the following way

$$a^2=\frac{\lambda+2\mu}{\rho};\quad b^2=\frac{\mu}{\rho}, \tag{3}$$

where λ, μ are Lame coefficients, ρ is a plane. Let's formulate the boundary conditions. It is assumed that the particles of elastic medium abutting to the inclusion are moved without isolation from him.

The pressure of medium to the inclusion is defined by the following way:

$$P=2\pi r^2\int_0^{\pi}\left(\sigma_r\sin\theta\cos\theta+\tau_{r\theta}\sin^2\theta\right)d\theta. \tag{4}$$

The equations of mentions of spherical inclusion of the mass M_1 and spring-loaded with it with shiffness of the mass M_2 will be

$$\begin{cases} M_1 \dfrac{d^2 x_1}{dt^2} = P + L(x_2 - x_1), \\ M_2 \dfrac{d^2 x_2}{dt^2} = -L(x_2 - x_1), \end{cases} \tag{5}$$

where x_1 and x_2 are the coordinates of positions of centers of masses.

We've the solutions of the equations (2) in the form

$$\varphi = \varphi_1 \cos\theta \,, \; \psi = \psi_1 \cos\theta \,. \tag{6}$$

The formula (1) will take the form

$$\begin{aligned} u &= \left(\varphi_1' + \frac{1}{r}\psi_1 \right)\cos\theta \,, \\ \mathrm{v} &= -\left(\psi_1' + \frac{1}{r}\varphi_1 \right)\sin\theta \,, \end{aligned} \tag{7}$$

and the solution (2) for the external boundary value problem in Carson-Laplace images will be

$$\begin{aligned} \overline{\varphi}_1 &= c\frac{1}{r^2}\left(\frac{p}{a}r + 1 \right)e^{-\frac{p}{a}r} \,, \\ \overline{\psi}_1 &= D\frac{1}{r^2}\left(\frac{p}{b}r + 1 \right)e^{-\frac{p}{b}r} \,. \end{aligned} \tag{8}$$

Defining stress component (8) with the help of (8) we'll find

$$\begin{aligned} \overline{P} = \frac{4\pi r_0^2}{3}\Bigg[& 2\mu D e^{-\frac{p}{b}r_0}\left(\frac{4p^3}{b^3}\frac{1}{r_0} + \frac{10p^2}{b^2}\frac{p}{r_0^2} + \frac{16p}{b}\frac{1}{r_0^3} + \frac{16}{r_0^4} \right) + \\ & + c e^{-\frac{p}{a}r_0}\left(\frac{\lambda p^3}{a^3}\frac{1}{r_0} + \frac{\lambda p^2}{a^2}\frac{1}{r_0^2} + \frac{2\mu p^3}{a^3}\frac{1}{r_0} + \frac{10\mu p^2}{a^2}\frac{1}{r_0^2} + \frac{24\mu p}{a}\frac{1}{r_0^3} + \frac{24\mu}{r_0^4} \right) \Bigg]. \end{aligned} \tag{9}$$

The replacement of particles of medium abutting to the inclusion is equal to

$$x_1 = u_r \cos\theta + u_\theta \sin\theta \,.$$

Allowing for the independence of x_1 of θ

$$x_1 = \frac{\partial\varphi_1}{\partial r} + \frac{\psi_1}{r} = -\left(\frac{\partial\psi_1}{\partial r} + \frac{\varphi_1}{r} \right), \tag{10}$$

whence subject to (8) it follows

$$-c e^{-\frac{p}{a}r}\left[\frac{p^2}{a^2}\frac{1}{r} + \frac{p}{a}\frac{1}{r^2} + \frac{1}{r^3} \right] = D e^{-\frac{p}{b}r}\left[\frac{p^2}{b^2}\frac{1}{r} + \frac{p}{b}\frac{1}{r^2} + \frac{1}{r^3} \right]. \tag{11}$$

