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LONGTERM STRENGTH OF ROTATABLE ANISOTROPIC RING

Abstract

On the basis of damageability conception the destruction process of cylindrical anisotropic ring rotating with constant angular velocity is investigated. The non-linear integral equation of destruction front is obtained and numerically solved. The obtained simulations allow to discover the difference of degree and kind in destruction process for anisotropic and isotropic rings.

Bodies of revolution manufactured from material with higher values of specific elastic and strength characteristics is used successfully in constructions working at higher rotational speed. Therefore they are used as flyweels accumulating mechanical energy. Because of low specific strength of metals and other materials having large specific mass we can't prepare from them the flyweels enduring the higher rotational speed. The rise of accumulating energy is attained by means of increase of moment of inertia of rotatable body for which it's required to increase the size of flyweels and complicate the technology of energy accumulation. By using flyweels from reinforced materials, the increase of accumulated energy is attained because of rise of rotational speed of flyweels.

For reeling flyweel the cylindrical curvilinear anisotropy, the transverse isotropy holds when the surfaces of isotropy are coaxial cylindrical surfaces. Besides the cylindrical symmetry also hold. For rotatable flyweel representing an elastic sylindrical anisotropic ring, the conditions ensuring the state of plane stress are realized.

Let's assume the following dimensionless quantities

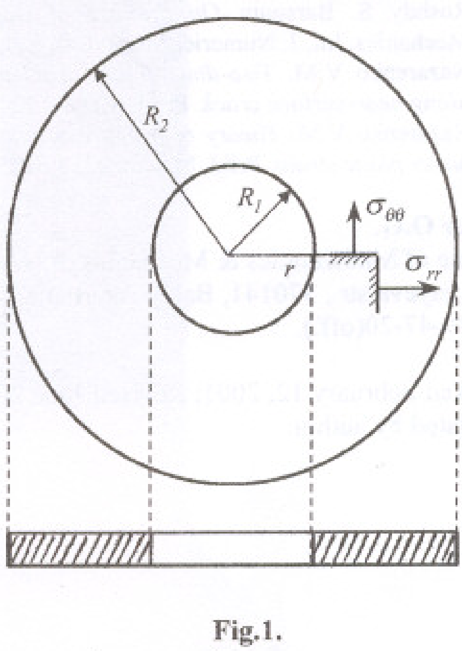

Fig.1.

$$\frac{R_1}{R_2}=\gamma;\ \frac{r}{R_2}=x;\ \frac{\sigma_r}{\rho\omega^2R_2^2}=\hat{\sigma}_r;\ \frac{\sigma_\theta}{\rho\omega^2R_2^2}=\hat{\sigma}_\theta;\ n^2=\frac{E_\theta}{E_r}. \qquad (1)$$

Then for dimensionless stress we obtain the expression in the form of

$$\begin{cases} \hat{\sigma}_r=\dfrac{3+\nu_{r\theta}n^2}{9-n^2}\left\{\dfrac{1-\gamma^{n+3}}{1-\gamma^{2n}}x^{n-1}-\dfrac{\gamma^{2n}\left(1-\gamma^{3-n}\right)}{1-\gamma^{2n}}\dfrac{1}{x^{n+1}}-x^2\right\}; \\ \hat{\sigma}_\theta=\dfrac{\left(3+\nu_{r\theta}n^2\right)n}{9-n^2}\left\{\dfrac{1-\gamma^{n+3}}{1-\gamma^{2n}}x^{n-1}+\dfrac{\gamma^{2n}\left(1-\gamma^{3-n}\right)}{1-\gamma^{2n}}\dfrac{1}{x^{n+1}}-\dfrac{\left(1+3\nu_{r\theta}\right)n}{3+\nu_{r\theta}n^2}x^2\right\}. \end{cases} \qquad (2)$$

It's easy to be convinced that for $n=1$ the expression (2) passes into the known formula for isotropic material. In fig.2 the comparative diagrams of distribution along the radius of ring of stresses for $\gamma=0,5;\ \nu_{r\theta}=0,3$ depending on the anisotropy parameter n are reduced.

