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DELAYED FRACTURE BY TENSIONING INHERENTLY ELASTIC THICK PLATE WITH TWO-SIDED EXTERIOR DEEP RECESS

Abstract

Analytic formulae for time preceding to damage and for time to failure of points of the narrowest section of the inherently elastic plate with two-sided exterior deep recess are obtained. H.Neuber's solution for the stress components is used. The graphics reflecting damage and durability processes from different geometric and mechanical parameters for two given types of loading are constructed.

Plate of sufficiently large thickness having two-sided exterior hyperbolic recess is exposed to tension by force P. Let $2a$ be the width of the narrowest section of the plate, d be thickness of plate. At that nominal stress over the narrowest cross-section of the plate will be $\sigma_n = P/2ad = p$. We will use Cartesian system of coordinates xyz. The origin will be at the center of narrow cross-section, x will be directed along the tension, y axis - along the line connecting the bottoms of recesses. At that z axis will be perpendicular to xy plane.

As is known components of stress tensor in the case of elastic plate are defined by the known formulae [1]. It is assumed that the material of the plate is mechanically incompressible, and if the plane-strain conditions are fulfilled, then in accordance with H.Neuber formulae, in the weakest section $x = 0$ the stress components will be

$$\sigma_x(0;y) = \Phi\left(\frac{a}{\rho}\right) F\left(\frac{a}{\rho},\frac{y}{a}\right)\left[2+\frac{a}{\rho}\left(1-\frac{y}{a}\right)\right] p \qquad (1)$$

$$\sigma_y\left(0;\frac{y}{a}\right) = \Phi\left(\frac{a}{\rho}\right) F\left(\frac{a}{\rho},\frac{y}{a}\right)\frac{a}{\rho}\left(1-\frac{y}{a}\right) p \qquad (2)$$

$$\sigma_z = \frac{1}{2}\left(\sigma_x+\sigma_y\right), \quad \sigma_{xy}\left(0,\frac{y}{a}\right)=0; \quad \sigma_{xz}=\sigma_{yz}=0 \qquad (3)$$

In formulae (1), (2) we denote by ρ the radius of curvature of recess. Besides,

$$\Phi\left(\frac{a}{\rho}\right) = \frac{\left(1+\frac{a}{\rho}\right)\sqrt{\frac{a}{\rho}}}{\sqrt{\frac{a}{\rho}}+\left(1+\frac{a}{\rho}\right) arctg\sqrt{\frac{a}{\rho}}} \qquad (4)$$

$$F\left(\frac{a}{\rho},\frac{y}{a}\right) = \left[1+\frac{a}{\rho}\left(1-\frac{y}{a}\right)\right]^{-\frac{3}{2}} \qquad (5)$$

Now we consider the case when material of the plate is physically linearly elastic whose defining relations are represented in the form of Volterra integral [2]. Let the tensile force be function of time t. In this case the components of stress tensor are also expressed by formulae (1)-(3). At that $p = p(t)$. Stress intensity σ_t in the narrow section of the plate is expressed by the formula

$$\sigma_t(0,y) = \frac{\sqrt{3}}{2}\left|\sigma_x(0,y) - \sigma_y(0,y)\right| .$$

Subject to formulae (1) and (2) we obtain

$$\sigma_t\left(0,\frac{y}{a}\right) = \sqrt{3}\Phi(a,\rho)\,F\left(\frac{a}{\rho},\frac{y}{a}\right)p(t). \tag{6}$$

At the point $y = a$ (bottom of the recess) the stress intensity attains its maximal value, since at this point, function $F\left(\frac{a}{\rho},\frac{y}{a}\right)$ accepts its maximal value. For $y = a$ we have $F(a,\rho) = 1$. Taking this into account, from (6) we find: $\sigma_t(0,1) = \sqrt{3}\Phi\left(\frac{a}{\rho}\right)p(t)$.

Questions of delayed fracture of inherently elastic thick plate will also be considered in narrow section ($x = 0$) connecting bottoms of recesses, because this section in the weakest one in terms of fracture. We will proceed from the concept of damage accumulation [3,4], i.e. we assume that fracture (surface discontinuity) occurs when during deformation process accumulated damages achieve the certain value. Besides, we will take into account the experimental information on the fact that damage accumulation process occurs some time later than beginning of deformation process [5]. According to data of paper [5], the mentioned time constitutes a significant part of durability.

