MECHANICS

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BOUNDARY CRACK IN I TYPE TWO-PLAY ORTHOTROPIC MATERIALS

Abstract

In this paper the boundary crack in I type two-ply orthotropic materials is investigated. The solution of this problem for I type orthotropic materials is reduced to the second kind Fredholm type integral equation and the stress intensity factor K_I is determined. The numerical analysis of the problem is given.

Let the band $x \in [0, H]$, $|y| < \infty$ composed of the first type two different elastic orthogonal materials is rigidly associated along plane $x = h < H$, $|y| < \infty$ contains at $y = 0$, $x \in [0, l < h]$ the boundary crack perpendicular to lateral surfaces $x = 0$, $|y| < \infty$ and $x = H$, $|y| < \infty$ (fig.1) free from stresses. The normal stress given to its surface is applied to the coast of the crack. The problem is plane and is assumed to be symmetric with respect to the plane $y = 0$, $x \in [0, H]$. At infinity $|y| \to \infty$ the stresses are absent, and displacements disappear.

The boundary conditions of the problem have the form

$$x = 0, \qquad |y| < \infty \qquad (\sigma_x)_1 = 0, \qquad (\tau_{xy})_1 = 0 \tag{1}$$

$$y = 0, \quad 0 \le x \le l \qquad (\sigma_y)_1 = 0, \qquad (\tau_{xy})_1 = 0 \tag{2}$$

$$y = 0, \quad l \le x \le h \qquad (\upsilon)_1 = 0, \qquad (\tau_{xy})_1 = 0 \tag{3}$$

$$y = 0, \quad h \le x \le H \qquad (\upsilon)_2 = 0, \qquad (\tau_{xy})_2 = 0 \tag{4}$$

$$\begin{aligned} x = h, \qquad |y| < \infty \qquad & (u)_1 = (u)_2\,, \qquad (\upsilon)_1 = (\upsilon)_2 \\ & (\tau_{xy})_1 = (\tau_{xy})_2\,, \ \ (\sigma_x)_1 = (\sigma_x)_2 \end{aligned} \tag{5}$$

$$x = H, \qquad |y| < \infty \qquad (\sigma_x)_2 = 0, \qquad (\tau_{xy})_2 = 0 \tag{6}$$

The conditions at infinity are the followings

$$0 \le x \le H, \quad |y| \to \infty \qquad (\sigma_x)_j\,, \quad (\sigma_y)_j\,, \ \ (\tau_{xy})_j \to 0. \tag{7}$$

$$(u)_j\,, (\upsilon)_j \sim O\left(r^{-\alpha}\right) \qquad \left(r = \sqrt{x^2 + y^2}, \quad \alpha > 0\right), \quad (j = 1, 2)$$

The condition in apex of the crack is

$$K_I = \lim_{x \to l+0} \left[\sqrt{2\pi\,(x - l)}\sigma\,(y)_1\,(x, 0)\right]. \tag{8}$$

Remark 1. In boundary conditions the notation is standard [1]. The function $p\,(x) \ge 0$, $\forall x \in [0, l]$ is the given continuous function. K_I is a stress intensity factor to be defined.

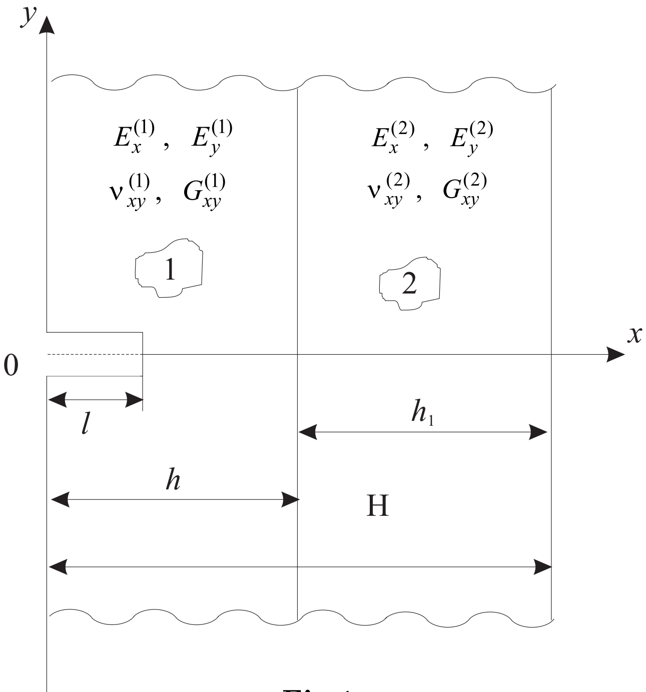

Fig.1.