From these it follows that as in isotropic case $(n=1)$ the tangential stress is prevailing, in addition its maximal tending value for all cases are attained on interior

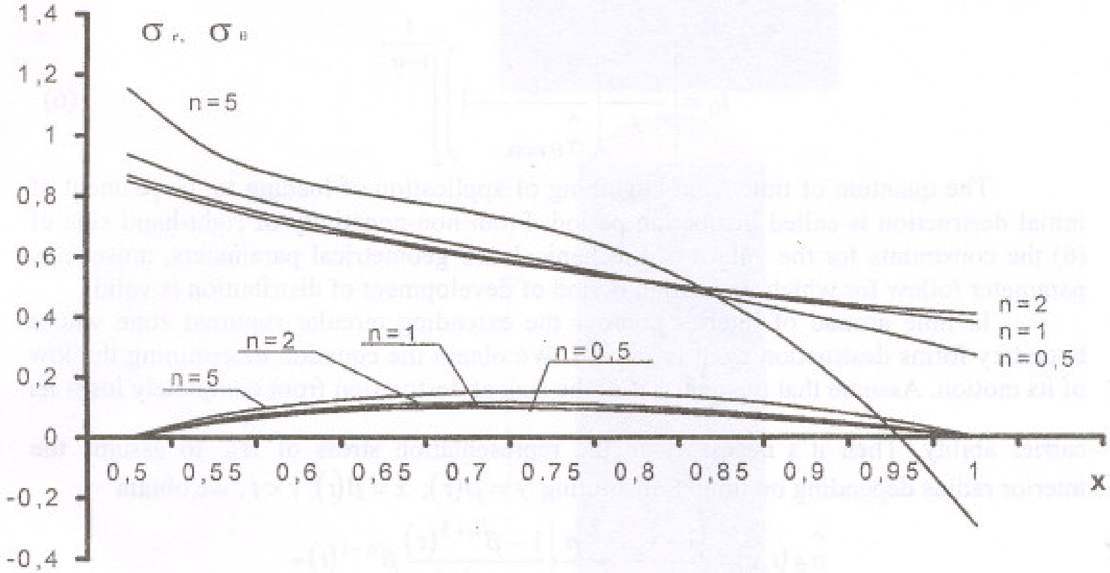

Fig.2.

contour of ring. This predetermines the use of destruction [2] criterion along the most tangential stress

$$\sigma_\theta + M^*\sigma_\theta = \sigma_0, \tag{3}$$

where M^* is an hereditary type damage time operator, σ_0 is instantaneous limit of strength of material. This damage operator is contained in deformation equations [2,3]. Under continuous active loading the damage operator behaves as an ordinary operator of hereditary elasticity that allows to use correspondence method for determination of stresses for the known elastic solution. Assuming the constancy of Poission coefficients and also supposing that the anisotropy is found only in distribution of instantaneous elasticity of modulus, we obtain that for the considered problem in the presence of damage the stress distribution is also given by the formulas (2), with only difference that the relations of instantaneous elasticity of modulus are contained in them.

As it was noted the prevailing tangential stress σ_θ attains its maximal tending value

$$\sigma_{\theta\max} = \frac{\left(3+\nu_{r\theta}n^2\right)n}{9-n^2}\left\{\frac{2-\gamma^{n+3}-\gamma^{3-n}}{1-\gamma^{2n}}\gamma^{n-1} + \frac{\left(1+3\nu_{r\theta}\right)n}{3+\nu_{r\theta}n^2}\gamma^2\right\} \tag{4}$$

on the interior contour $x=\gamma$. It means that destruction will begin on interior contour. In addition the destruction time t_0, is determined from the destruction (3)

$$\int_0^{t_0} M(\tau)d\tau = \frac{\hat{\sigma}_0}{\hat{\sigma}_{\theta\max}} - 1 \tag{5}$$

Here $\hat{\sigma}_0 = \sigma_0 / \rho\omega^2 R_2^2$, and $M(t)$ is a difference kernel of damage operator.