In L.Kh.Talybly's paper [6] the kinetic equation of damage accumulation in inherently elastic body was constructed which takes into account the existence of incubation time preceding to damage of material of the body. In particular case it has the form

$$\Pi(t) = H\left(t - t'\right)\left[-\frac{t_1^{1+\lambda}(\sigma_*,T)}{t_0^{1+\lambda}(\sigma_*,T) - t_1^{1+\lambda}(\sigma_*,T)}+\right.$$

$$\left.+(1+\lambda)\int_0^t \frac{(t-\tau)^{\lambda}d\tau}{t_0^{1+\lambda}(\sigma_*(\tau),T(\tau)) - t_1^{1+\lambda}(\sigma_*(\tau),T(\tau))}\right] \tag{7}$$

Here quantity $\Pi(t)$ which except of time t also depends on the coordinates of the points of body, characterizes the extent of accumulated damages; σ_* is some equivalent stress for which, e.g., the stress intensity can be taken; T is temperature counted off from the initial temperature; λ is experimentally defined constant of material; $H(t)$ Heaviside unit function: $H(t) = 0$ for $t < 0$, $H(t)$ for $t > 0$; $t_0 = t_0\ (\sigma_*,T)$ is experimentally defined function of material - durability at various constants $\sigma_* = const$, $T = const$; $t_1 = t_1(\sigma_*,T)$ function of material introduced by L.Kh.Talybly - time preceding to damage of points of the body at various constants $\sigma_* = const$, $T = const$. Function $t_1 = t_1(\sigma_*,T)$ can be defined, for example, by acoustic emission method, t' - is desired time preceding to damage of points of the body at arbitrary $\sigma_* = \sigma_*(t)$ and $T = T(t)$. It is defined from condition $\Pi(t') = 0$, given in [6] by damage condition. If we denote desired durability for arbitrary $\sigma_* = \sigma_*(t)$ and $T = T(t)$ by t_*, then this time is defined from condition $\Pi(t_*) = 1$.

Now we put $t_1(\sigma_*, T)/t_0(\sigma_*, T) \approx A = const$. This condition is usually satisfied at isothermal loading. According to the experimental data of paper [5], constant A depending on material and type of loading changes on the segment $0,35 \le A \le 0,7$. In the general case $0 \le A < 1$. If we use this condition, equation (7) will be reduced to the form

$$\Pi(t) = H\left(t - t'\right)\left[\frac{1}{1 - A^{1+\lambda}}\left(-A^{1+\lambda} + (1+\lambda)\int_0^t \frac{(t-\tau)^\lambda d\tau}{t_0^{1+\lambda}(\sigma_*(\tau), T(\tau))}\right)\right]. \quad (8)$$

Damage conditions $\Pi(t') = 0$ and durability condition $\Pi(t_*) = 1$ by using (8) are written in the following form

$$(1+\lambda)\int_0^{t'} \frac{(t'-\tau)^\lambda d\tau}{t_0^{1+\lambda}(\sigma_*, T)} = A^{1+\lambda} \qquad (0 \le A < 1), \quad (9)$$

respectively

$$(1+\lambda)\int_0^{t_*} \frac{(t_*-\tau)^\lambda d\tau}{t_0^{1+\lambda}(\sigma_*, T)} = 1. \quad (10)$$

Condition (10) coincides with V.V.Moskvitin durability condition being obtained in the case $t_1(\sigma_*, T) = 0$ [2].

Function $t_0(\sigma_*, T)$ can be represented by the following formulae:

$$t_0(\sigma_*, T) = t_{00}\exp\left[\beta\left(1 - \frac{\sigma_*}{\sigma_0}\right) + \mu\left(1 - \frac{T}{T_0}\right)\right] \ (\sigma_* \neq 0), \quad (11)$$

$$t_0(\sigma_*, T) = t_{00}\left(\frac{T_0}{T}\right)^\gamma\left(\frac{\sigma_*}{\sigma_0}\right)^{-\alpha}. \quad (12)$$

Here σ_0 and T_0 are stress and reduction temperature which are independent of time and are chosen from turn-down of σ_* and T; t_{00} is time to failure of material at $\sigma_* = \sigma_0$ and $T = T_0$; $\beta, \mu, \gamma, \alpha$ are material constants. In formulae (11), (12) σ_* is taken for σ_t

a) Suppose $p(t) = p_0 \ln\left(1 + \frac{t}{t_a}\right)$, where $0 \le t \le t_a$; $t' \le t_a \le t_*$; $p_0 = const$. At that representation of $p(t)$ it is expedient to use formula (11). In the case of considered problem, $T = T_0$, and formula (11) takes the form

$$t_0(\sigma_t) = t_{00}\exp\left[\beta\left(1 - \frac{\sigma_t}{\sigma_0}\right)\right]. \quad (13)$$

