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STABILITY LOSS AROUND AN INTERFACE CRACK IN A SANDWICH PLATE

Abstract

A stability loss problem around an interface cracks in a clamped plate is studied. The material of the layers in this plate assumed to be elastic, isotropic and homogeneous. The investigations are carried out in the framework of the piece-wise homogeneous body model with the use of Three Dimensional Linearised Theory of Stability. Corresponding boundary value problems are solved numerically by employing FEM. The numerical results, which show the critical (failure) values of the external compression force are presented for various problem parameters.

1. Introduction. A stability loss of the equilibrium form around a crack is taken as a beginning of the fracture or delamination of the composite materials under compression of those along the crack. The first investigation in this field had been made in [1] and up to now the numerous studying was carried out in this field. The review of these investigations are given in [2] and [3] in which the corresponding stability loss problems for the part of the material which occupies the region between the free plane are studied in the framework of the approximate stability theories of plates or beams. Therefore the results of these investigations can't be applied in the cases where the thickness of this plate or bar is equal or greater than the length of the crack. Moreover approximate delamination theories do not take into account the singularity of the stresses and strains at tips of the crack. According to this situation, it was arisen necessity for the development of the delamination theories in the framework of the Three Dimensional Linearized Theory of Stability (TDLTS) of the deformable solid body mechanics, which has been carried out in [4-8] relate to the cases where the crack length is significantly greater than the size of components (i.e. the thickness of layers or the diameters of fibers) of composites. Otherwise it is necessary to apply the piece-wise homogeneous body model for the investigation of the corresponding stability loss problems. It should be noted that in [9, 10 and others] the piece-wise homogeneous body model has been employed for investigation of the stability loss problems around the cracks on the interfaces of t he half-planes. However up to now the piece-wise homogeneous body model has not been employed for the stability loss problems of the near interface cracks on the structural elements from composites. Taking this situation into account in the present paper the first attempt is made in this field and in the framework of the piece-wise homogeneous body model with the use of the version of TDLTS developed in [11-13] the stability loss around the crack in the interface of the layers of the sandwich clamped plate is studied. The numerical results on the critical compressive force are presented. For the obtaining of these results the FEM is employed.

2. Formulation of the problem and solution method.

Consider the plate-strip with the geometry shown in Fig.1. Assume that the materials of the upper and lower layers of the plate are the same and isotropic, homogeneous elastic ones. The material of the middle (matrix) layer is also isotropic, homogeneous elastic one. Moreover, between the matrix and upper as well as and lower layer of the plate there is a crack the location of which is shown in Fig.1.

We associate with the plate Lagrangian coordinate system $Ox_1x_2x_3$ which in its natural state coincides with Cartesian coordinates. Note that the Ox_3 axis is perpendicular to the plane Ox_1x_2 and does not shown in Fig.1. We assume that the plate occupies the region $\Omega = \{0 \le x_1 \le l, \ -h/2 \le x_2 \le +h/2, \ -\infty < x_3 < +\infty\}$. Moreover we assume that in the initial (natural) state the crack edges are slightly and symmetrical open. In Fig.1 these edges are denoted by $S_u^+(S_u^-)$, $S_w^+(S_w^-)$ (upper (lower) edge of the upper and lower crack respectively). The equations of these surfaces are selected as follows:

For $S_w^{\pm}$:

$$x_2^{\pm} = \frac{h}{2} - h_u \pm L\sin^2\left(\frac{\pi(x_1 - (l/2 - l_0))}{2l_0}\right), \quad x_1 \in [l/2 - l_0, l/2 + l_0]. \tag{1}$$

For $S_u^{\pm}$:

$$x_2^{\pm} = -\left(\frac{h}{2} - h_u\right) \pm L\sin^2\left(\frac{\pi(x_1 - (l/2 - l_0))}{2l_0}\right), \quad x_1 \in [(l/2) - l_0, (l/2) + l_0]. \tag{2}$$

We suppose that $L << l_0$ and introduce a small parameter $\varepsilon = L/l_0$.

