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ON SOLVING A SYSTEM OF THE FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS IN DISTRIBUTIONS

Abstract

In the paper the motion of vibro-correctness was introduced and the conditions were obtained under which the system of the first order differential equations with generalized effects is vibro-correct.

Let it be required to find a solution of the system

$$\begin{aligned} x_t &= f_1(x,y,u,\vartheta,t,s) + \varphi_1(x,u,t,s)\dot{u}(t), \\ y_s &= f_2(x,y,u,\vartheta,t,s) + \varphi_2(y,\vartheta,s,t)\dot{\vartheta}(s) \end{aligned} \tag{1}$$

in the rectangular $G=(t_0,t_1)\times(s_0,s_1)$, satisfying the following conditions

$$\begin{aligned} x(t_0,s) &= \psi_1(s), \ \ s\in S=(s_0,s_1), \\ y(s_0,t) &= \psi_2(t), \ \ t\in T=(t_0,t_1), \end{aligned} \tag{2}$$

where derivatives are understood in the sense of generalized functions theory, f_i,ψ_i are n_i-dimensional functions, $i=1,2$, φ_i are $n_i\times m_i$ matrices, $(u(t),\vartheta(s))$ is m_1+m_2 dimensional function of bounded variation, $(\dot{u}(t),\dot{\vartheta}(s))$ are distributions of zero order [1, p.203-208].

In the case, when $\varphi_i\equiv 0$, $i=1,2$ the analogous problems were studied in [2] and in the case, when $\varphi_i=b_i(t,s)$, $i=1,2$ in [3].

Definition 1. *For the absolutely continuous function* $(u(t),\vartheta(s))\in AC_{m_1}(T)\times\times AC_{m_2}(S)$ *the function* $(x(t,s),y(s,t))\in C(T;L^{n_1}(S))\times C(S;L^{n_2}(T))$ *is called a solution of the problem (1), (2), if it satisfies almost everywhere in* G *the system of integral equations*

$$\begin{aligned} x(t,s) &= \psi_1(s)+\int_{t_0}^{t}\left[f_1(x(\tau,s),y(s,\tau),u(\tau),\vartheta(s),\tau,s)+\varphi_1(x(\tau,s),u(\tau),\tau,s)\dot{u}(\tau)\right]d\tau, \\ y(s,t) &= \psi_2(t)+\int_{s_0}^{s}\left[f_2(x(t,\sigma),y(\sigma,t),u(t),\vartheta(\sigma),t,\sigma)+\varphi_2(y(\sigma,t),\vartheta(\sigma),\sigma,t)\dot{\vartheta}(\sigma)\right]d\sigma, \end{aligned} \tag{3}$$

where $C(T;L^n(S))$ *is the space of continuous mappings* $T\to L^n(S)$.

Let the following conditions be fulfilled:

a) $\psi_1(s)\in L^{n_1}(S),\ \psi_2(t)\in L^{n_2}(T)$;

b) vector functions $f_i(x,y,u,\vartheta,t,s),\ i=1,2$ are continuous on $(x,y,u,\vartheta)\in R^{n_1+n_2+m_1+m_2}$ for a.e. $(t,s)\in G$, measurable on (t,s) for all (x,y,u,ϑ);

c) matrix functions $\varphi_1(x,u,t,s)$ and $\varphi_2(y,\vartheta,s,t)$ are continuous $(x,u,t)\in R^{n_1}\times R^{m_1}\times T$ and $(y,\vartheta,s)\in R^{n_2}\times R^{m_2}\times S$ for a.e. $s\in S$ and $t\in T$, measurable on $s\in S$ and $t\in T$ for all (x,u,t) and (y,ϑ,s) respectively;

d) for the fixed function $(u(t),\vartheta(s))\in AC_{m_1}(T)\times AC_{m_2}(S)$ and for any functions $(x(t,s),y(s,t))\in S_R(G)=\{(x,y)\in C(T;L^{n_1}(S))\times C(S;L^{n_2}(T)):\|x-\psi_1(s)\|_{L^{n_1}(S)}\le R,$ $t\in T;\ \|y-\psi_2(t)\|_{L^{n_2}(T)}\le R,\ s\in S,\ R>0\}$:

