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ON ONE GENERALIZATION OF THE WINKLER MODEL IN DYNAMICS PROBLEMS

Abstract

In the given article the generalization of the Winkler model for dynamics problems is suggested. The generalization is in the introduction of dependence of coefficient on the frequency of oscillation. For the elucidation of an influence of this dependence, the problem on oscillation of an spring beam situated on elastic base, has been considered. It was shown that the discount of "dynamical" coefficient brings to decreasing of eigen frequency values.

Winkler's model gives the results acceptable for practice by solving statistical problems in linear statement. As was shown in the work [1], in dynamics problems, with determined frequencies Winkler's model gives an error. It is connected with taking the coefficients of the bed K independently from frequency [1]. In [2] it was attempted to establish the dependence K on the frequency.

It is obvious that this dependence can be written only for the model representations and acceptable only for some problems

1. The essence of the model problem leading to a model representation is the following: The oscillation of the elastic layer with the thickness H, lying on a rigid base is considered. Proportionality factor of the acting loading and vertical replacement of restricting surface is taken as a stiffness coefficient. From the statement of the problem it follows that a system of determining equations has the form

$$\frac{\partial\sigma}{\partial x}=\rho_g\frac{\partial^2 u}{\partial t^2};\ \sigma=E_g\frac{\partial u}{\partial x} \tag{1.1}$$

for $x=0$ $\sigma=-q_0\sin\omega t$, for $x=H$ $u=0$, where E_g is the modulus of elasticity of ground ρ_g is the density of ground.

The solution of the problem we will seek in the form $u=u_0\sin\omega t_0$

$$c^2\frac{\partial^2 u_0}{dx^2}+\omega^2 u_0=0;\ c^2=\frac{E_g}{\rho_g} \tag{1.2}$$

for $x=0$ $\frac{du_0}{dx}=-\frac{q_0}{E_g}$, for $x=H$; $u_0=0$.

Let's take the solution of the equation in the form:

$$u_0=A_1\sin\frac{\omega}{c}x+A_2\cos\frac{\omega}{c}x$$

The unknown parameters A_i are determined from the following system of equations, based on the boundary conditions:

$$A_1\frac{\omega}{c}=-\frac{q_0}{E_g}\qquad A_1\sin\frac{\omega}{c}x+A_2\cos\frac{\omega}{c}x=0.$$

Hence follows

$$u_0=-\frac{q_0}{E_g}\frac{c}{\omega}\sin\frac{\omega}{c}x+\frac{q_0}{E_g}tg\frac{\omega}{c}H\cos\frac{\omega}{c}x.$$

Let's define the replacement of points restricting the surfaces $x = 0$. It is defined by the following expression:

$$u_0\Big|_{x=0} = +\frac{q_0}{E_g}\frac{c}{\omega}tg\frac{\omega}{c}H \approx \frac{q_0}{E_g}H\frac{1-\frac{1}{6}\alpha^2\omega^2}{1-\frac{1}{2}\alpha^2\omega^2}; \ \alpha^4 = \frac{H}{c}. \qquad (1.3)$$

For small values of ω we'll obtain

$$u_0\big|_{x=0} = \frac{q_0}{E_g}H = K_0^{-1}q_0; \quad K_0^{-1} = \frac{H}{E_g}$$

One can interpret the obtained correlation as the characteristic correlation of Winkler's model $W = k^{-1}q$. In the case of accounting dynamical effects, in the first approximation one can represent the relation (1.3) in the form:

$$q = u_0\big|_{x=0} = K_0\left(1 - \frac{1}{3}\alpha^2\omega^2\right).$$

This equality permits to introduce the "dynamical" Winker's model:

$$q = K(\omega)W = K_0\Omega(\omega)W = K_0\left(1-\beta^2\omega^2\right)W. \qquad (1.4)$$

Hence it follows the main deduction: the proportionality factor of a decreasing function of frequency, in addition is quadratic. One can introduce the "dynamical" Winkler's model basing on Dalambert principle and exactly

$$q - \rho_g\frac{\partial^2 W}{\partial t^2} = kW, \qquad (1.5)$$

where ρ_g is the density of ground, unit of area of surface is $\left([\rho_g] = \frac{k^2}{m^2}\right)$.

If we'll take $q = q_0 \sin\omega t$ and $W = W_0 \sin\omega t$, then from (1.5) we obtain:

$$q_0 = W_0\left(k - \rho_g\omega^2\right) = W_0k\left(1-\omega^2\gamma^2\right); \ \gamma^2 = \frac{\rho_g}{K}$$

i.e. the character of dependence of the bed coefficient on frequency doesn't depend on the way of introduction of the "dynamical" model.

