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THE SCATTERING DESTRUCTION OF HOLLOW SPHERICAL CONTAINER UNDER THE ACTION OF INTERNAL PRESSURE

Abstract

On the base of conception of material heredity damage in loading process the process of scattering destruction of hollow spherical container is investigated under the action of internal pressure. The analytical expression is obtained for the initial time destruction. Non-linear integral motion equation of front destruction in assumption of full carrying capacity of material of front destruction is obtained. For constant kernel of damage operator the evident analytical expression low on motion of front destruction is received; for weak singular Abell kernel the integral equation was realized on the base on numerical algorithm.

The hollow spherical container is considered, whose internal radius is R_0, and external is R. On internal surface of hollow sphere acts equally distributed loading of the intensity P. Material of container we take as elastically damaging [1]:

$$э_j = \frac{1}{2\mu}\left(1 + M^*\right)s_{ij} \; ; \; \varepsilon = \frac{1}{3K}\sigma , \tag{1}$$

where M^* is a damage operator, s_{ij} and $э_{ij}$ are deviators of stress and deformation tensors, σ and ε are their sphere parts, μ and K are instant shear moduluses and volume pressing.

In the capacity of destruction we accept:

$$\left(1 + M^*\right)\sigma_u = \sigma_0 , \tag{2}$$

where σ_u is intensity of stresses; σ_0 is a strength of defectless material.

As known [1] at monotone loading which holds in considered problem the damage operator M^* behaves as a hereditary type ordinary operator, and then the stress distribution in an hollow sphere will be as in an elastic case, i.e.

$$\begin{cases} \sigma_\rho = \dfrac{Pc^3}{1-c^3}\left[1 - \left(\dfrac{R}{\rho}\right)^3\right], \\ \sigma_\theta = \sigma_\varphi = \dfrac{Pc^3}{1-c^3}\left[1 + \dfrac{1}{2}\left(\dfrac{R}{\rho}\right)^3\right]. \end{cases} \tag{3}$$

Here σ_ρ, σ_θ and σ_φ are correspondingly radical and tangential stresses; ρ is a current radius; $c = R_0/R$.

Then by virtue of

$$\sigma_u = \frac{1}{\sqrt{2}}\left[\left(\sigma_\rho - \sigma_\theta\right)^2 + \left(\sigma_\rho - \sigma_\varphi\right)^2 + \left(\sigma_\theta - \sigma_\varphi\right)^2\right]^{\frac{1}{2}}$$

we'll get

$$\sigma_u = \left|\sigma_p - \sigma_\theta\right| = \frac{1,5Pc^3}{1-c^3}\left(\frac{R}{\rho}\right)^3 . \tag{4}$$

From this formula it follows that intensity of stresses σ_u gets its greatest value on internal surface of hollow sphere at $\rho = R_0$

$$\sigma_{u\max} = \frac{1{,}5P}{1-c^3}. \tag{5}$$

Allowing for this expression to criterion of destruction (2) we'll get the formula for initial time destruction of internal layer of container:

$$\int_0^{t_0} M(\tau)d\tau = \frac{\sigma_0\left(1-c^3\right)}{1{,}5P} - 1. \tag{6}$$

For example for weak singular kernel of the damage operator $M(t) = mt^{-\alpha}$; $0 < \alpha < 1$, we'll have:

$$t_0 = \left\{\frac{1-\alpha}{m}\left[\frac{\sigma_0\left(1-c^3\right)}{1{,}5P} - 1\right]\right\}^{\frac{1}{1-\alpha}}. \tag{7}$$

After the destruction of internal surface of hollow sphere redistribution of stresses happen in it, with the next destruction of adjoined to internal surface layer and so on. Thus again formed internal bound, front of destruction moves to the external one. Character of its motion and velocity determines the main parameters of scattering destruction of hollow sphere, and its long-term strength.

For deriving the destruction equation of front motion we accept the notations:

$$\frac{\rho}{R} = \beta(t); \quad c = \frac{R_0}{R} = \beta(\tau). \tag{8}$$

Then according to (4) we'll have:

$$\sigma_u(t,\tau) = 1{,}5P\frac{\beta^3(\tau)}{\beta^3(t)\left[1-\beta^3(\tau)\right]}. \tag{9}$$

Allowing for the last in the destruction criterion (2) we'll get the following non-linear Volterra equation of the second order for the dimensionless radial coordinate $\beta(t)$ of destruction front:

$$\frac{1}{1-\beta^3(t)} + \frac{1}{\beta^3(t)}\int_0^t M(t-\tau)\frac{\beta^3(\tau)}{1-\beta^3(\tau)}d\tau = \frac{\sigma_0}{1{,}5P}. \tag{10}$$

Here it is necessary to take into account that $\beta(t) = \beta_0$ when $t \le t_0$.

