RAMAZANOV T.K., NAMAZOV M.A.

FLAT FILTRATION OF LIQUID IN LINEAR-ELASTIC POROUS MEDIA

Abstract

Filtration equations of weakly compressed liquid having linear-elastic porous matkices and subjected to flat deformation are deduced. The influence of filtrational dynamics of deformation of a matrix is investigated at a flow to the central well.

In the classical papers [1-4] the mathematical modeling of underground hydrodynamics problems, have been constructed on the basis of continuity and motion for the liquid phase of saturated porous medium equations. On the one hand introduction of continuity and motion, deformation low of porous medium matrix equations to the filtration process increases the number of equations which complicate the solution of the problem [5 ,6] and on the other hand their joined solution more exactly describes the dynamics of filtration.

1. Let's consider the flat flow of weakly compressed liquid in linear elastic saturated medium and the flat-radial problem of filtration on simultaneous work of the well with constant production rate.

In this case the linearized equations of conservation of mass and momentum for the rigid selection of medium will get the form [4,5]

$$\frac{\partial m}{\partial t}+\beta_1\frac{\partial \sigma^f}{\partial t}-\beta_1(1-m_0)\frac{\partial P}{\partial t}-(1-m_0)\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)=0, \tag{1.1}$$

$$\begin{aligned}&\frac{\partial \sigma^f_{xx}}{\partial x}+\frac{\partial \sigma^f_{xy}}{\partial y}-\frac{\partial P}{\partial x}=0;\\ &\frac{\partial \sigma^f_{yx}}{\partial x}+\frac{\partial \sigma^f_{yy}}{\partial y}-\frac{\partial P}{\partial y}=0.\end{aligned} \tag{1.2}$$

The linearized equation of conservation of mass for liquid phase is written analogously

$$\frac{\partial m}{\partial t}+m_0\beta_2\frac{\partial P}{\partial t}+m_0\left(\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}\right)=0; \tag{1.3}$$

and the filtration low

$$\begin{aligned}&\frac{k}{\mu}\frac{\partial P}{\partial x}=-m_0(w_x-v_x),\\ &\frac{k}{\mu}\frac{\partial P}{\partial y}=-m_0(w_y-v_y);\end{aligned} \tag{1.4}$$

The equations (1.1)-(1.4) are closed by independence ration of porosity (pressure) and Hook's generalized low [4]

$$k=k(m) \tag{1.5}$$

$$(1-m_0)Ee_{xx}=\sigma^f_{xx}-\varepsilon P-v\left(\sigma^f_{yy}+\sigma^f_{ss}-2\varepsilon P\right), \tag{1.6}$$

$$(1-m_0)Ee_{xx}=\sigma^f_{zz}-\varepsilon P-v\left(\sigma^f_{xx}+\sigma^f_{yy}-2\varepsilon P\right)=0. \tag{1.7}$$

Here the deformation of matrix of porous medium and the filtration of liquid take place on the plane $xy(x;y=i,j)$, w_i,v_i are rates of displacement of particles of liquid and matrix σ_{ij}^f are effective tensions, e_{ij} are complete deformations of matrix, m is a porosity of matrix, P is pore pressure. The mentioned parameters correspond to deviations of their values with respect to the initial values: $w_{i0} = v_{i0} = 0, \sigma_{ij0}^f$, $e_{ij0} = 0$, m_0, P_0, $\varepsilon = (1-m_0)\beta_1 K$ is relative rigidity of matrix, K, G, E are module of comprehensive compression, and elasticity of matrix, β_1, β_2 are compressibilities of solid and liquid phases, k is a coefficient of permeability of matrix, μ is viscosity of liquid

$$\frac{\partial v_x}{\partial x} = \frac{\partial e_{xx}}{\partial t},\ \frac{\partial v_y}{\partial y} = \frac{\partial e_{yy}}{\partial t},\ K = \frac{(1-m_0)E}{3(1-2v)},\ 3\sigma^f = \sigma_{xx}^f + \sigma_y^f + \sigma_{zz}^f .$$

Substituting (1.7) into (1.6) we'll obtain

$$e_{xx} = \frac{1-v^2}{(1-m_0)E}\left[\sigma_{xx}^f - \varepsilon P - \frac{v}{1-v}(\sigma_{yy} - \varepsilon P)\right], \tag{1.8}$$

One can simplify the solution of flat problem of elasticity of saturated porous medium reducing it to discovery of the Ery function $\Phi(x, y, t)$

$$\sigma_{xx}^f = \frac{\partial^2 \Phi}{\partial y^2} + P,\ \sigma_{yy}^f = \frac{\partial^2 \Phi}{\partial x^2} + P,\ \ \sigma_{xy}^f = -\frac{\partial^2 \Phi}{\partial x \partial y}. \tag{1.9}$$

The expressions (1.9) fulfill the equations (1.2) identically.

