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OSCILLATIONS OF A NON-HOMOGENEOUS BEAM FROM THE MOTION OF CARGO ALLOWING FOR THE INFLUENCE OF NON-HOMOGENEOUS RESISTANCE

Abstract

The problem about forced oscillation in the case of cargo motion on non-homogeneous elastic beam allowing for resistance of the non-homogeneous basis is considered. It is supposed, that an elasticity module is a continuous function of coordinate of the length and height of the beam. The problem is solved by the method of separation of variables, with further use the Bubnov-Galerkin's method. The calculation has been done at concrete values of non-homogenity.

The results of calculation are represented in the form of graph of dependence between characteristic values.

As is known, the question about action of a mobile cargo on a beam is one of important problems in the engineering practice in connection with construction of a different kind of bridges, tunnels and buildings. The first research on this question belongs to Villis [1].

As in the last years in different branches of modern technics and construction the materials, having a different kind of non-homogeneity are widely used and there is a necessity of the account of influence of environment, then there is a question of solution of problems of stability, oscillations and etc. structural elements in view of the above-stated specific singularities.

For such materials one of the important factors is, that the elasticity module E and specific density - "ρ" and coefficient of Poisson may be functions of the coordinates of body points. It is necessary to note, that the account of the above-stated singularities much complicates a mathematical solution of problems, but if we ignore them this may lead to essential errors. If we to add to here and influences of the resistance of an environment (elastic, inelastic, non-homogeneous, non-linear) then the analysis of frequency-amplitude characteristics is doubly complicated. In this connection by solving the above-stated and some other problems approximate-analytical methods are applied or they are solved by numerical methods by using computer.

At solving the given problem, we'll suppose, that the elasticity module and specific density are functions of coordinate of length and heights of a rod. (fig.1)

$$E = E_0 f_1(x) f_2(z); \quad \rho = \rho_0 \psi_1(x) \psi_2(z); \tag{1}$$

here E_0 and ρ_0 correspond to the homogeneous material. The function $f_1(x)$ with its derivatives, and $\psi_1(x)$, $f_1(z)$ and $f_2(z)$ are continuous functions

Let's consider a problem on a cargo moving on a beam with constant speed, and the mass of a cargo rather is less than the mass of a beam.

Let's denote a speed of cargo by V.

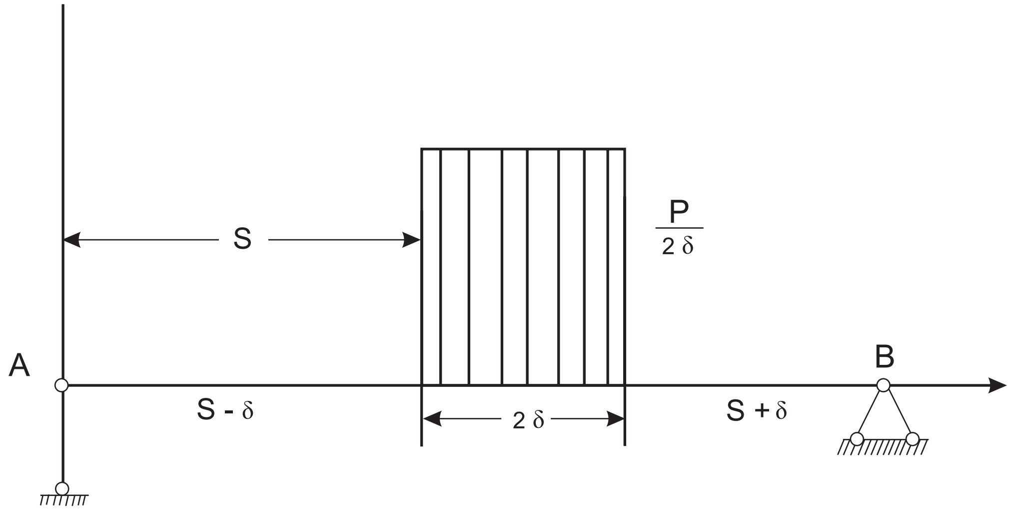

Fig. 1.

