

ON EXAMPLES OF NONEQUIVALENCE OF WEIGHT ESTIMATES FOR RIESZ POTENTIALS AND EMBEDDING THEOREMS

Abstract

In this paper the problems on equivalence of Sobolev type inequality

$$\|u\|_{L^q_{v dx}(R^n)} \leq C \|\nabla u\|_{L^p_{\omega dx}(R^n)}, \quad \forall u \in C_0^\infty(R^n) \quad (1)$$

to the estimate

$$\|I_1(f)\|_{L^q_{v dx}(R^n)} \leq C \|f\|_{L^p_{\omega dx}(R^n)} \quad (2)$$

$\forall f \in L^p_{\omega dx}(R^n)$ *for the Riesz potentials*

$$I_\alpha(f) = \int_{R^n} |x-y|^{\alpha-n} f(y) dy \quad (\alpha = 1)$$

are studied.

It is shown that if $v \in RD$ (is inverse doubling), $\omega \in A_p$ (is a Makkenkhaupt class) and $q > p \geq 1$ or $v \in A_\infty$ is a Makkenkhaupt class and $q = p$, $\omega \in A_p$, then these two estimations are equivalent to the A_{pq} -Sawyer condition.

The examples on weight v, ω for which estimate (1) holds, but estimate (2) is broken for some function $f \in L^p_{\omega dx}(R^n)$ are also cited.