So for example for the weak-singular damage operator $M(t) = \lambda t^{-\alpha}$;α; $\lambda = const$; $0 < \alpha < 1$, we obtain

$$t_0 = \left[\frac{1-\alpha}{\lambda} \left(\frac{\hat{\sigma}_0}{\hat{\sigma}_{\theta\max}} - 1 \right) \right]^{\frac{1}{1-\alpha}} . \tag{6}$$

The quantum of time from beginning of application of loading to the moment of initial destruction is called incubation period. From non-negativity of right-hand side of (6) the constraints for the values of mechanical and geometrical parameters, anisotropy parameter follow for which incubation period of development of distribution is valid.

In time around of interior contour the extending circular ruptured zone whose boundary forms destruction front is formed. We obtain the equation determining the low of its motion. Assume that the material of the ring at destruction front completely loses its carrier ability. Then it's necessary in the representation stress of $\hat{\sigma}_\theta$ to assume the interior radius depending on time. Substituting $\gamma = \beta(\tau)$; $x = \beta(t)$, $\tau < t$, we obtain

$$\hat{\sigma}_\theta(t,\tau) = \frac{\left(3 + v_{r\theta} n^2\right) n}{9 - n^2} \left\{ \frac{1 - \beta^{n+3}(\tau)}{1 - \beta^{2n}(\tau)} \beta^{n-1}(t) + \right.$$
$$\left. + \frac{\beta^{2n}(\tau)\left(1 - \beta^{3-n}(\tau)\right)}{1 - \beta^{2n}(\tau)} \frac{1}{\beta^{n+1}(t)} - \frac{\left(1 + 3v_{r\theta}\right) n}{3 + v_{r\theta} n^2} \beta^2(t) \right\}. \tag{7}$$

This formula is for stress in time τ, when the destruction front has the coordinate $\beta(\tau)$ in layer, where the destruction front attains in time $t > \tau$. In this layer in the time t when destruction front approaches to it, the tangential stress will have the following form

$$\hat{\sigma}_\theta(t,t) = \frac{\left(3 + v_{r\theta} n^2\right) n}{9 - n^2} \left\{ \frac{2 - \beta^{n+3}(t) - \beta^{3-n}(t)}{1 - \beta^{2n}(t)} \beta^{n-1}(t) - \frac{\left(1 + 3v_{r\theta}\right) n}{3 + v_{r\theta} n^2} \beta^2(t) \right\}. \tag{8}$$

Then according to the fracture criterion (3) we obtain the next motion equation of destruction front

$$\hat{\sigma}_\theta(t,t) + \int_0^t M(t-\tau) \hat{\sigma}_\theta(t,\tau) d\tau = \hat{\sigma}_0 \tag{9}$$

representing itself subject to the formulas (7) and (8) the second Volterra non-linear integral equation with respect to the function $\beta(t)$. Where $\beta(\tau) = \gamma$ for $0 < \tau \le t_0$. In view of complexity of analytic solution we solve this equation numerically for the following values of numerical parameters: $v_{r\theta} = 0{,}3$; $\hat{\sigma}_0 = 2$; $\alpha = 0$; 0,025, $\beta_0 = 0{,}3$; 0,5 for mentioned above singular kernel of damage operator. In fig. 3-5 the simulations characterizing the low of motion of destruction front for the values of anisotropy parameter $n = E_\theta / E_r$ equal 0,5 ; 1 (isotropy): 2; 2,25 are given.