For qualitative investigation of process as kernel of damage operator we accept the simplest: $M(t-\tau) = m = const$. Then integral equation (10) is led to the following differential equation of the fist order with initial conditions:

$$\begin{cases} \dfrac{d\beta}{dt} = \dfrac{m}{3}\dfrac{\beta\left(1-\beta^3\right)}{g\left(1-\beta^3\right)^2 - 1}, \\ \beta\big|_{t=t_0} = \beta_0. \end{cases} \tag{11}$$

Here β_0 is initial relatively thickness of spherical container; t_0 is incubation time, determined according to the formula (7) when $\alpha = 0$ and also it is accepted

$$g = \frac{\sigma_0}{1{,}5P}. \tag{12}$$

The scattering destruction process holds and continues till the condition $d\beta/dt > 0$ is fulfilled. Since according to the introduced notation (8) $b < 1$ then this impose the conditions on the value of internal pressure P and especially:

$$P < P_* = \frac{1}{1,5}\sigma_0\left(1-\beta^3\right)^2. \quad (13)$$

At the same time the since $t_0 \geq 0$ then from (7) when $\alpha = 0$ we have

$$P < P_{**} = \frac{1}{1,5}\sigma_0\left(1-\beta^3\right). \quad (14)$$

At pressure of big P_{**} the destruction process happens instantly by applying the internal pressure. For the pressure in the internal $P_* < P < P_{**}$ some lag period to instant destruction is observed, moreover this lag period is determined according to (2). For the pressure less than P_* step-by-step scattering destruction of hollow sphere holds, the degree which is determined by the position at the given moment of the destruction front period, the equation which according to (11) has the form

$$m(t-t_0) = \frac{\sqrt{3}}{3}\left\{arctg\left[\frac{2\sqrt{3}}{3}\left(\beta+\frac{1}{2}\right)\right] - arctg\left[\frac{2\sqrt{3}}{3}(\beta_0+1)\right]\right\} -$$
$$- g\left(\beta^3-\beta_0^3\right) - \ln\left[\left(\frac{\beta}{\beta_0}\right)^{2-3g} \cdot \frac{1-\beta_0}{1-\beta}\sqrt{\frac{(2\beta_0+1)^2+3}{(2\beta+1)^2+3}}\right]. \quad (15)$$

The period of full destruction (critical time) is determined as period of motion destruction front internal surface of spherical container to external, provided that at all period of this motion velocity of destruction front is finite or as period when the velocity becomes infinitely great.

In the last case from (11) $d\beta/dt = \infty$ is determined corresponding dimensionless coordinate of front destruction

$$\beta_{kp} = \left(1-g^{-1/2}\right)^{1/3}. \quad (16)$$

At this it is necessary: $\beta_0 < \beta_{kp} < 1$.

For weak singular kernel damage operator the non-linear integral equation (10) must be solved directly. For this the numerical algorithm based on step calculation of integral member is used.

The results of numerical computation are given in fig. 1, 2, 3. In fig. 1 the curves of motions of destruction front in dependence of singularity parameter of kernel integral member are cited.

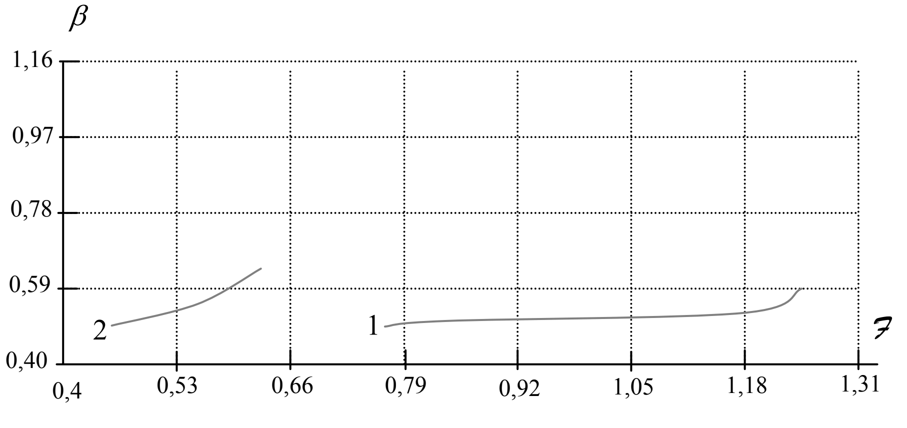

Fig. 1: $g = 2$; $\beta_0 = 0,5$; $\alpha = 0$ (1); $\alpha = 0,2$ (2).