Solution of boundary value problem for I type orthotropic materials in composite

By symmetry the solution of boundary value problem (1)-(8) is considered only in the domain $x \in [0, H]$, $y \geq 0$. In the paper [2] the general solutions of equilibrium equations in projections of displacement for the case of plane elasticity theory of I type orthotropic material layer with crack and without crack are constructed. We have [2]:

at $0 \leq x \leq h, \quad y \geq 0$ (the first medium)

$$(u)_1(x,y) = \sqrt{\frac{2}{\pi}}\int_0^\infty \left\{ A_0^{(1)}(t)\, e^{s_1 tx} + B_0^{(1)}(t)\, e^{-s_1 tx} + C_0^{(1)}(t)\, e^{s_2 tx} + \right.$$

$$\left. + D_0^{(1)}(t)\, e^{-s_2 tx} \right\} \cos ty dt + \sqrt{\frac{2}{\pi}}\int_0^\infty \left\{ B_1^{(1)}(\tau)\, e^{k_1 \tau y} + D_1^{(1)}(\tau)\, e^{-k_2 \tau y} \right\} \sin \tau x d\tau,$$

$$(\upsilon)_1(x,y) = \sqrt{\frac{2}{\pi}}\int_0^\infty \left\{ q_4^{(1)} \left[A_0^{(1)}(t)\, e^{s_1 tx} - B_0^{(1)}(t)\, e^{-s_1 tx} \right] + q_5^{(1)} \left[C_0^{(1)}(t)\, e^{s_2 tx} + \right. \right.$$

$$\left. \left. + D_0^{(1)}(t)\, e^{-s_2 tx} \right] \right\} \sin ty dt - \sqrt{\frac{2}{\pi}}\int_0^\infty \left\{ q_2^{(1)} B_1^{(1)}(\tau)\, e^{-k_1 \tau y} + q_9^{(1)} D_1^{(1)}(\tau)\, e^{-k_2 \tau y} \right\} \cos \tau x d\tau, \tag{9}$$

at $h \leq x \leq H$ (the second medium)

$$(u)_2(x,y) = \sqrt{\frac{2}{\pi}}\int_0^\infty \left\{ B_0^{(2)} \left[R_4^{(2)} e^{s_1^* t(x-H)} + e^{-s_1^* tx} + 2R_7^{(2)} e^{s_2^* t(x-H)} \right] + \right.$$

$$+D_0^{(2)}\left[R_6^{(2)}e^{s_1^*t(x-H)}+e^{-s_2^*tx}+R_5^{(2)}e^{s_2^*t(x-H)}\right]\Big\}\cos tydt,$$

$$(\upsilon)_2(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\Big\{B_0^{(2)}\left[q_4^{(2)}R_4^{(2)}e^{s_1^*t(x-H)}-q_4^{(2)}e^{-s_1^*tx}+2q_5^{(2)}R_7^{(2)}e^{s_2^*t(x-H)}\right]+$$

$$+D_0^{(2)}\left[q_4^{(2)}R_6^{(2)}e^{s_1^*t(x-H)}+q_5^{(2)}e^{-s_2^*tx}+q_5^{(2)}R_5^{(2)}e^{s_2^*t(x-H)}\right]\Big\}\sin tydt. \tag{10}$$

Here $q_4^{(j)},q_5^{(j)}\quad(j=1,2)\,,\quad R_4^{(2)},\;R_5^{(2)},\;R_6^{(2)},\;R_7^{(2)},\;q_8^{(1)},q_9^{(1)}$

Here

$$\varphi(x)=\frac{1}{q_{10}^{(1)}\left(\nu_{xy}^{(1)}+k_2q_9^{(1)}\right)-\left(\nu_{xy}^{(1)}+k_1q_8^{(1)}\right)}\times\left\{\frac{1-\nu_{yx}^{(1)}\nu_{xy}^{(1)}}{E_y^{(1)}}p(x)+\right.$$
$$+\sqrt{\frac{2}{\pi}}\int\limits_0^\infty t\left[\left(s_1\nu_{xy}^{(1)}+q_4^{(1)}\right)\left(A_0^{(1)}e^{s_1tx}-B_0^{(1)}e^{-s_1tx}\right)+\right.$$
$$\left.\left.+\left(s_2\nu_{xy}^{(1)}+q_5^{(1)}\right)\left(C_0^{(1)}e^{s_2tx}-D_0^{(1)}e^{-s_2tx}\right)\right]dt\right\}$$

$$q_{10}^{(1)}=\left(q_8^{(1)}-k_1\right)/\left(q_9^{(1)}-k_2\right).$$

The obtained dual integral equations are solved as in [2,5]. We define the stress intensity factor K_I.