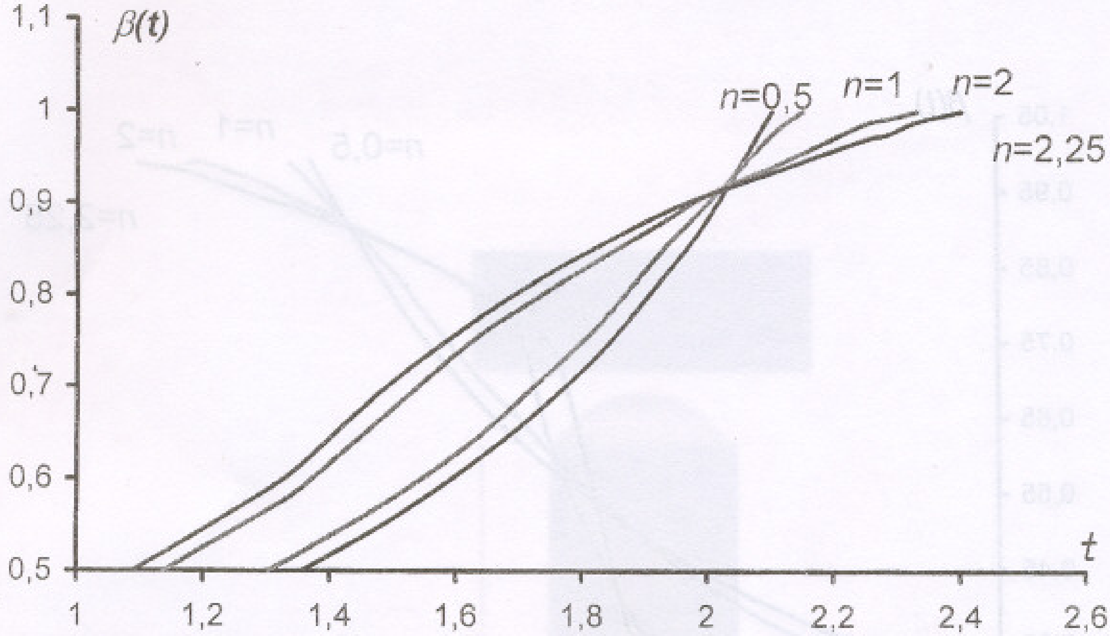

Fig.3. $\beta_0 = 0{,}5$; $\alpha = 0$.