For quantity σ_0 we take

$$\sigma_0 = \sqrt{3}\beta \left(1+\lambda\right) \Phi \left(\frac{a}{\rho}\right) F \left(\frac{a}{\rho}, \frac{y}{a}\right) P_0. \tag{14}$$

By using (13) and (14) the following relation holds:

$$J_1\left(t_a\right) = \int_0^{t_0} \frac{\left(t_a-\tau\right)^\lambda d\tau}{t_0^{1+\lambda}\left(\sigma_t\left(\tau\right)\right)} = \frac{\left(1+\lambda\right) t_a^{1+\lambda}}{\left(1+\lambda\right)\left(2+\lambda\right) e^{(1+\lambda)\beta} t_{00}^{1+\lambda}\left(\sigma_0\right)}. \tag{15}$$

Quantity t_{00}, included here, is independent of time t, but depends on the coordinate y. Therefore for this quantity, the formula (13) can be used,

$$t_{00}\left(\sigma_0\right) = t_{0s} \exp\left[\beta\left(1-\frac{\sigma_0}{\sigma_{0s}}\right)\right]. \tag{16}$$

At that $\sigma_{os} = const$ is some stress from turn-down of σ_0 with respect to y, $t_{os} = const$ is time to failure at $\sigma_0 = \sigma_{0s}$. Subject to (14), relation (15) is transforms

$$J_1\left(t_a\right) = \frac{\left(3+\lambda\right) t_a^{1+\lambda} \exp\left[\left(1+\lambda\right)\beta\frac{\sigma_0}{\sigma_s}\right]}{\left(1+\lambda\right)\left(2+\lambda\right) t_{0s}^{1+\lambda}\left(\sigma_{0s}\right) \exp\left[2\left(1+\lambda\right)\beta\right]} \tag{17}$$

Taking into account (17) at $t_a = t'$ in damage condition (9), for time $t'\left(0, \frac{y}{a}\right)$ preceding to damage of points of the considered plate at $x = 0$, we obtain

$$t_0'\left(0, \frac{y}{a}\right) = t_{0s}\left(\sigma_{os}\right)\ A \left(\frac{2+\lambda}{3+\lambda}\right)^{1/(1+\lambda)} \exp\left[\beta\left(2-\frac{\sigma_0}{\sigma_{os}}\right)\right]. \tag{18}$$

Using (17) at $t_a = t_*$ long term strength condition (10) defines time to failure of points which lie on the narrowest section of the plate:

$$t_*\left(0, \frac{y}{a}\right) = t_{0s}\left(\sigma_{0s}\right) \left(\frac{2+\lambda}{3+\lambda}\right)^{1/(1+\lambda)} \exp\left[\beta\left(2-\frac{\sigma_0}{\sigma_{0s}}\right)\right]. \tag{19}$$

In relations (18) and (19) σ_0 is defined by formula (14).

Graphic of dependencies $\ln\frac{t'}{t_{os}}\tilde{}\frac{y}{a}$ (dotted lines) and $\ln\frac{t_*}{t_{os}}\tilde{}\frac{y}{a}$ (firm lines) constructed in accordance with formulae (18) and (19) subject to (14), (4), (5) are represented in fig.1. At that the following numerical data are used: $\frac{a}{\rho} = 4; 25$; $\lambda = -0,3; \beta = 1,8; A = 0,4; T = T_0 = const; \frac{p_0}{\sigma_{os}} = \frac{1}{3}$. These graphics show that failure process starts at the bottom of recess and occurs intensively, but when approaching to center - to the point $(x = 0,\ y = 0)$ this process becomes stabilized. Since in this problem point $(0, a)$ - bottom of recess is of great interest, hence, in fig.2 damage curve (dotted line) and durability (firm line) of point $x = 0, y = a$ depending on value $\frac{a}{\rho}$ are represented. Note that the case $\frac{a}{\rho} = 0$ corresponds to the plate without recess. To construct the curves represented in fig.2 we used the mentioned above numerical data. It is clear from fig.2 that durability decreases as ration $\frac{a}{\rho}$ increases.