The replacement in images will be

$$\overline{x}_1 = -\frac{1}{r}\left[e^{-\frac{p}{a}r}\left(\frac{p^2}{a^2} + \frac{2p}{a}\frac{1}{r} + \frac{2}{r^2} \right) - \frac{e^{-\frac{p}{a}r_0}\left(\dfrac{p^2}{a^2}\dfrac{1}{r_0} + \dfrac{p}{a}\dfrac{1}{r_0^2} + \dfrac{1}{r_0^3} \right)}{e^{-\frac{p}{b}r_0}\left(\dfrac{p^2}{a^2}\dfrac{1}{r_0} + \dfrac{p}{b}\dfrac{1}{r_0^2} + \dfrac{1}{r_0^3} \right)} \times \right.$$

$$\times\left[\frac{p}{b}\frac{1}{r}+\frac{1}{r^2}\right]e^{-\frac{p}{b}r}\Bigg]c\,. \qquad (12)$$

Allowing for (9) and (11) we'll have

$$\overline{P}=\frac{3\pi r_0^2}{4}ce^{-\frac{p}{a}r_0}\left[-\frac{\frac{p^2}{a^2}+\frac{p}{a}\frac{1}{r_0}+\frac{1}{r_0^2}}{\frac{p^2}{b^2}+\frac{p}{b}\frac{1}{r_0}+\frac{1}{r_0^2}}\left\{\frac{1}{r_0}\left(\frac{8\mu p^3}{b^3}+\frac{20\mu p^2}{b^2 r_0}+\frac{32\mu p}{b}\frac{1}{r_0^2}+\frac{32\mu}{r_0^3}\right)\right\}+\right.$$

$$\left.+\frac{1}{r_0}\left\{\frac{\lambda p^3}{a^3}+\frac{\lambda p^2}{a^2}\frac{1}{r_0}+\frac{24\mu p^3}{a^3}+\frac{10\mu p^2}{a^2 r_0}+\frac{24\mu p}{a}\frac{1}{r_0^2}+\frac{24\mu}{r_0^3}\right\}\right]. \qquad (13)$$

The equations (5) in images will be

$$\begin{cases} PM_1(\varphi_1'-\dot{x}_0)+\frac{4\pi}{3}\rho r_0^2 p\overline{\varphi}_1=L(\overline{x}_2-\overline{x}_1), \\ M_2\left(p^2\overline{x}_2-p\dot{x}_0\right)+L(\overline{x}_2-\overline{x}_1)=0\,, \end{cases} \qquad (14)$$

where $\dot{x}_0$ is the initial velocity of inclusion.

Substituting the values $\overline{P}$ from (13) and $\overline{x}_1$ we'll obtain from (12)

$$c=\frac{\dot{x}_0 p}{p^2\left[-\frac{1}{r}\left(e^{-\frac{p}{a}r_0}\left\{\frac{p^2}{a^2}+\frac{2p}{a}\frac{1}{r_0}+\frac{2}{r_0^2}\right\}-\frac{e^{-\frac{p}{a}r_0}\left(\frac{p^2}{a^2}\frac{1}{r_0}+\frac{p}{a}\frac{1}{r_0^2}+\frac{1}{r_0^3}\right)}{e^{-\frac{p}{b}r_0}\left(\frac{p^2}{b^2}\frac{1}{r_0}+\frac{p}{b}\frac{1}{r_0^2}+\frac{1}{r_0^3}\right)}\times\right.\right.}$$

$$\left.\left.\times\frac{1}{\left[\frac{p}{b}\frac{1}{r_0}+\frac{1}{r_0^2}\right]e^{-\frac{p}{b}r_0}}\right)\right]+\left(\frac{p^2}{L}+\frac{1}{M_2}\right)\left(\frac{M_*}{r_0}e^{-\frac{p}{a}r_0}(A+B)\right), \qquad (15)$$