The values related to the upper and lower layers we will denote by upper index (1) and (3), however the values related to the matrix (middle) layers by (2). Within each layer in the geometrical non-linear statement we write the following equilibrium equations, mechanical and geometrical relations.

$$\frac{\partial}{\partial x_j}\left[\sigma_{jn}^{(k)}\left[\delta_i^n + \frac{\partial u_i^{(k)}}{\partial x_n}\right]\right] = 0, \ \sigma_{11}^{(k)} = A_{11}^{(k)}\varepsilon_{11}^{(k)} + A_{12}^{(k)}\varepsilon_{22}^{(k)},$$

$$\sigma_{22}^{(k)} = A_{12}^{(k)}\varepsilon_{11}^{(k)} + A_{22}^{(k)}\varepsilon_{22}^{(k)}, \ \sigma_{12}^{(k)} = 2A_{66}^{(k)}\varepsilon_{12}^{(k)},$$

$$2\varepsilon_{ij}^{(k)} = \frac{\partial u_i^{(k)}}{\partial x_j} + \frac{\partial u_j^{(k)}}{\partial x_i} + \frac{\partial u_n^{(i)}}{\partial x_i}\frac{\partial u_n^{(1)}}{\partial x_j}, \quad A_{ij}^{(1)} = A_{ij}^{(3)}, \ i; j; n = 1,2, \ k = 1,2,3. \tag{3}$$

In (3) the conventional notation is used.

According to the above-stated and to the Fig.1 the following boundary and contact conditions are satisfied.

$$\left[\sigma_{2n}^{(1)}\left(\delta_i^n + \frac{\partial u_i^{(1)}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2 = h/2 \\ x_1 \in (0,l)}} = 0, \quad \left[\sigma_{2n}^{(3)}\left(\delta_i^n + \frac{\partial u_i^{(3)}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2 = -h/2 \\ x_1 \in (0,l)}} = 0,$$

$$\left[\sigma_{2n}^{(1)}\left(\delta_i^n + \frac{\partial u_i^{(1)}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}} = \left[\sigma_{2n}^{(2)}\left(\delta_i^n + \frac{\partial u_i^{(2)}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}},$$

$$u_i^{(1)}\Big|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}} = u_i^{(2)}\Big|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}},$$

$$\left[\sigma_{2n}^{(3)}\left(\delta_i^n + \frac{\partial u_i^{(3)}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2 = -(h/2 - h_u) \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}} = \left[\sigma_{2n}^{(2)}\left(\delta_i^n + \frac{\partial u_i^{(2)}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2 = -(h/2 - h_u) \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}},$$

$$u_i^{(3)}\Big|_{\substack{x_2=-(h/2-h_u)\\ x_1\in(0,l/2-l_0)\cup(l/2+l_0,l)}} = u_i^{(2)}\Big|_{\substack{x_2=-(h/2-h_u)\\ x_1\in(0,l/2-l_0)\cup(l/2+l_0,l)}},$$

$$\left[\sigma_{jn}^{(1)}\left(\delta_i^n+\frac{\partial u_i^{(1)}}{\partial x_n}\right)\right]\Bigg|_{S_u^+} n_j^+=0,\quad \left[\sigma_{jn}^{(2)}\left(\delta_i^n+\frac{\partial u_i^{(2)}}{\partial x_n}\right)\right]\Bigg|_{S_u^-} n_j^-=0,$$

$$\left[\sigma_{jn}^{(2)}\left(\delta_i^n+\frac{\partial u_i^{(2)}}{\partial x_n}\right)\right]\Bigg|_{S_w^+} n_j^+=0,\quad \left[\sigma_{jn}^{(3)}\left(\delta_i^n+\frac{\partial u_i^{(3)}}{\partial x_n}\right)\right]\Bigg|_{S_w^-} n_j^-=0. \qquad (4)$$

Furthermore, we assume that the considered plate is compressed through a rigid body, which holds the ends by the normal forces with intensity p along the Ox_1 axis and it is supposed that

$$u_1^{(k)}(0,x_2)=-u_1^{(k)}(l,x_2)=U,\quad u_2^{(k)}(0,x_2)=u_2^{(2)}(l,x_2)=0. \qquad (5)$$