Let at $t = 0$ a cargo with weight P enters into the beam. Then at moment t it will be at a distance $S = V \cdot t$ from the support A. It is necessary to note, that unlike the homogeneous medium, in the given case at oscillation a neutral axis doesn't coincide with a central line (even in the case, when a cross-section has two axes of symmetry).

Using conditions of absence of the axial force it is easy to determine, that a motion equation in view of resistance of the non-homogeneous medium takes the following form:

$$\frac{\partial}{\partial x}\left[f_1(x)\frac{\partial^2 W}{\partial x^2}\right] - \tilde{k}\left(1+\varepsilon\varphi(x)\right)W + m^2\psi_1(x)\frac{\partial^2 W}{\partial t^2} = P(x,t) \tag{2}$$

Here W is deflection,

$$\tilde{k} = k\,(E_0 J)^{-1}; \qquad m_0^2 = \tilde{\rho}\ \ (E_0 J)^{-1};$$

$$\tilde{\rho} = \frac{\rho_0}{2n}\int_{-h}^{+h}\psi_2(z)\,dz\ ;$$

$$J = \int_{-h}^{+h} f_2(z)\,z^2 dz\ ; \tag{3}$$

$$a_1 = \int_{-h}^{+h} b(z)\,f(z)\,dz\ ; \quad a_2 = \int_{-h}^{+h} b(z)\,f(z)\,z dz\ ;$$

$$a_3 = \int_{-h}^{+h} b(z)\,f(z)\,z^2 dz\ ;$$

Here k is a coefficient of a linear resistance; $0 \le \varepsilon \le 1$, $\varphi(x)$ is a continuous function, $P(x,t)$ expresses the action of cargo P at moment t at the point x.

For giving $P(x,t)$ the extended form, we'll expand it in a Fourier series by sines supposing that a cargo is uniformly distributed at small region of length 2δ from $S-\delta$ to $S+\delta$ (fig.1).

Then we'll have the following dependence for

$$at \quad x < S-\delta; \quad P(x,t) = 0$$

$$at \quad S-\delta \le x \le S+\delta; \quad P(x,t) = \frac{P_0}{2\delta} \tag{4}$$

$$at \quad S+\delta < x; \quad P(x,t) = 0$$

Fourier coefficients will be expressed by the following way:

$$P(x,t) = \sum_n P_n(t) \sin \frac{n\pi}{l} x \tag{5}$$

It is easy to show, that $P_n(t)$ is defined by the following formula:

$$P_n(t) = \frac{2P}{l} \sin \frac{n\pi V}{l} t \tag{6}$$

Then (5) is defined in the following form:

$$P(x,t) = \frac{2P_0}{l} \sum_{n=1}^{\infty} \sin \frac{n\pi V}{l} t \sin \frac{n\pi}{l} x \tag{7}$$

Putting (7) in (2) we'll obtain:

$$\begin{gathered} \frac{\partial^2}{\partial x^2}\left[f_1(x)\frac{\partial^2 W}{\partial x^2}\right] - \tilde{k}(1+\varepsilon\varphi(x))W + \\ +m^2\psi_1(x)\frac{\partial^2 W}{\partial t^2} = \frac{2P_0}{Jl}\sum_{n=1}^{\infty}\sin\frac{n\pi V}{l}t\sin\frac{n\pi}{l}x \end{gathered} \tag{8}$$

Having kept in (8) only one term by sines, we'll obtain:

$$\begin{gathered} f_1(x)\frac{\partial^4 W}{\partial x^4} + 2f_1''(x)\frac{d^3 W}{\partial x^3} + f_1'(x)\frac{d^2 W}{dx^2} - \\ -\tilde{k}(1+\varepsilon\varphi(x))W - m^2\psi_1(x)\frac{\partial^2 W}{\partial t^2} = \\ = \frac{2P_0}{Jl}\sin\frac{n\pi V}{l}t\sin\frac{n\pi}{l}x \end{gathered} \tag{9}$$