2. For elucidation of influence of dependence of the bed coefficient on frequency we'll consider the oscillation of straight-forward pivot with the length L, hinge supported on end walls situated in elastic medium. The equation of oscillations of points of the pivot has the form:

$$EI\frac{\partial^4 W}{\partial x^4} + \rho F\frac{\partial^2 W}{\partial x^2} + bq = 0, \qquad (2.1)$$

where E-is Young's modulus of material of the pivot, b is the characteristic width (width of contact with ground), I is the inertion moment of cross-section, ρ is the density of material of the pivot, F is the area of cross-section, W is deflection, q is the intensity of pressure acting on pivot from ground's side.

The equation (2.1) is solved with the following boundary conditions

$$W = 0, \ \frac{\partial^2 W}{\partial x^2} = 0$$

for $x = 0; L$.

Assume that one can neglect the action of ground to pivot i.e. $q = 0$. Then taking $W = W_0 \sin\omega t$ from (2.1) follows

$$EI\frac{\partial^4 W_0}{\partial x^4} - \rho F\omega^2 W_0 = 0 .$$

The characteristic equation of the obtained equation has the form:

$$EI\alpha^4 - \rho F\omega^2 = 0 \text{ or } \frac{\rho F}{EI}\omega^2 = \alpha^4 .$$

Hence it follows that

$$W_0 = c_1 sh\alpha x + c_2 ch\alpha x + c_3 \sin\alpha x + c_4 \cos\alpha x .$$

Subject to the boundary conditions, the system for determination of c_i has the form:

$$c_2 + c_4 = 0; \; c_2 - c_4 = 0 .$$

For the existence of non-trivial values c_i it is necessary that $\sin\alpha L = 0$ or $\alpha L = \pi n$. Hence it follows that the eigen frequency is defined by the following equality

$$\omega = \frac{\pi^2}{L^2}\sqrt{\frac{EI}{\rho F}} . \tag{2.2}$$

In the limits of the Winkler statistical model the equation (2.1) takes the form:

$$EI\frac{\partial^4 W}{\partial x^4} + \rho F\frac{\partial^2 W}{\partial t^2} + bkW = 0 \tag{2.3}$$

or for amplitude

$$EI\frac{\partial^4 W_0}{\partial x^4} - \rho F\omega^2 W_0 + bkW_0 = 0 .$$

Not constructing all calculations we've

$$\omega = \sqrt{\frac{1}{\rho F}\left[bk + \left(\frac{\pi}{L}\right)^4 EI\right]} . \tag{2.4}$$

Let's consider the Winkler dynamical model also by using D'Alambert principle and subject to (1.5) from the equation (2.1) we'll obtain:

$$EI\frac{\partial^2 W}{\partial x^4} + \rho F\frac{\partial^2 W}{\partial t^2} + b\rho_\Gamma\frac{\partial^2 W}{\partial t^2} + bkW = 0$$

hence it follows, that

$$\omega = \sqrt{\frac{1}{\rho F + b\rho_g}\left[bk + \left(\frac{\pi}{L}\right)^4 EI\right]} . \tag{2.5}$$

Comparing the equations (2.5) and (2.3) it follows that the application of D'Alambert principle on constructing the "dynamical" Winkler model brings to the increasing of density of material of pivot i.e. "weighting".

In physics point of view this model is equivalent to the following: some boundary layer of liquid oscillate together with pivot.

Let's consider the elastic approach of introduction of the "dynamical" Winkler model. In limits of this approach for harmonic oscillations from the equation (2.3) we obtain the equation for determining the amplitude of oscillation. It has the form:

$$EI\frac{d^4 W}{dx^4} - \rho F\omega^2 W_0 + bk(\omega)W_0 = 0 .$$

For the small eigen form of oscillation we've:

$$EI\left(\frac{\pi}{L}\right)^4 - \rho F\omega^2 + bk(\omega) = 0 .$$

The given equation permits to define the value of an eigen frequency.

In particular, in the case of quadratic dependence K of ω we'll obtain

$$EI\left(\frac{\pi}{L}\right)^4 - \rho F\omega^2 + bk_0\left(1-\beta^2\omega^2\right) = 0 ,$$

$$EI\left(\frac{\pi}{L}\right)^4 - \left(\rho F + bk_0\beta^2\right)\omega^2 + bk_0 = 0 .$$

Hence it follows that

$$\omega = \sqrt{\frac{1}{\rho F + k_0\beta^2 b}\left[bk_0 + EI\left(\frac{\pi}{L}\right)^4\right]} . \qquad (2.6)$$

Comparing the obtained values of ω if follows that the physical base of generalization of the Winkler model brings to decreasing of values of an eigen frequency in comparison with calculations by the classical model.

References

[1]. Latifov F.S. *The oscillations of covers by elastic and liquid medium.* 1999, Baku, Elm, 164 p.

[2]. Eyubov Y.A., Alizadeh A.N., Achundov M.B. *The Winkler method on calculations of ground bases.* Baku, 1999, 126 p.

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