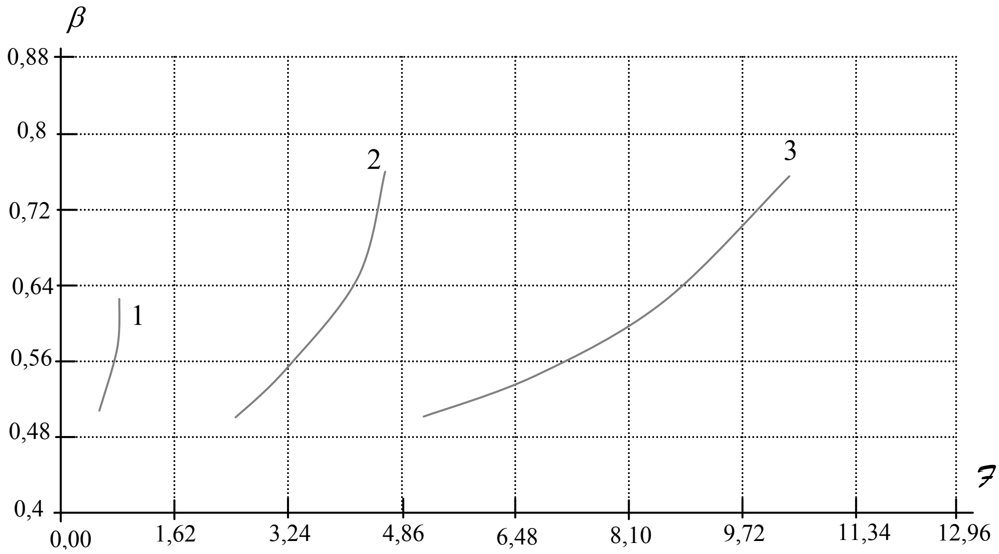

Fig. 2: $a = 0{,}3$; $b = 0{,}5$, $g = 2(1)$, $g = 4(2)$, $g = 6(3)$.

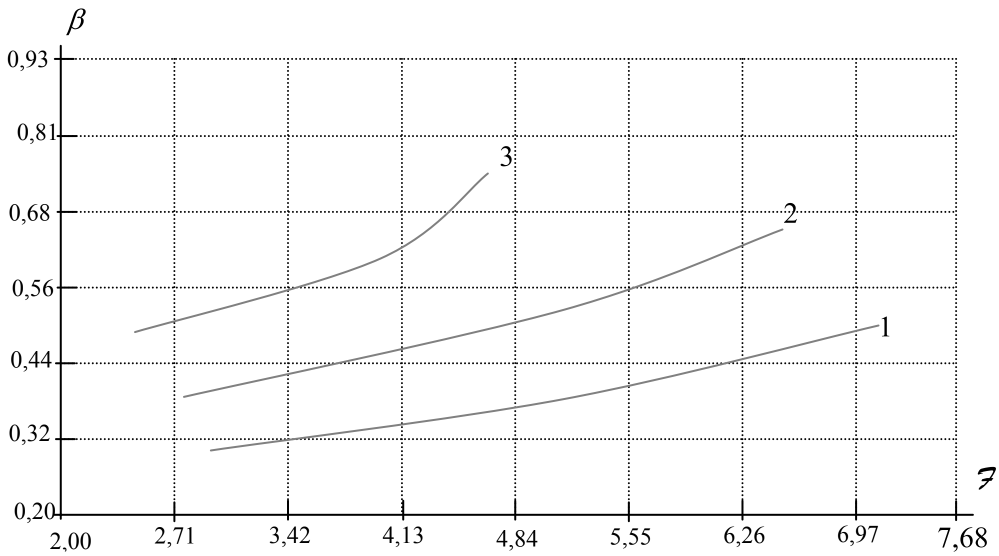

Fig. 3: $g = 4$; $\alpha = 0{,}2$; $\beta_0 = 0{,}3(1)$; $\beta_0 = 0{,}4(2)$; $\beta_0 = 0{,}5(3)$.

In fig. 2 such curves map their dependence on the values of internal pressure, and in fig. 3- from thickness of spherical container. Their analysis show that increasing of internal pressure leads to both decreasing of initial time destruction and period of all destruction process. For thick spherical container the destruction process begins later and continues any longer. Besides the presence of singularity of damage kernel leads to considerable reduction of time of destruction process.

References

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