b) suppose now $p(t) = p_0 \left(\frac{t}{t_a}\right)^{\gamma}$ where $\gamma = const > 0$, $t \in [0, t_a]$, $t' \le t_a \le t_*$, $p_0 = const$. At that formula (6) will be written in the form

$$\sigma_t\left(0, \frac{y}{a}\right) = \sqrt{3}\Phi\left(\frac{a}{\rho}\right) F\left(\frac{a}{\rho}, \frac{y}{a}\right) P_0 \left(\frac{t}{t_a}\right)^{\gamma}. \tag{20}$$

In this case it is expedient to use approximation (12) at $T = T_0 = const$:

$$t_0 = t_{os}\left(\frac{\sigma_t}{\sigma_{os}}\right)^{-\alpha}, \ (\alpha > 0) \tag{21}$$

where $\sigma_{0s} = const$ is stress reduction from turn down σ_t, t_{os} - is time to failure at $\sigma_t = \sigma_{0s}$; $\alpha = const$.

By using (20) and (21) the following relation holds.

$$J_2(t_a) = \int_0^{t_a} \frac{(t_a - \tau)^{\lambda} d\tau}{t_0^{1+\lambda}(\sigma_t(\tau))} = 3^{\frac{\alpha}{2}}\left(\frac{t_a}{t_{os}}\right)\left(\frac{\Phi\left(\frac{a}{\rho}\right) F\left(\frac{a}{\rho}, \frac{y}{a}\right) p_0}{\sigma_s}\right)^{\alpha} \times$$

$$\times B(1 + \alpha\gamma; 1 + \lambda), \tag{22}$$

where $B(1 + \alpha\gamma; 1 + \lambda)$ is betta-function which can be expressed by gamma-function Γ in the following form $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

We will use relation (22) at $t_a = t'$ in damage condition (9). At that for defining time $t'\left(0, \frac{y}{a}\right)$ preceding to damage of point $x = 0$ at loading of plate by the programm mentioned in point b):

$$t'\left(0, \frac{y}{a}\right) = \frac{A\left[\Phi\left(\frac{a}{\rho}\right) F\left(\frac{a}{\rho}, \frac{y}{a}\right)\frac{p_0}{p_s}\right]^{-\frac{\alpha}{1+\lambda}}}{\left[3^{\frac{\alpha}{2}}(1+\lambda) B(1 + \alpha\gamma; 1 + \lambda)\right]^{1/(1+\lambda)}} t_{os}. \tag{23}$$

Taking into account (22) at $t_a = t_*$long term strength condition (10) we define time to failure of points which are on the narrowest section of the plate

$$t_*\left(0, \frac{y}{a}\right) = \frac{\left[\Phi\left(\frac{a}{\rho}\right) F\left(\frac{a}{\rho}, \frac{y}{a}\right)\frac{p_0}{\sigma_s}\right]^{-\frac{\alpha}{1+\lambda}}}{\left[3^{\frac{\alpha}{2}}(1+\lambda) B(1 + \alpha\gamma; 1 + \lambda)\right]^{1/(1+\lambda)}} t_{os}. \tag{24}$$

Damage curves of points of the narrowest section of the plate constructed in accordance with analytic formula (24) for values $\gamma = 0; \frac{1}{2}; 1; 2$ are represented in fig.3. Damage curves of the weak point $(0, a)$ in terms of fracture are represented in fig.4. To construct durability curves we also used formula (24) at $F\left(\frac{a}{\rho}, 1\right) = 1$. Curves in fig.3 correspond to values $\frac{a}{\rho} = 4$; $\frac{p_0}{\sigma_s} = \frac{1}{3}$; $\alpha = 2, \lambda = -0,3$. And curves in fig.4 are constructed at data $\frac{y}{a} = 1$; $\frac{p_0}{\sigma_s} = \frac{1}{3}$; $\alpha = 2$; $\lambda = -0,3$. In these figures stress $p = p_0 = const$ corresponds to value. It is clear from fig.3 and 4 that durability increases as parameter γ increase and it decreases as value $\frac{a}{\rho}$ increases.

Note that using formula (23) we can construct damage curves $t'\left(0,\frac{y}{a}\right)$. Moreover, note that comparison of formula (23) and (24) gives relation defining the formula of time $t'\left(0,\frac{y}{a}\right)$ by durability: $t\left(0,\frac{y}{a}\right)=At_*\left(0,\frac{y}{a}\right)$ where $0\le A<1$.

Finally, note that analytic formulae obtained in this paper for defining time preceding to damage and also durability, allow to find these quantities also for other values of parameters of materials.