$$\| f_i(x(t,s),y(s,t),u(t),\vartheta(s),t,s)\|\le m_i(t,s),\ i=1,2;$$

$$\|\varphi_1(x(t,s),u(t),t,s)\|\le n_1(t,s),\ \|\varphi_2(y(s,t),\vartheta(s),s,t)\|\le n_2(s,t),$$

where $m_i(t,s)\in L(G)$, $i=1,2$; $n_1(t,s)\in L_\infty(T,L(S))$, $n_2(s,t)\in L_\infty(S,L(T))$, besides, for $(\tilde{x}(t,s),\tilde{y}(s,t)),(x(t,s),y(s,t))\in S_R(G)$ and $(u(t),\vartheta(s))\in AC_{m_1}(T)\times AC_{m_2}(S)$: $\| f_i(\tilde{x}(\tau,\sigma),$ $\tilde{y}(\sigma,\tau),u(\tau),\vartheta(\sigma),\tau,\sigma)-f_i(x(\tau,\sigma),y(\sigma,t),u(\tau),\vartheta(\sigma),\tau,\sigma)\|_{L^{n_i}(G_{ts})}\le \int_{t_0}^{t}\gamma_i(u(\tau),\tau)\|\tilde{x}(\tau,\cdot)-$ $-x(\tau,\cdot)\|_{L^{n_1}(s_0,s)}d\tau+\int_{s_0}^{s}\rho_i(\vartheta(\sigma),\sigma)\|\tilde{y}(\sigma,\cdot)-y(\sigma,\cdot)\|_{L^{n_2}(t_0,t)}d\sigma$, $\|\varphi_1(\tilde{x}(t,\sigma),u(t),t,\sigma)-$ $-\varphi_1(x(t,\sigma),u(t),t,\sigma)\|_{L^{n_1\times m_1}(s_0,s)}\le r_1(u(t),t)\|\tilde{x}(t,\cdot)-x(t,\cdot)\|_{L^{n_1}(s_0,s)}$, $\|\varphi_2(\tilde{y}(s,\tau),\vartheta(s),s,\tau)-$ $-\varphi_2(y(s,\tau),\vartheta(s),s,\tau)\|_{L^{n_2\times m_2}(t_0,t)}\le r_2(\vartheta(s),s)\|\tilde{y}(s,\tau)-y(s,\tau)\|_{L^{n_2}(t_0,t)}$, holds, where $\gamma_i(u(t),t)\in L(T)$, $\rho_i(\vartheta(s),s)\in L(T)$, $i=1,2$, $r_1(u(t),t)\in L_\infty(T)$, $r_2(\vartheta(s),s)\in$ $\in L_\infty(S)$, $G_{ts}=(t_0,t)\times(s_0,s)$, $t_0\le t\le t_1$, $s_0\le s\le s_1$.

Theorem 1. *Under the conditions a)-d) for the fixed absolutely continuous functions $u(t)$, $\vartheta(s)$ there exists a unique local solution of the problem (1), (2).*

The theorem is proved with the help of contracted mappings principle.

In the case when $(u(t),\vartheta(s))$ (or at least one of them) are functions of bounded variation the defining of solution of the problem (1), (2) in the integral form (3) meets difficulties connected with extension of a definition of multiplication operation of the singular generalized function $\dot{u}(t)$ on the discontinuous function $\varphi_1(x(t,s),u(t),t,s)$ [1, p.214-215].

Definition 2. *Let the sequence $(u_k(t),\vartheta_k(s))\in AC_{m_1}(T)\times AC_{m_2}(S)$, $k=1,2$ in *-weak topology of the space $VB_{m_1}(T)\times VB_{m_2}(S)$ converge to the function $(u(t),\vartheta(s))\in$ $\in VB_{m_1}(T)\times VB_{m_2}(S)$. If the corresponding solution $(x_k(t,s),y_k(s,t))$ in *-weak topology of the space $VB(T;L^{n_1}(S))\times VB(S;L^{n_2}(T))$ converges to some function $(x(t,s),y(s,t))\in$ $\in VB(T;L^{n_1}(S))\times VB(S;L^{n_2}(T))$ and limit doesn't depend on the choice of sequence $(u_k(t),\vartheta_k(s))$, then the limit is called vibro-solution and problem (1), (2) is called vibro-correct on input of bounded variation [4, p.36-57].*