where

$$A=-\frac{\frac{p^2}{a^2}+\frac{p}{a}\frac{1}{r_0}+\frac{1}{r_0^2}}{\frac{p^2}{b^2}+\frac{p}{b}\frac{1}{r_0}+\frac{1}{r_0^2}}\left[\frac{1}{r_0}\left\{\frac{8\mu p^3}{a^3}+\frac{20\mu p^2}{b^2}\frac{1}{r_0}+\frac{32\mu p}{b}\frac{1}{r_0^2}+\frac{21\mu}{r_0^3}\right\}\right];$$

$$B=\frac{1}{r_0}\left[\frac{\lambda p^3}{a^3}+\frac{\lambda p^2}{a^2}\frac{1}{r_0}+\frac{24\mu p^3}{a^3}+\frac{10\mu p^2}{a^2}\frac{1}{r_0}+\frac{24\mu p}{a}\frac{1}{r_0^2}+\frac{24\mu}{r_0^3}\right];$$

$$M_*=\frac{4\pi\rho}{3}r_0^3\,,\quad \omega_1=\frac{a}{r_0},\ \omega_2=\sqrt{\frac{L}{M_*}}\,;$$

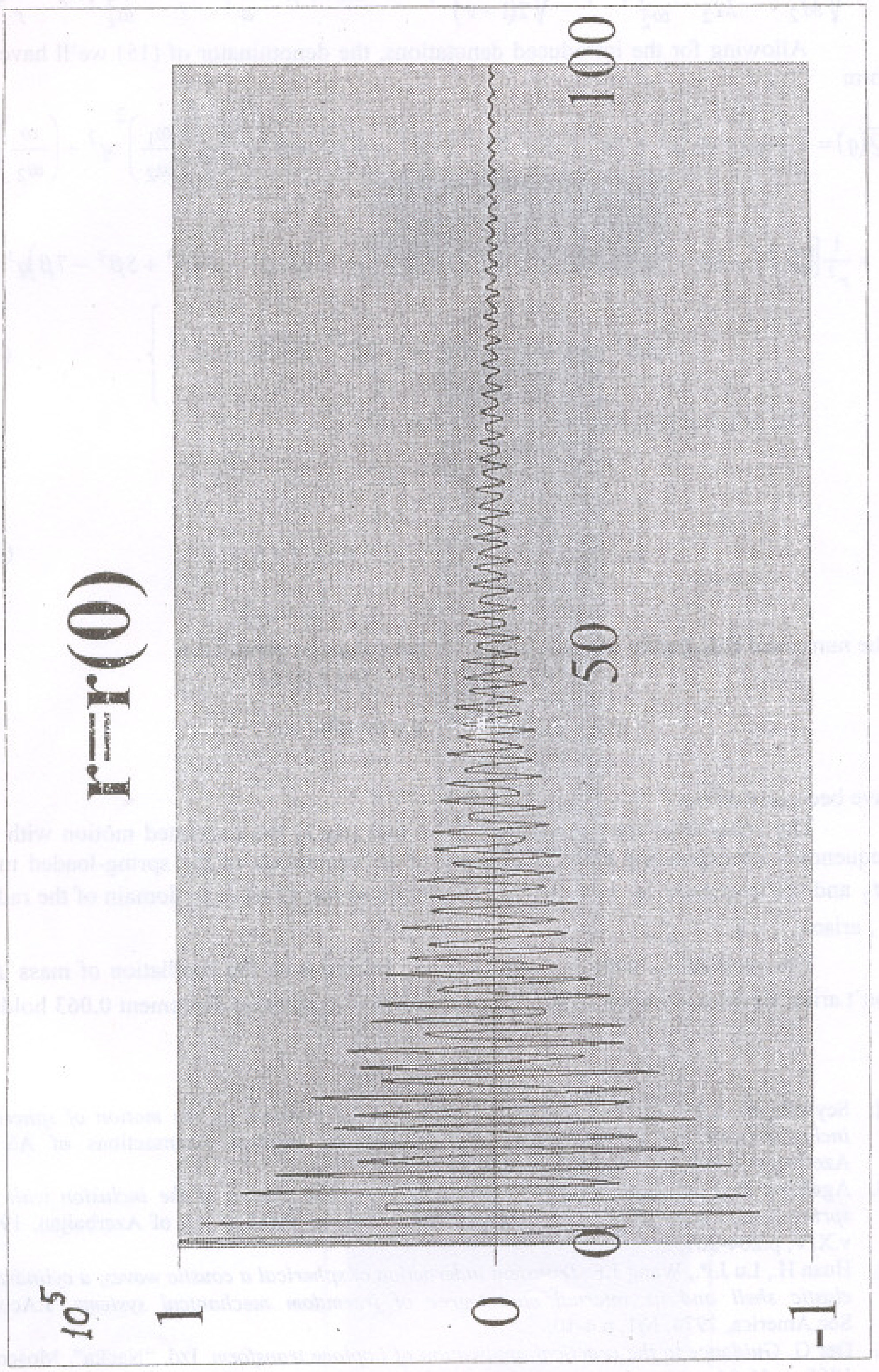

Fig.1.

$$\omega=\sqrt{\frac{L}{M_2}};\quad \frac{M_*}{M_2}=\frac{\omega^2}{\omega_2^2};\quad \beta=\sqrt{\frac{1-2\nu}{2(1-\nu)}}\ ;\ \nu=0{,}25,\ \beta=\frac{b}{a};\quad \frac{M_*}{L}=\frac{1}{\omega_2^2};\quad p=\frac{a}{r}q\,.$$

Allowing for the introduced denotations, the denominator of (15) we'll have the form

$$\overline{Q}(q)=\frac{1}{\beta^2 r^5}\left\{-\omega_1\left(q^6+2q^5+(\beta+2)q^4+\left(\beta^2+\beta\right)q^3+\beta^2q^2\right)+\left[\left(\frac{\omega_1}{\omega_2}\right)^2q^2+\left(\frac{\omega}{\omega_2}\right)^2\right]\times\right.$$