Note that the relation between U and p (Fig.1) is determined after solution procedure from the equation

$$hp=\int_{-h/2}^{-h/2+h_u}\sigma_{11}^{(3)}\Big|_{x_1=const}dx_2+\int_{-h/2+h_u}^{h/2-h_u}\sigma_{11}^{(2)}\Big|_{x_1=const}dx_2+\int_{h/2-h_u}^{h/2}\sigma_{11}^{(1)}\Big|_{x_1=const}dx_2. \qquad (6)$$

Thus in the framework of the equations (1)-(6) we consider the plane strain state in the platestrip and investigate the development of the insignificant initial imperfection of the crack edges (1), (2) with increasing U (i.e. with increasing p) in (6). As it follows from the above stated that in the initial state there is a symmetry of the problem with respect to $x_1=l/2$ and $x_2=0$. We will assume that this symmetry will remain in all period of the development of the initial imperfection.

Now we consider the solution method of the problem. According to [11-13], values characterizing stress-deformation state in the strip we represent in the series form in the small parameter ε:

$$\left\{\sigma_{ij}^{(k)};\varepsilon_{ij}^{(k)};u_i^{(k)}\right\}=\sum_{q=0}^{\infty}\varepsilon^q\left\{\sigma_{ij}^{(k),q};\varepsilon_{ij}^{(k),q};u_i^{(k),q}\right\}. \qquad (7)$$

Substituting the equation (7) into (3) and comparing equal powers of ε to describe each approximation, we obtain the corresponding closed system equations. Owing to the linearity of the mechanical relations considered, it will be satisfied for every approximation in equation (7) separately. The remaining relation obtained from equation (3) for every q-th approximation contain the values of all previous approximation. Moreover, substituting the equation (7) in conditions (4) and doing some operations, we obtain from equation (4) for each approximation the corresponding boundary, contact conditions and the conditions on the crack edges. In the similar manner we obtain from (5) the corresponding conditions for each approximation at the ends of the plate, which satisfies each approximation separately. To simplify the exposition, we consider the relations for the zeroth and some subsequent approximations. Note that in the zeroth approximation the equation (3) and the conditions (4) (without the conditions on the crack edges) holds for $\sigma_{ij}^{(k),0}$, $\varepsilon_{ij}^{(k),0}$, $u_i^{(k),0}$ and from the corresponding equations in (4) we obtain the following ones on the cracks edges.

$$\left[\sigma_{2n}^{(1),0}\left(\delta_i^n+\frac{\partial u_i^{(1),0}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2=h/2-h_u\\ x_1\in(l/2-l_0,\,l/2+l_0)}}=0,\quad \left[\sigma_{2n}^{(2),0}\left(\delta_i^n+\frac{\partial u_i^{(2),0}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2=h/2-h_u\\ x_1\in(l/2-l_0,\,l/2+l_0)}}=0,$$

$$\left[\sigma_{2n}^{(2),0}\left(\delta_i^n+\frac{\partial u_i^{(2),0}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2=-h/2+h_u\\ x_1\in(l/2-l_0,\,l/2+l_0)}}=0,$$

$$\left[\sigma_{2n}^{(3),0}\left(\delta_i^n+\frac{\partial u_i^{(3),0}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2=-h/2+h_u\\ x_1\in(l/2-l_0,\,l/2+l_0)}}=0. \tag{8}$$

As in [11-13], for the values of the first and subsequent approximations we obtain the linear inhomogeneous equations, boundary and contact conditions the homogeneous parts of which coincide with corresponding ones of TDLTS. Thus, the investigation of the growth of the initial insignificant imperfections of the crack edges in the plate with the growing of the external compressive forces are reduced to the solutions of the series of boundary-value problems. In this case the values of the zeroth approximation are determined from the non-linear equations (3), (4), (8) and the values of the subsequent approximations are determined from the boundary value problems for equations of TDLTS.