Investigating the vibro-correctness we'll assume that besides the above mentioned the following conditions are fulfilled:

e) functions $f_i(x,y,u,\vartheta,t,s)$, $i=1,2$, $\varphi_1(x,u,t,s)$, $\varphi_2(y,\vartheta,s,t)$ satisfy the growth condition on infinity with respect to x, y

$$\| f_i(x,y,u,\vartheta,t,s)\|=M_{f_i}(u,t)\|x\|+N_{f_i}(\vartheta,s)\|y\|+C_{f_i}(u,\vartheta,t,s),$$

$$\|\varphi_1(x,u,t,s)\|=M_{\varphi_1}(u,t)\|x\|+C_{\varphi_1}(u,t,s),$$

$$\|\varphi_2(y,\vartheta,s,t)\|=M_{\varphi_2}(\vartheta,s)\|y\|+C_{\varphi_2}(\vartheta,s,t),$$

for $x\in R^{n_1}$, $y\in R^{n_2}$, $u\in R^{m_1}$, $\vartheta\in R^{m_2}$, t, $s\in R$, where $M_{f_i}(u(t),t)\in L(T)$,

$N_{f_i}(\vartheta(s),s)\in L(s),\ M_{\varphi_1}(u(t),t)\in L_\infty(T),\ M_{\varphi_2}(\vartheta(s),s)\in L_\infty(S), C_{f_i}(u(t),\vartheta(s),t,s)\in$
$\in L(G),\ C_{\varphi_1}(u(t),t,s)\in L_\infty(T,L(S)),\ C_{\varphi_2}(\vartheta(s),s,t)\in L_\infty(S;L(T)),\ u(t)\in AC_{m_1}(T),$
$\vartheta(s)\in AC_{m_2}(S)$;

f) functions $\varphi_1(x,u,t,s)$ and $\varphi_2(y,\vartheta,s,t)$ are continuous together with partial derivatives φ_{1x}, φ_{1t} and φ_{2y}, φ_{2s} at $x\in R^{n_1}$, $y\in R^{n_2}$, $u\in R^{m_1}$, $\vartheta\in R^{m_2}$, $t\in T$, $s\in S$. Besides, the functions $\varphi_1(x,u,t,s), \varphi_{1x}(x,u,t,s)$, $\varphi_{1t}(x,u,t,s)$ and $\varphi_2(y,\vartheta,s,t)$, $\varphi_{2y}(y,\vartheta,s,t)$, $\varphi_{2s}(y,\vartheta,s,t)$ locally satisfy Lipshitz condition with respect to x and y respectively;

g) systems of the first order partial differential equations

$$\frac{dk}{dp}=\varphi_1(k,p,\tau,\sigma),\ k(u)=\xi\,, \tag{4}$$

$$\frac{dh}{dq}=\varphi_2(h,q,\sigma,\tau),\ h(\vartheta)=\eta\,, \tag{5}$$

are locally solvable for $\xi\in R^{n_1}$, $\eta\in R^{n^2}$, $p,u\in R^{m_1}$, $q,\vartheta\in R^{m_2}$, $\tau\in T$, $\sigma\in S$, where τ,s take part of parameters.

Denote local solutions of the systems (4), (5) by $k(\xi,p,u,\tau,\sigma)$, $h(\eta,q,\vartheta,\sigma,\tau)$. By the theorem on continuous dependence and differentiability on initial conditions and parameters [4, p.44-45; 5; 6] it follows that the solution $k(\xi,p,u,\tau,\sigma)$ of the problem (4) is continuous with the partial derivatives k_ξ, k_τ for $\xi\in R^{n_1}$, $p,u\in R^{m_1},\tau\in T$, $\sigma\in S$, where $\|p-u\|$ is sufficiently small, besides functions k,k_ξ,k_τ locally satisfy Lipschitz condition with respect to ξ. Analogous statements take place for the function $h(\eta,q,\vartheta,\sigma,\tau)$.