The equation of jointness of deformation

$$\frac{\partial^2 e_{xx}}{\partial y^2} + \frac{\partial^2 e_{yy}}{\partial x^2} = 2\frac{\partial^2 e_{xy}}{\partial x \partial y}. \tag{1.10}$$

in the effective stresses gets the form

$$(1-v)\left(\frac{\partial^2}{\partial y^2}\left[\sigma_{xx}^f - \varepsilon P - \frac{v}{1-v}(\sigma_{yy}^f - \varepsilon P)\right] + \right.$$
$$\left. + + \frac{\partial^2}{\partial x^2}\left[\sigma_{yy}^f - \varepsilon P - \frac{v}{1-v}(\sigma_{xx}^f - \varepsilon P)\right]\right) = 2\frac{\partial^2 \sigma_{xx}^f}{\partial x \partial y}. \tag{1.11}$$

Substituting the expressions (1.9) in the jointness equation (1.11) we'll obtain

$$\frac{\partial^4 \Phi}{\partial x^4} + 2\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} + a\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) = 0, \tag{1.12}$$

where $a = \dfrac{(1-2v)(1-\varepsilon)}{1-v}$.

Integrating the equation (1.1) with respect to t we'll find

$$m = \beta_1(1-m_0)P + (1-m_0)e - \beta_1\sigma^f. \tag{1.13}$$

We'll use the low (1.7) to determine the volume deformation

$$\sigma^f = K + \varepsilon P. \tag{1.14}$$

One can write the relation (1.13) in the form

$$m = (1-m_0)(1-\beta_1 K)(e + \beta_1 P), \tag{1.15}$$

$$e = \frac{1-2v}{2G}\left(\sigma_{xx}^f + \sigma_{yy}^f - 2\varepsilon P\right) = \frac{1-2v}{2G}\left[\Delta\Phi + 2(1-\varepsilon)P\right]. \tag{1.16}$$

Substituting the expressions of velocity of liquid phase and porosity subject to the volume deformation (1.6), from (1.4) and (1.5) into (1.3), we'll obtain

$$b\frac{\partial}{\partial t}\left(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}\right)+\frac{\partial P}{\partial t}=\gamma\left[\frac{\partial}{\partial x}\left(k\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial P}{\partial y}\right)\right], \qquad (1.17)$$

where

$$b=\frac{1}{2}\frac{(1-2v)[(1-m_0)(1-\beta_1 K)+m_0]}{\beta},$$

$$\gamma=\frac{G}{\mu\beta},\ \beta=(1-2v)(1-\varepsilon)[(1-m_0)(1-\beta_1 K)+m_0]+$$

$$+G[(1-m_0)(1-\beta_1 K)\beta_1+m_0\beta_2].$$

After integrating of the equation (1.12) we'll find

$$\Delta\Phi+aP=f(x,y,t). \qquad (1.18)$$

For $P=0$ the alternations of effective stresses in porous medium are absent, i.e. follows from (1.9)

$$\frac{\partial^2\Phi}{\partial x^2}=\frac{\partial^2\Phi}{\partial y^2}=0.$$

Now one can write the equation (1.18) in the form

$$\Delta\Phi+aP=0. \qquad (1.19)$$

From the equations (1.17) and (1.18) we'll obtain the following equation for pressure

$$\frac{\partial P}{\partial t}=\gamma_0\left[\frac{\partial}{\partial x}\left(k(m)\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(m)\frac{\partial P}{\partial y}\right)\right], \qquad (1.20)$$

where

$$\gamma_0=\frac{\gamma}{1-ab}.$$

With the help of the formulae (1.13), (1.16) and the equation (1.19) we'll express the alternation of porosity (1.15) by P

$$\frac{m}{m_0}=\alpha P, \qquad (1.21)$$

where $\alpha=\dfrac{(1-m_0)(1-\beta_1 K)[2G\beta_1+(1-2v)][2(1-\varepsilon)-a]}{2Gm_0}$

With the help of (1.21) the ratio between the coefficient of permeability and porosity (1.5) will get the following form

$$\frac{k}{k_0}=(1+\alpha P)^n,\ n=\frac{\alpha_k^0}{\alpha_m^0}, \qquad (1.22)$$

where α_k^1, α_m^0 are coefficients of alternation of permeability and porosity.

The substitution of (1.22) into the (1.20) leads it to the form

$$\frac{\partial p}{\partial t}=\lambda\Delta p^{n+1}, \qquad (1.23)$$

where $p=1+\alpha P$; $\lambda=\dfrac{\gamma_0 k_0}{1+n}$.

The equation (1.23) describes the flat elastic filatration of weakly compressed liquid in saturated porous medium. Solving (1.23) at initial and boundary conditions we'll find the alternation of pore pressure. After the determination of P from (1.12) we find the equation relative to the Ery function, whose solution at boundary condition determines the stress strain state of stratum.

2. Let in flat-radial stratum be immediately opened the well with radius r_c working with constant production rate. On the external reservoir boundary R_k the constant pressure P_{k_0} is supported.