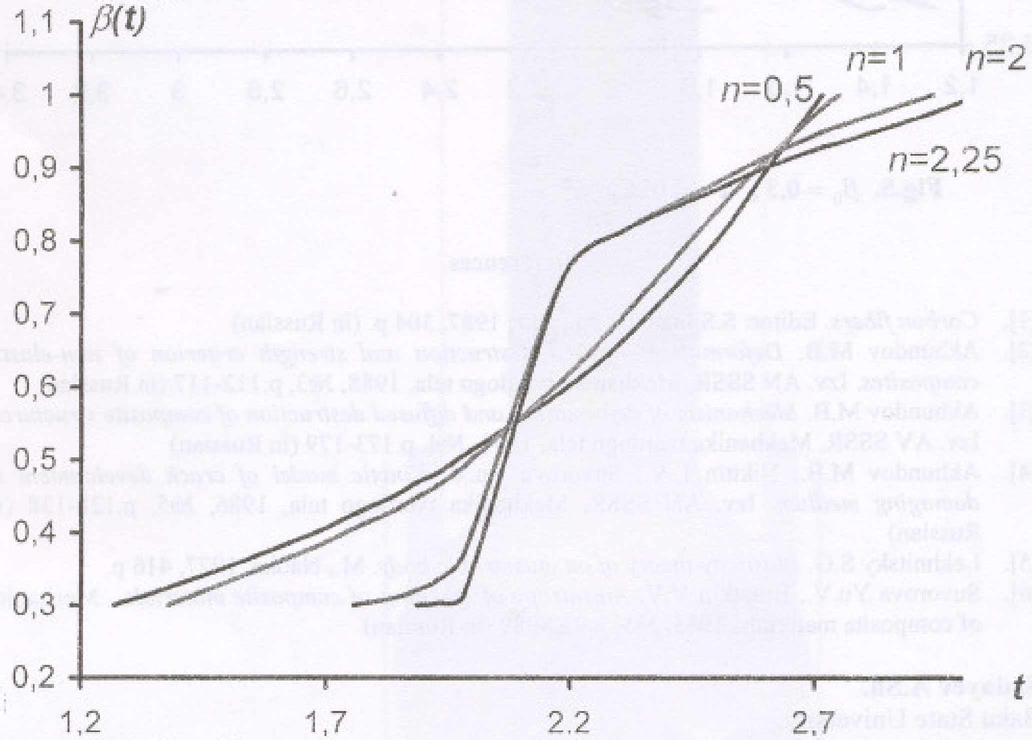

Fig.4. $\beta_0 = 0{,}3$; $\alpha = 0$.

From them it follows the increase of non-linear character of motion of destruction front with increase of the parameter n and the late or the early depending on the relative thickness of β_0 on set of development of destruction process, which on initial stage occurs sufficiently slowly, but then it acquires avalanche character.

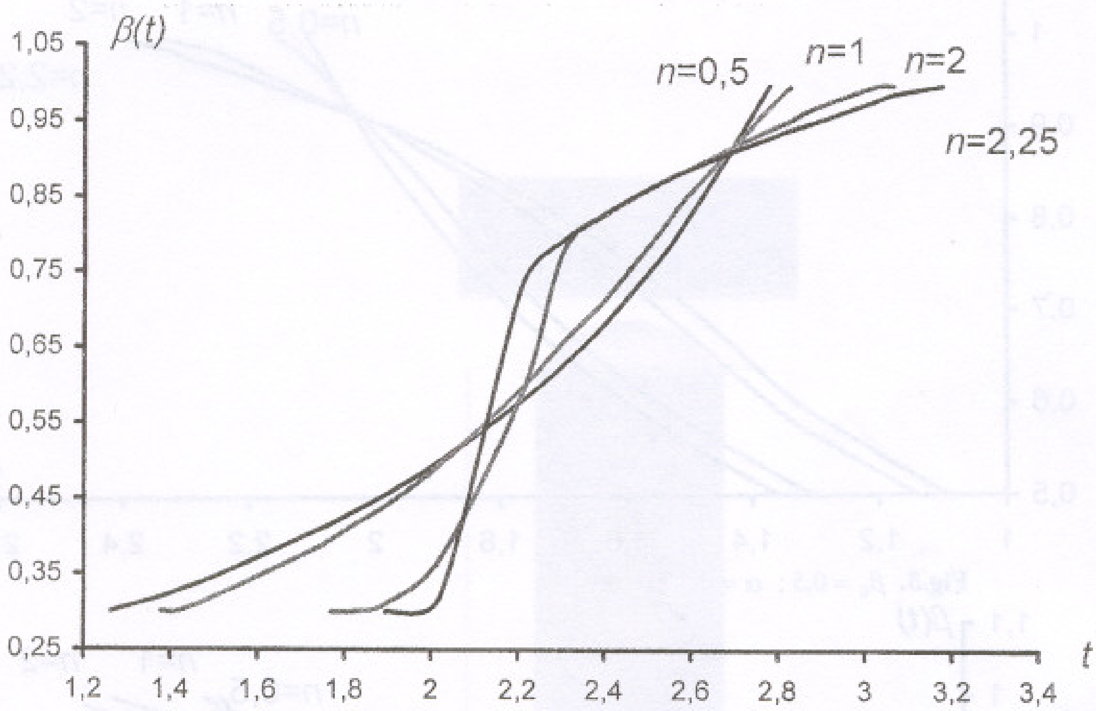

Fig.5. $\beta_0 = 0,3$; $\alpha = 0,025$.

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