We'll search solution (9) in the following form:

$$W(x,t) = V(x)\sin\beta t \quad \left(\beta = \frac{m\pi V}{l}\right) \tag{10}$$

Putting (10) in (9) we'll obtain

$$\begin{gathered} f_1(x)\frac{\partial^4 W}{\partial x^4} + 2f_1''(x)\frac{d^3 V}{\partial x^3} + f_1'(x)\frac{d^2 V}{dx^2} - \\ -\tilde{k}(1+\varepsilon\varphi(x))V - m_0^2\psi_1(x)\frac{\partial^2 V}{\partial t^2} = \frac{2P_0}{Jl}\sin\frac{n\pi}{l}x \end{gathered} \tag{11}$$

Solution (11) can be constructed by one of approximate-analytical, or a numerical method.

In the given case we'll apply the Bubnov-Galerkin's method, at that we'll search $V(x)$ in the following form:

$$V(x) = \sum_{i=1}^{m} C_m \theta_m (x) \tag{12}$$

Here each θ_m satisfies the corresponding boundary conditions.

Using above-indicated module in view of (11) and (12) we'll obtain:

$$\sum_{i=1}^{m} C_i \int_0^l \left[f_1(x) \frac{d^4\theta_i}{dx^4} + 2f_1''(x)\frac{d^3\theta_i}{dx^3} + f_1'(x)\frac{d^2\theta_i}{dx^2} \right] \theta_k dx - m_0^2 \beta^2 \ . \tag{13}$$

$$\sum_{i=1}^{m} C_i \int_0^l \left[\psi_i(x)\theta_i(x) \right] \theta_k(x)\,dx -$$

$$-\tilde{k}\left(\sum_{i=1}^{m} C_i \int_0^l \left[1+\varepsilon\varphi(x) \right] \theta_i(x)\,\theta_k(x)\,dx \right) = 0$$

$$k = 1, 2, ...$$

It would be desirable to note, that from equation (13) it is possible to obtain the solutions of the following analogous problems:

a) $\varphi(x) = 0$, linear elastic resistance,

b) solution of a problem without resistance of environment $k = 0$,

c) solution of a problem for homogeneous case

$$f_1(x) = 1; \quad f_2(z) = 1; \quad \psi_1(x) = 1; \quad \psi_2(z) = 1$$

For double-sided pinning of a function $\theta_i(x)$ we'll choose the following form:

$$\theta_i(x) = \sin \frac{i\pi}{l} x \tag{14}$$

For simplicity of an analysis we'll confine to one term (14). Then we'll obtain:

$$\beta^2 = \frac{1}{m^2} \frac{\int_0^l \left\{ \left[\frac{m\pi}{l}\right]^4 \left[f_1(x)\sin^2\frac{m\pi}{l}x \right] - 2\left(\frac{m\pi}{l}\right)^3 \left(\sin^2\frac{m\pi}{l}x\right) f_1'(x) - f''(x)\left(\frac{m\pi}{l}\right)^2 \times \right.}{\int_0^l \left(\psi_1(x)\sin^2\frac{m\pi}{l}x\right)dx}$$

$$\frac{\left. \times \sin^2\frac{m\pi x}{l} \right] dx}{\int_0^l \left(\psi_1(x)\sin^2\frac{m\pi}{l}x\right)dx} - \frac{\tilde{k}}{m_0^2}\frac{\int_0^l (1+\varepsilon\varphi(x))\sin^2\frac{m\pi}{l}x dx}{\int_0^l \left(\psi_1(x)\sin^2\frac{m\pi}{l}x\right)dx} -$$

$$-\frac{2P_0}{Jlm_0^2}\frac{\int\limits_0^l \sin^2\frac{m\pi}{l}xdx}{\int\limits_0^l\left(\psi_1(x)\sin^2\frac{m\pi}{l}x\right)dx} \tag{15}$$

From formula (15) at $f_1(x)=1$ and $\psi_1(x)=1$ we'll obtain a solution of the analogous problem for the case, when a beam is made of a non-homogeneous by thickness material, i.e.

