

ON COMPARATIVE INCREASE OF RELATIONS OF CONVEX FUNCTIONS
AND THEIR DERIVATIVES

Abstract

For differentiable on $[0; b)$, $b \leq \infty$ and infinitesimal and infinitely large when $x \rightarrow b -$ functions $f(x)$ and $g(x)$, the following relations are well-known

$$\liminf \frac{f'(x)}{g'(x)} \leq \liminf \frac{f(x)}{g(x)}, \quad \overline{\lim} \frac{f(x)}{g(x)} \leq \overline{\lim} \frac{f'(x)}{g'(x)} \quad (*)$$

(all the limits unless otherwise stated, are considered under the condition $x \rightarrow b -$).

In the present paper some methods of obtaining the estimations of the inverse sense, and also estimations of some quantities closely connected with mutual increase of functions and their derivatives, are suggested.

One of the methods described in §1 admits to get an once two sided estimations of relations $\frac{f'(x)}{g'(x)}$ on the known estimations $\frac{f(x)}{g(x)}$. The estimations obtained by this method are revisions of the known estimations and it is shown in this paper that they are exact.

In §2 the most important cases commonly used in the theory of entire and meromorphic functions, are considered.

The other method (see §3) is the obtaining the lower (or upper) bound of one of quantities, "massive" property of the set on which the upper (lower) bound of this quantity is reached, is known. In this direction only some partial results concerning to entire functions and obtained by E.Lindelöf and G.Valiron [1] is known to the author.