$$\times\frac{1}{r^2}\left[\left(22\beta^2-8\beta+1\right)q^5+\left(1-12\beta^2-7\beta+22\beta^3\right)q^4+\left(22\beta^4-24\beta^3+5\beta^2-7\beta\right)q^3+\right.$$

$$\left.\left.+\left(5\beta^2-8\beta^3-24\beta^4\right)q^2+\left(-8\beta^4-8\beta^3\right)q-8\beta^4\right]\right\}. \qquad (16)$$

The original (16) has the following form [4]:

$$\frac{1}{Q(t)}=\sum_{m=1}^{7}\frac{1}{\overline{Q}_m(a_m)}e^{a_m t};$$
$$\overline{Q}_m(q)=\frac{\overline{Q}(q)}{q-a_m}\quad a_i\neq a_k;\ \ i\neq k; \qquad (17)$$
$$\overline{Q}(q)=(q-a_1)(q-a_2)\cdots(q-a_7).$$

The numerical calculation of the problem for the values of parameters

$$\omega_2=10\ hc \qquad \omega=161{,}241\ hc$$
$$\omega_1=280\ hc \qquad a=1400\,\mathrm{m/sec}^2$$
$$r_0=5\mathrm{m}$$

have been carried out.

The analysis of the calculation shows that (fig.1) the oscillated motion with the frequencies corresponding to the frequency of ω oscillation of the spring-loaded mass M_2 and the frequency ω_1 with the less amplitude of run of wave in domain of the radius r_0, arises.

The oscillations with the frequency ω_2 related with the oscillation of mass M_* don't arise. Besides attention of oscillations with the logarithmic decrement 0.063 holds.

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