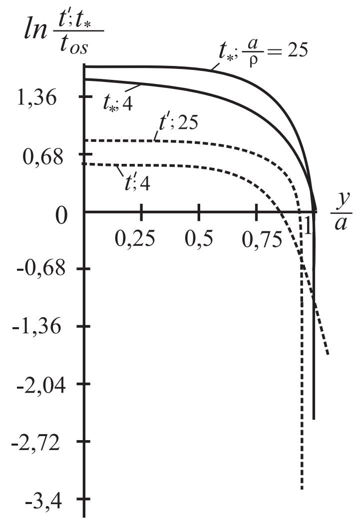

Fig.1. Damage curves (dotted lines) and durability curves (firm lines) of points of the narrowest section of the plate at loading by program $p\left(t\right)=p_0\ln\left(1+\frac{t}{t_a}\right)$; $0\le t\le t_a$; $t'\le t_a\le t_*$; $p_0=const$.

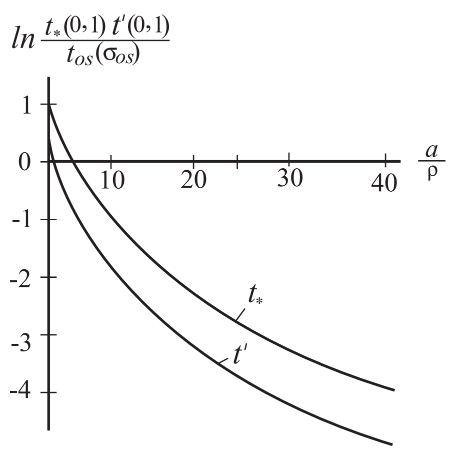

Fig.2. Damage curves (dotted lines) and durability curve (firm lines) of point $x=0,\ y=a$ (bottom of recess) of the plate loading by program $p\left(t\right)=p_0\ln\left(1+\frac{t}{t_a}\right)$; $0\le t\le t_a$; $t'\le t_a\le t_*$; $p_0=const$.

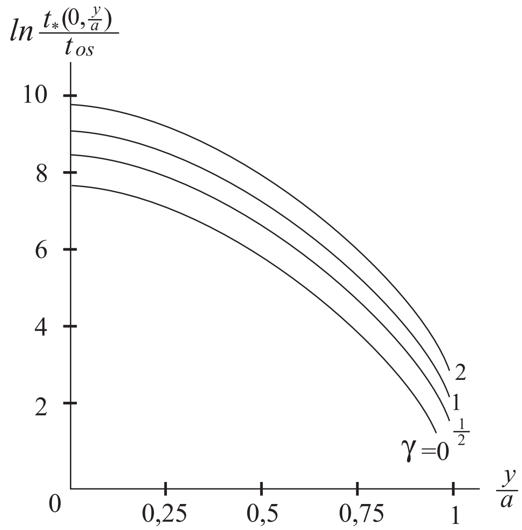

Fig.3. Durability curves of points of the narrowest section of plate at loading by program $p\,(t) = p_0 \left(\frac{t}{t_a}\right)^{\gamma}; \gamma > 0;\ 0 \le t \le t_a;\ t' \le t_a \le t_*;\ p_0 = const;\ \frac{a}{\rho} = 4.$

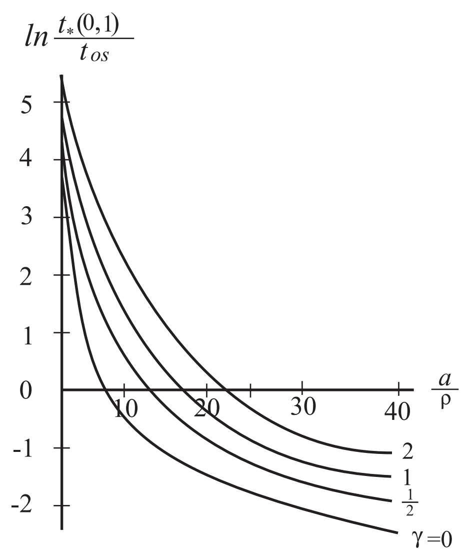

Fig.4. Durability curves of point $x = 0,\ y = a$ of the plate at loading by program $p\,(t) = p_0 \left(\frac{t}{t_a}\right)^{\gamma};\ \gamma > 0; 0 \le t \le t_a;\ t' \le t_a \le t_*;\ p_0 = const.$

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