We now consider the determination of the values of the zeroth and first approximations and investigate the case where the plate material is comparatively rigid, i.e. the inequalities $\partial u_i^{(0)}/\partial x_j \ll 1$ occur. Taking this situation into account we neglect the non-linear terms in (3) and (4), (8) under determination of the values of the zeroth approximation. Moreover, taking this situation into account that, in the equations related to the first approximation we replace the values $\left(\delta_i^n+\partial u_i^{(0)}/\partial x_n\right)$ with δ_i^n. Thus we obtain the following equations, boundary and contact conditions for the first approximation.

$$\frac{\partial}{\partial x_j}\left[\sigma_{ij}^{(k),1}+\sigma_{jn}^{(k),0}\frac{\partial u_i^{(k),1}}{\partial x_n}\right]=0,\quad \varepsilon_{ij}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_i^{(k),1}}{\partial x_j}+\frac{\partial u_j^{(k),1}}{\partial x_j}\right), \tag{9}$$

$$\sigma_{2i}^{(1),1}\Big|_{\substack{x_2=h/2\\ x_1\in(0,l)}}=0,\quad \sigma_{2i}^{(3),1}\Big|_{\substack{x_2=-h/2\\ x_1\in(0,l)}}=0,\quad u_i^{(k),1}\Big|_{x_1=0,l}=0,\quad k=1,2,3,$$

$$\left[\sigma_{2i}^{(1),1}+\sigma_{2n}^{(1),0}\frac{\partial u_i^{(1),1}}{\partial x_n}\right]\Bigg|_{\substack{x_2=h/2-h_u\\ x_1\in(0,\,l/2-l_0)\cup(l/2+l_0,\,l)}}=$$

$$=\left[\sigma_{2i}^{(2),1}+\sigma_{2n}^{(2),0}\frac{\partial u_i^{(2),1}}{\partial x_n}\right]\Bigg|_{\substack{x_2=h/2-h_u\\ x_1\in(0,\,l/2-l_0)\cup(l/2+l_0,\,l)}},$$

$$\left[\sigma_{2i}^{(2),1}+\sigma_{2n}^{(2),0}\frac{\partial u_i^{(2),1}}{\partial x_n}\right]\Bigg|_{\substack{x_2=-h/2+h_u\\ x_1\in(0,\,l/2-l_0)\cup(l/2+l_0,\,l)}}=$$

$$= \left[\sigma_{2i}^{(3),1} + \sigma_{2n}^{(3),0} \frac{\partial u_i^{(3),1}}{\partial x_n} \right] \Bigg|_{\substack{x_2 = -h/2 + h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}},$$

$$u_i^{(1),1} \Big|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}} = u_i^{(2),1} \Big|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}},$$

$$u_i^{(2),1} \Big|_{\substack{x_2 = -h/2 + h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}} = u_i^{(3),1} \Big|_{\substack{x_2 = -h/2 + h_u \\ x_1 \in (0, l/2 - l_0) \cup (l/2 + l_0, l)}},$$

$$\sigma_{2i}^{(1),1} \Big|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (l/2 - l_0, l/2 + l_0)}} = \sigma_{2i}^{(2),1} \Big|_{\substack{x_2 = h/2 - h_u \\ x_1 \in (l/2 - l_0, l/2 + l_0)}} = 0,$$

$$\sigma_{2i}^{(2),1} \Big|_{\substack{x_2 = -h/2 + h_u \\ x_1 \in (l/2 - l_0, l/2 + l_0)}} = \sigma_{2i}^{(3),1} \Big|_{\substack{x_2 = -h/2 + h_u \\ x_1 \in (l/2 - l_0, l/2 + l_0)}} = 0. \quad (10)$$

Due to linearity, the mechanical relations remain as in (3) for each approximation.