Then the initial and boundary conditions will have the form:

$$p=1,\ \ (P=0)\ \ \text{for}\ \ t=0, \tag{2.1}$$

$$\frac{\partial p}{\partial r}=-\frac{\mu Q\alpha}{2\pi r_c hk_0}\ \ \text{for}\ \ r=r_c,\ \ p=1\ \ \text{for}\ \ r=R_k. \tag{2.2}$$

Here contour and initial pressure are taken identically $(P_k=P_0)$. If $r_c\to 0,\ R_k\to\infty$, then the equation (1.23) in infinity startum admits the automodel solution

$$\frac{\partial p}{\partial t}=\lambda\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p^r}{\partial r}\right) \tag{2.3}$$

at the following initial and boundary conditions

$$p(0,r)=1, \tag{2.4}$$

$$r\frac{\partial p}{\partial r}=-\frac{Q\mu\alpha}{2\pi hk_0}=Q^*\ \ \text{for}\ \ r\to 0,\ \lim_{r\to\infty}p(r,t)=1. \tag{2.5}$$

The transition to the dimensionless variable

$$\xi=\frac{r}{\sqrt{2\gamma t}} \tag{2.6}$$

reduces (2.3) to the ordinary nonlinear equation

$$\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{dp^{n+1}}{d\xi}\right)+\xi\frac{dp}{d\xi}=0 \tag{2.7}$$

at the boundary conditions

$$\xi\frac{\partial p}{\partial\xi}=Q^*\ \ \text{for}\ \ \xi\to 0,\ \lim_{r\to\infty}p=1. \tag{2.8}$$

From the numerical solution of the problem (2.7) and (2.8) it is obvious that the cone of depression (the rise of absolute value of pressure drop of pressure) is formed in the interval $\xi\in(0,2)$. Then from (2.6) it follows

$$r(t)=\sqrt{8\lambda t}\,. \tag{2.9}$$

When the moving front reaches on the external boundary of stratum R_k, the automodel solution stops to exist and in the next stage the deviation from automodelness brings to separation of variables ξ and r/R_k.

The numerical solutions of the problem (2.7)-(2.8) are given in fig.1 for

$$Q=-0{,}005\,n=0;1;4;8.$$

Calculations show that for linear $(n=0)$ and nonlinear ($n>1$ points in fig.1) selection of liquid through well from infinite flat stratum the nonlinearity doesn't almost influence to the cone of depression and by its increasing the curves coincide. However the influence of n is more significant in finite stratum.

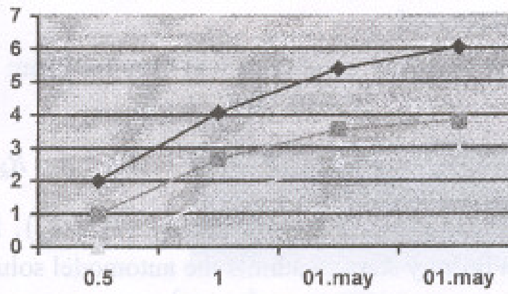

Fig.1

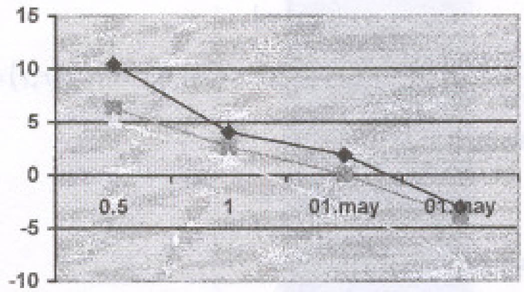

Fig.2

References

[1]. Brot M.A. *Theory of propagation of elastic waves in a fluid saturated porous solids.* J. Acoustic Soc. Amer., v.28, 1956, p.168-186.

[2]. Masket M. *Flow of homogeneous liquids in porous medium.* Moscow-Leningrad, 1949, 628 p. (Trans. From eng)

[3]. Schelkachev V.N. *Exploitations of the oil and water-bearing formation at elastic regime.* Moskow, Gostechizdat, 1959, 467 p.

[4]. Nikolayevskiy V.N., Basniyev K.S., Gorbunov A.T., Zotov G.A. *Mechanics of a saturated porous medium.* Moskow, Nedra, 1970, 339 p.

[5]. Ramazanov T.K. *Axially symmetric problem of filtration of a liquid in stratum in elastic mountain mass.* Dok. AN Azerb. SSR, Baku, Elm, 1979, v.XXXV, №11, p.17-20.

[6]. Nikolayevskiy V.N., Ramazanov T.K. *Stress strain state of stratum and repressuring in well.* Mech. of Defor. Body. Strength and viscous-elastic-plasticity. Moscow. Nauka, 1986, p.94-105.

Ramazanov T.K., Namazov M.A.
Baku State University.
23, Z.I. Khalilov str., 370148, Baku, Azerbaijan.

Received January 26, 2001; Revised June 23, 2001.
Translated by Nazirova S.H.