Functions $k(\xi,p,u,\tau,\sigma)$, $h(\eta,q,\vartheta,\sigma,\tau)$ have the properties [4, p.43]:

$k(\xi,u,u,\tau,\sigma)=\xi$, $h(\eta,\vartheta,\vartheta,\sigma,\tau)=\eta$,
$k(\xi,p_1+p,u,\tau,\sigma)=k(k(\xi,p_1,u,\tau,\sigma),p_1,p_1+p,\tau,\sigma)$,
$h(\eta,q_1+q,\vartheta,\sigma,\tau)=h(h(\eta,q_1,\vartheta,\sigma,\tau),q_1,q_1+q,\sigma,\tau)$,
$k(k(\xi,p,u,\tau,\sigma),u,p,\tau,\sigma)=\xi$,
$h(h(\eta,q,\vartheta,\sigma,\tau),\vartheta,q,\sigma,\tau)=\eta$.

Solutions $x(t,s)$, $y(s,t)$ of the system (1) responding to absolutely continuous inputs $u(t)$, $\vartheta(s)$; $u^0=u(t_0)$, $\vartheta^0=\vartheta(s_0)$ and satisfying the condition (2) we'll seek in the following form

$$\begin{aligned} x(t,s)&=k(z(t,s),u(t),u^0,t,s),\\ y(s,t)&=h(\omega(s,t),\vartheta(s),\vartheta^0,s,t). \end{aligned} \tag{5'}$$

Then, taking into account the last properties of functions $k(\xi,p,u,\tau,\sigma)$, $h(\eta,q,\vartheta,\sigma,\tau)$, we have

$$\begin{aligned} z(t,s)&=k(x(t,s),u^0,u(t),t,s),\\ \omega(s,t)&=h(y(s,t),\vartheta(s),\vartheta^0,s,t). \end{aligned} \tag{6}$$

From here it particularly follows, that

$$z(t_0,s)=k\left(x(t_0,s),u^0,u^0,t_0,s\right)=x(t_0,s)=\psi_1(s),$$
$$\omega(s_0,t)=h\left(y(s_0,t),\vartheta^0,\vartheta^0,s_0,t\right)=y(s_0,t)=\psi_2(t).$$

Further, taking into account properties of functions k and h subject to (1), we obtain that functions $z(t,s)$, $\omega(s,t)$ are solutions of the system

$$\begin{aligned} z_t &= \Psi_1\left(z,\omega,u,\vartheta,u^0,\vartheta^0,t,s\right),\\ \omega_s &= \Psi_2\left(z,\omega,u,\vartheta,u^0,\vartheta^0,t,s\right) \end{aligned} \tag{7}$$

and satisfy the conditions

$$z(t_0,s)=\psi_1(s),\ s\in S,\ \omega(s_0,t)=\psi_2(t),\ t\in T, \tag{2'}$$

where $\Psi_1\left(z,\omega,u,\vartheta,u^0,\vartheta^0,t,s\right)=k_\xi\left(k\left(z,u,u^0,t,s\right),u^0,u,t,s\right)f_1\left(k\left(z,u,u^0,t,s\right),h\left(\omega,\vartheta,\vartheta^0,s,t\right),\right.$ $\left.u,\vartheta,t,s\right)+k_\tau\left(k\left(z,u,u^0,t,s\right),u^0,u,t,s\right)$, $\Psi_2\left(z,\omega,u,\vartheta,u^0,\vartheta^0,t,s\right)=h_\eta\left(h\left(\omega,\vartheta,\vartheta^0,s,t\right),\vartheta^0,\vartheta,\right.$ $\left.s,t\right)f_2\left(k\left(z,u,u^0,t,s\right),h\left(\omega,\vartheta,\vartheta^0,s,t\right),u,\vartheta,t,s\right)+h_\sigma\left(h\left(\omega,\vartheta,\vartheta^0,s,t\right),\vartheta^0,\vartheta,s,t\right)$.

Theorem 2. *Under conditions a)-g) for arbitrary function* $\left(u(t),\vartheta(s)\right)\in VB_{m_1}(T)\times$ $\times VB_{m_2}(S)$ *such that* $\left\|u(t)-u^0\right\|\le r$, $\forall t\in T$, $\left\|\vartheta(s)-\vartheta^0\right\|\le r, \forall s\in S$ *there exists a unique solution* $\left(z(t,s),\omega(s,t)\right)\in C\left(T';L^{n_1}(S')\right)\times C\left(S';L^{n_2}(T')\right)$ *of the problem (7), (2'), where* $s'-s_0$, r, $t'-t_0$ *are sufficiently small,* $T'=(t_0,t')$, $S'=(s_0,s')$.