As it has been noted above the boundary-value problems related to the zeroth and first approximations are investigated by employing FEM. In this case taking the problem symmetry into account we consider only the region $\{0 \le x_1 \le l/2, 0 \le x_2 \le h/2\}$ which in turn is divided into 120 rectangular Lagrange family quadratic elements, with 533 nodes and 1027 NDOF. More detail description of the FEM version which is employed here is given in [14]. Moreover in addition to this in the present investigation we apply also the technique presented [15] and others for keeping the singularity order stresses and strains at the cracks tips. As a result of these procedure we evaluate the crack edge displacement values with respect to p. In this case the value of p for which $u_2^{(1)}(l/2, h/2 - h_u) \to +\infty$ is taken as critical or failure compression force. Note that the examinations which are not detailed here show that taking into account the second and the subsequent approximations does not change the values of the critical force. Therefore under determination of this critical force we restrict ourselves with the consideration of the zeroth and first approximations only. Moreover note that all PC programs, in the framework of which all numerical investigations are carried out, have been composed by the author in the FTN77.

2. Numerical results.

Assume that $E^{(1)} = E^{(3)}$, $\nu^{(1)} = \nu^{(2)} = \nu^{(2)} = 0.3$, $h/l = 0.15$ and consider the values of $\overline{p}_{cr.} = p/E$, (where $E = \min\{E^{(1)}, E^{(2)}\}$ given in Tables 1 and 2. The results shown in Table 1 are obtained for various l_0/l, $E^{(1)}/E^{(2)}$ under $h_u/l = 0.025$, however the results given in Table 2 are obtained for various h_u/l, $E^{(1)}/E^{(2)}$ under $l_0/l = 0.25$. It follows from these results that the values of $\overline{p}_{cr.}$ increase monotonically with growing $E^{(1)}/E^{(2)}$, h_u/l and with decreasing l_0/l. This concluding agrees with the well-known mechanical considerations. Moreover in the case where $E^{(1)}/E^{(2)} = 1$ the values of $\overline{p}_{cr.}$ obtained here coincide with corresponding ones obtained in [16, 17] for near-surface crack. This situation guarantees the trustiness of the numerical algorithms and programs used in the present investigations.

According to the Saint-Venant principle, in the cases where $l/2 - l_0 >> h$, $h_u << h/2$ the obtained results can be also considered as results related to the stability loss problem around a crack in the interface of the half-plane and stratified layer. The last problem has not been investigated up to now in the framework of the piecewise homogeneous body model with the use of TDLTS. This situation increases also the importance of the obtained numerical results and their application areas.

Table 1

l_0/l	$E^{(1)}/E^{(2)}$						
	0.05	0.10	0.20	1.0	5.0	10.0	20.0
0.3612	0.00147	0.00161	0.00188	0.0024	0.0133	0.0226	0.0358
0.3056	0.00201	0.00221	0.00260	0.0055	0.0184	0.0311	0.0493
0.2500	0.00244	0.00320	0.00376	0.0081	0.0269	0.0456	0.0722
0.1950	0.00455	0.00505	0.00596	0.0129	0.0430	0.0727	0.1153
0.1389	0.00807	0.00901	0.01072	0.0234	0.0786	0.1330	0.2107

Table 2

h_u/l	$E^{(1)}/E^{(2)}$						
	0.05	0.10	0.20	1.0	5.0	10.0	20.0
	0.00244	0.00320	0.00376	0.0081	0.0269	0.0456	0.0722
0.0250							
0.0375	0.00783	0.00848	0.00952	0.0165	0.0471	0.0796	0.1304
0.0500	0.01572	0.01686	0.01836	0.0266	0.0608	0.0981	0.1595

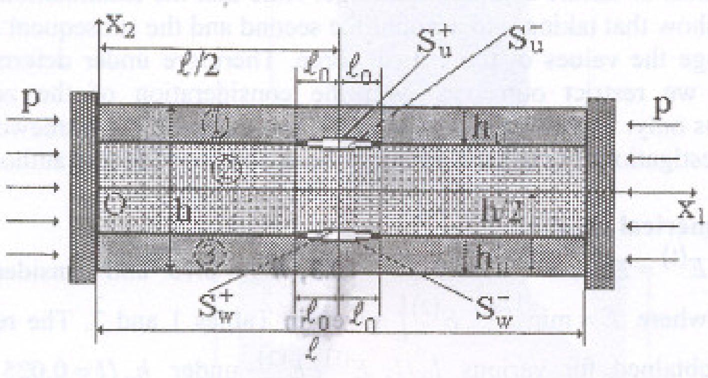

Fig.1

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