$$\beta_1^2=\frac{1}{m_0^2}\frac{\int\limits_0^l\left(\frac{m\pi}{l}\right)^4\sin^2\left(\frac{m\pi}{l}x\right)dx}{\int\limits_0^l\sin^2\left(\frac{m\pi}{l}\right)^2dx}-\tilde{k}\frac{\int\limits_0^l(1+\varepsilon\varphi(x))\sin^2\frac{m\pi}{l}xdx}{\int\limits_0^l\sin^2\frac{m\pi}{l}xdx}-\frac{2P_0}{m_0^2l}$$

or

$$\beta_1^2=\frac{1}{m_0^2}\left(\frac{m\pi}{l}\right)^4-\frac{2P_0}{m_0^2l}-\frac{k}{m^2}\left(1+\frac{\int\limits_0^l\varphi(x)\sin^2\frac{m\pi}{l}dx}{\int\limits_0^l\psi_1(x)\sin^2\frac{m\pi}{l}xdx}\right)$$

At linear resistance we have

$$\beta_2^2=\frac{1}{Jm_0^2}\left(\frac{m\pi}{l}\right)^4-\frac{2P_0}{m_0^2lJ}-\frac{k}{E_0Jm_0^2}$$

Ignoring the resistance, we'll obtain

$$\beta_3^2=\frac{1}{m_0^2}\left[\left(\frac{m\pi}{l}\right)^4-\frac{2P_0}{JE_0l}\right]$$

or

$$\left(\frac{m\pi}{l}\right)^2V^2=\frac{1}{m_0^2}\left[\left(\frac{m\pi}{l}\right)^4-\frac{2P_0}{JE_0l}\right]$$

So, giving concrete values of the function, which characterize a non-homogeneity of the parameters of external resistance, it is easy to find a connection between partial oscillation and speed of cargo's motion.

The calculation was carried out for the cases:

$$f_1(x)=1+\mu_0xl^{-1};\qquad \varphi_1=1+\mu_2xl^{-1}$$

$$f_2(z)=1+\mu_1e^{zh^{-1}};\qquad \psi_2=1+\mu_3e^{zh^{-1}}$$

$$f(\rho)=1+\alpha_1\rho+\alpha_2\rho^2;\quad \psi(\rho)=1+\beta_1\rho+\beta_2\rho^2$$

The results of calculation have been represented in the form of table and graphs of dependence between characteristic parameters.

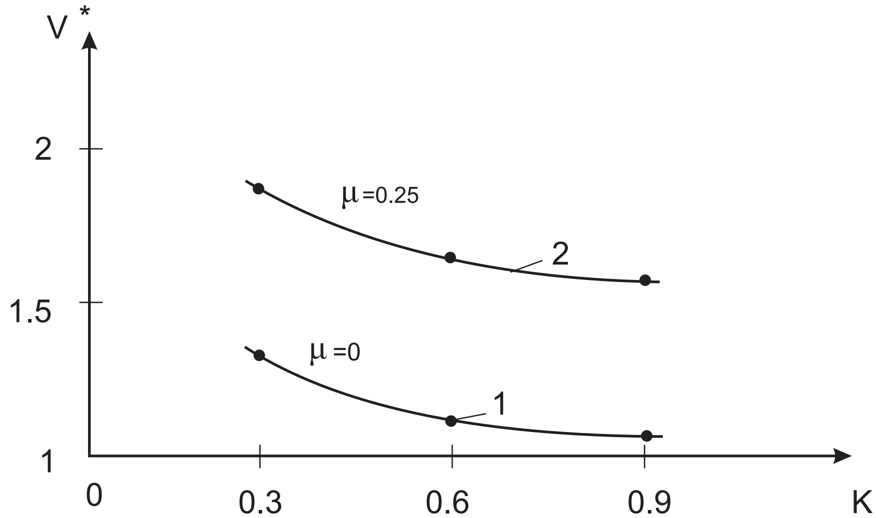

Fig.2

1) $f = 1 + k\rho; \quad \psi = 1 + \mu e^{\rho}$
2) $f = 1 + ke^{\rho}; \quad \psi = 1 + \mu e^{\rho}$

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