Proof of the theorem is led by the generalized principle of contracted mappings [1, p.82-83].

Theorem 3. *Let conditions a)-g) be fulfilled. Then there exists local vibro-solution of problem (1), (2) on inputs of bounded variation.*

Proof. Consider a sequence of absolutely continuous functions $\left(u_k(t),\vartheta_k(s)\right)$, $k=1,2,\ldots$, approximating the function $\left(u(t),\vartheta(s)\right)$ of bounded variation, i.e. *-$\lim\limits_{k\to\infty}u_k(t)=u(t)$, $t_0\le t\le t'$, *- $\lim\limits_{k\to\infty}\vartheta_k(s)=\vartheta(s)$, $s_0\le s\le s'$, where $\left\|u(t)-u(t_0)\right\|<r$, $t\in T'$, $\left\|\vartheta(s)-\vartheta(s_0)\right\|<r$, $s\in S'$, r is sufficiently small. By theorem 2 for each absolutely continuous function $\left(u_k(t),\vartheta_k(s)\right)$, $k=1,2,\ldots$ there exists a unique solution $\left(x_k(t,s),y_k(s,t)\right)\in C\left(T',L^{n_1}(S')\right)\times C\left(S',L^{n_2}(T')\right)$ of problem (1), (2):

$$\begin{aligned} x_k(t,s)&=\psi_1(s)+\int_{t_0}^{t}\left[f_1\left(x_k(\tau,s),y_k(s,\tau),u_k(\tau),\vartheta_k(s),\tau,s\right)+\right.\\ &\left.+\varphi_1\left(x_k(\tau,s),u_k(\tau),\tau,s\right)\dot u_k(\tau)\right]d\tau,\\ y_k(s,t)&=\psi_2(t)+\int_{s_0}^{s}\left[f_2\left(x_k(t,\sigma),y_k(\sigma,t),u_k(t),\vartheta_k(\sigma),t,\sigma\right)+\right.\\ &\left.+\varphi_2\left(y_k(\sigma,t),\vartheta_k(\sigma),\sigma,t\right)\dot\vartheta_k(\sigma)\right]d\sigma,,\quad k=1,2,\ldots,\ (t,s)\in G'. \end{aligned} \tag{8}$$

Suppose

$$X_k(t,s)=\int_{s_0}^{s}\left\|x_k(t,\sigma)\right\|d\sigma,\quad Y_k(s,t)=\int_{t_0}^{t}\left\|y_k(s,\tau)\right\|d\tau.$$

From (8) we obtain

$$X_k(t,s)=\int_{t_0}^{t}\alpha_k^{(1)}(\tau)X_k(\tau,s)d\tau+\int_{s_0}^{s}\beta_k^{(1)}(\sigma)Y_k(t,\sigma)d\sigma+\omega_k^{(1)}, \tag{9}$$

$$Y_k(s,t)=\int_{t_0}^{t}\alpha_k^{(2)}(\tau)X_k(\tau,s)d\tau+\int_{s_0}^{s}\beta_k^{(2)}(\sigma)Y_k(t,\sigma)d\sigma+\omega_k^{(2)}\,, \tag{10}$$

where $\alpha_k^{(1)}(t)=M_{f_1}(u_k(t),t)+M_{\varphi_1}(u_k(t),t)\|\dot{u}_k(t)\|$, $\alpha_k^{(2)}(t)=M_{f_2}(u_k(t),t)$, $\beta_k^{(1)}(s)=$
$=N_{f_1}(\vartheta_k(s),s)$, $\beta_k^{(2)}(s)=N_f(\vartheta_k(s),s)+M_{\varphi_2}(\vartheta_k(s),s)\|\dot{\vartheta}_k(s)\|$, $\omega_k^{(1)}=\|\psi_1\|_{L^{n_1}(S')}+$
$+\iint_{G'}\left[C_{f_1}(u_k(t),\vartheta_k(s),t,s)+C_{\varphi_1}(u_k(t),t,s)\|\dot{u}_k(t)\|\right]dsdt$, $\omega_k^{(2)}=\|\psi_2\|_{L^{n_2}(T')}+$
$+\iint_{G'}\left[C_{f_2}(u_k(t),\vartheta_k(s),t,s)+C_{\varphi_2}(\vartheta_k(s),t,s)\|\dot{\vartheta}_k(s)\|\right]dsdt$.

Applying to (9), (10) Cronwall lemma [7, p.10], we obtain

$$X_k(t,s)\le\left(\int_{s_0}^{s}\beta_k^{(1)}(\sigma)Y_k(\sigma,t)d\sigma+\omega_k^{(1)}\right)\exp\int_{T'}\alpha_k^{(1)}(t)d\tau\,, \tag{11}$$

$$Y_k(s,t)\le\left(\int_{t_0}^{t}\alpha_k^{(2)}(\tau)X_k(\tau,s)d\tau+\omega_k^{(2)}\right)\exp\int_{S'}\beta_k^{(2)}(s)d\tau s\,,\ (t,s)\in G\,. \tag{12}$$

Substituting (12) into inequality (11) and again applying Cronwall lemma [7, p.68], we obtain

$$X_k(t,s)\le\eta_k^{(1)}\exp M_k\iint_{G'}\alpha_k^{(2)}(t)\beta_k^{(1)}(s)dsdt\,. \tag{13}$$

Analogously we have

$$Y_k(s,t)\le\eta_k^{(2)}\exp M_k\iint_{G'}\alpha_k^{(2)}(t)\beta_k^{(1)}(s)dsdt\,. \tag{14}$$

where $\eta_k^{(2)}=\omega_k^{(2)}\exp\int_{S'}\beta_k^{(2)}(s)ds+\omega_k^{(1)}M_k\int_{T'}\alpha_k^{(2)}(t)dt$,

$\eta_k^{(1)}=\omega_k^{(1)}\exp\int_{T'}\alpha_k^{(1)}(t)dt+\omega_k^{(2)}M_k\int_{S'}\beta_k^{(1)}(s)ds$,

$M_k=\exp\left[\int_{T'}\alpha_k^{(1)}(t)dt+\int_{S'}\beta_k^{(2)}(s)ds\right]$.

From definition of *-weak convergence of the sequence $(u_k(t),\vartheta_k(s))$, $k=1,2,...$ it follows that

$$\sup_k\int_{T'}\|\dot{u}_k(t)\|dt=\sup_k Var_{t_0}^{t}\|u_k(t)\|<+\infty\,,$$

$$\sup_k\int_{S'}\|\dot{\vartheta}_k(s)\|ds=\sup_k Var_{s_0}^{s}\|\vartheta_k(s)\|<+\infty\,.$$

From properties of functions k and h it follows that $z_k(t,s)=k(x_k(t,s),u_k(t_0),u_k(t),t,s)$, $\omega_k(s,t)=h(y_k(s,t),\vartheta_k(s_0),\vartheta_k(s),s,t)$, $k=1,2,...$ are uniformly bounded in $C(T',L^{n_1}(S'))\times$ $\times C(S',L^{n_2}(T'))$. Functions $(z_k(t,s),\omega_k(s,t))$ almost everywhere in G' satisfy the integral equations

$$\begin{gathered}z_k(t,s)=\psi_1(s)+\int_{t_0}^{t}\Psi_1(z_k(\tau,s),\omega_k(s,\tau),u_k(\tau),\vartheta_k(s),u_k(t_0),\vartheta_k(s_0),\tau,s)d\tau\,,\\ \omega_k(s,t)=\psi_2(t)+\int_{s_0}^{s}\Psi_2(z_k(t,\sigma),\omega_k(\sigma,t),u_k(t),\vartheta_k(\sigma),u_k(t_0),\vartheta_k(s_0),t,\sigma)d\sigma\,,\end{gathered} \tag{15}$$

$k=1,2,...$, $(t,s)\in G'$.

We'll show that sequences $(z_k(t,s),\omega_k(s,t))$, $k=1,2,...$ converge to function $(z(t,s),\omega(s,t))$ in $C(T',L^{n_1}(S'))\times C(S';L^{n_2}(T'))$, where $(z(t,s),\omega(s,t))$ is a solution of problem (7), (2') at $(u(t),\vartheta(s))$:

$$
\begin{aligned}
z(t,s)&=\psi_1(s)+\int_{t_0}^{t}\Psi_1(z(\tau,s),\omega(s,\tau),u(\tau),\vartheta(s),u(t_0),\vartheta(s_0),\tau,s)d\tau ,\\
\omega(s,t)&=\psi_2(t)+\int_{s_0}^{s}\Psi_2(z(t,\sigma),\omega(\sigma,t),u(t),\vartheta(\sigma),u(t_0),\vartheta(s_0),t,\sigma)d\sigma .
\end{aligned}
\tag{16}
$$

For this we estimate the quantity $\delta z_k(t,s)=z_k(t,s)-z(t,s),\delta\omega_k(s,t)=\omega_k(s,t)-$ $-\omega(s,t)$. We introduce functions

$$
\delta Z_k(t,s)=\int_{s_0}^{s}\|\delta z_k(t,\sigma)\|d\sigma,\ \delta\Omega_k(s,t)=\int_{t_0}^{t}\|\delta\omega_k(s,\tau)\|d\tau .
$$

Analogously to obtaining estimates (13), (14) from (15), (16) the following

$$
\begin{aligned}
\delta Z_k(t,s)&=O\Bigg(\iint_{G'}\|\Psi(z,\omega,u_k(t),\vartheta_k(s),u_k(t_0),\vartheta_k(s_0),t,s)-\\
&\qquad-\Psi(z,\omega,u(t),\vartheta(s),u(t_0),\vartheta(s_0),t,s)\|dsdt\Bigg),\\
\delta\Omega_k(s,t)&=O\Bigg(\iint_{G'}\|\Psi(z,\omega,u_k(t),\vartheta_k(s),u_k(t_0),\vartheta_k(s_0),t,s)-\\
&\qquad-\Psi(z,\omega,u(t),\vartheta(s),u(t_0),\vartheta(s_0),t,s)\|dtds\Bigg),
\end{aligned}
\tag{17}
$$

where $\Psi=(\Psi_1,\Psi_2)$, $\lim\limits_{\varepsilon\to 0}\frac{O(\varepsilon)}{\varepsilon}=l$ are derived.

From (17) by virtue of Lebesgue theorem on bounded convergence it follows that $\lim\limits_{k\to\infty}\delta Z_k(t,s)=0$, $\lim\limits_{k\to\infty}\delta\Omega_k(s,t)=0$, $(t,s)\in G'$. Consequently, sequences $(z_k(t,s),$ $\omega_k(s,t))$, $k=1,2,...$ converge to function $(z(t,s),\omega(s,t))$ in $C(T';L^{n_1}(S'))\times C(S';L^{n_2}(T'))$. Then subject to transformations (5') we have

$$
\begin{aligned}
&x_k(t,s)=k(z_k(t,s),u_k(t),u_k(t_0),t,s), x(t,s)=k(z(t,s),u(t),u(t_0),t,s),\\
&y_k(s,t)=h(\omega_k(s,t),\vartheta_k(s),\vartheta_k(s_0),s,t), y(s,t)=h(\omega(s,t),\vartheta(s),\vartheta(s_0),s,t),\\
&k=1,2,....
\end{aligned}
\tag{18}
$$

Using boundedness of sequences $(u_k(t),\vartheta_k(s))$ and $(x_k(t,s),y_k(s,t))$, $k=1,2,...$ in $VB_{m_1}(T')\times VB_{m_2}(S')$ and $VB(T',L^{n_1}(S'))\times VB(S';L^{n_2}(T'))$ we obtain, that

$$
\sup_k Var_{t_0}^{t'}\|x_k(t,\cdot)\|_{L^{n_1}(S')}<+\infty,\ \sup_k Var_{s_0}^{s'}\|y_k(s,\cdot)\|_{L^{n_2}(T')}<+\infty .
$$

From (18) we obtain that sequences $\{x_k(t,s),y_k(s,t)\}$ *-weak converge to the function $(x(t,s),y(s,t))$ in $VB(T';L^{n_1}(S'))\times VB(S''L^{n_2}(T'))$. The theorem is proved.

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