With the help of (9) and (11) we find

$$(\sigma_y)_1(x,0)=\frac{E_y^{(1)}}{1-\nu_{xy}^{(1)}\nu_{yx}^{(1)}}\left\{\sqrt{\frac{2}{\pi}}\int\limits_0^\infty t\left[\left(s_1\nu_{xy}^{(1)}+q_4^{(1)}\right)\left(A_0^{(1)}e^{s_1tx}-B_0^{(1)}e^{-s_1tx}\right)+\right.\right.$$
$$\left.+\left(s_2\nu_{xy}^{(1)}+q_5^{(1)}\right)\left(C_0^{(1)}e^{s_2tx}-D_0^{(1)}e^{-s_2tx}\right)\right]dt+ \tag{14}$$
$$+\sqrt{\frac{2}{\pi}}\left[\left(\nu_{xy}^{(1)}+k_1q_8^{(1)}\right)-q_{10}^{(1)}\left(\nu_{xy}^{(1)}+k_2q_9^{(1)}\right)\right]\int\limits_0^\infty\tau B_1^{(1)}(\tau)\cos\tau x d\tau.$$

Let $l<x<h$. Consider the integral

$$\sqrt{\frac{2}{\pi}}\int\limits_0^\infty\tau B_1^{(1)}(\tau)\cos\tau x d\tau=\frac{d}{dx}\int\limits_0^l\xi\Psi(\xi)\int\limits_0^\infty J_0(\tau\xi)\sin\tau x d\tau d\xi=$$
$$=\frac{d}{dx}\int\limits_0^l\frac{\xi\Psi(\xi)}{\sqrt{x^2-l^2}}d\xi=-\frac{x}{\sqrt{x^2-l^2}}\Psi(l)+\Psi(0)+x\int\limits_0^l\frac{\Psi'(\xi)}{\sqrt{x^2-\xi^2}}d\xi. \tag{15}$$

$$\Psi'(\xi)=d\Psi/d\xi.$$

We recall that $\Psi(\xi)\in C^1(0,l)$.

With the help of (14), (15) and (8) finally we find the stress intensity factor

$$K_I=\sqrt{\pi l}\Psi(l)\frac{E_y^{(1)}}{1-\nu_{xy}^{(1)}\nu_{yx}^{(1)}}\left[q_{10}^{(1)}\left(\nu_{xy}^{(1)}+k_2q_9^{(1)}\right)-\left(\nu_{xy}^{(1)}+k_1q_8^{(1)}\right)\right].$$

The function $\Psi(l)$ is determined from Fredholm equation [2].

Analysis of solution

We note that

$$p_0(x)=p(x)D,\quad D=\frac{1-\nu_{xy}^{(1)}\nu_{yx}^{(1)}}{E_y^{(1)}\left[\left(q_{10}^{(1)}\nu_{xy}^{(1)}+k_2q_9^{(1)}\right)-\left(\nu_{xy}^{(1)}+k_1q_8^{(1)}\right)\right]}.$$

We introduce the function

$$\Psi\left(x\right) = \Psi_*\left(x\right) D. \tag{16}$$

Allowing for (16) we find

$$\Psi_*\left(x\right) = \frac{2}{\pi}\int\limits_0^l \frac{p\left(\tau\right)}{\sqrt{x^2-\tau^2}}d\tau \; + \int\limits_0^l \Psi_*\left(x\right) K\left(x,\xi\right) d\xi. \tag{17}$$

At that

$$K_I = \sqrt{\pi l}\Psi_*\left(x\right) \qquad \left(l < h\right). \tag{18}$$

Let $p\left(x\right) = \sigma \equiv const, \quad x \in \left(0, l\right)$. After some computation we find

$$K_I = \sigma\sqrt{\pi l}\Psi_{**}\left(x\right) \qquad \left(l < h\right).$$

$$\Psi_{**}\left(l\right) = 1 + \int\limits_9^l \Psi_{**}\left(x\right) K\left(x,\xi\right) d\xi. \tag{19}$$

The computxtional program for IBM in which the solution of the second kind Fredholm equation (19) is led to the solution of system of $N\times N$ order linear algebraic equations

$$\Psi_i = 1 + \tfrac{1}{N}\sum_{j=1}^{N}\Psi_j k_{ij} \tag{20}$$
$$\left(i = \overline{1,N},\; k_{ij} = k\left(s_i, x_j\right), \Psi_i = \Psi_{**}\left(x_i\right),\; 0 < x_4 < x_6 < ... < x_N = l\;\right)$$

is developed.

The numerical analysis shows that for obtaining the stable decimal digits in values $\Psi_{**}\left(l\right)$ it is sufficient that $N = 21$.

Remark. The results of the given problem as $H \to \infty$ and $E_x^{(1)} = E_x^{(2)},\; E_y^{(1)} = E_y^{(2)},\; G_{xy}^{(1)} = G_{xy}^{(2)}, \nu_{xy}^{(1)} = \nu_{xy}^{(2)},\; \nu_{yx}^{(1)} = \nu_{yx}^{(2)}$ coincide with the results of the paper [6] obtained by tha Wiener-Hopf method.

Let (see [6]):

$$\begin{array}{lcc} (I) & E_x^{(1)} = 3.5\times 10^6 psi & E_{xy}^{(0)} = 3.24\times 90^7 psi \\ & \nu_{yx}^{(1)} = 0.23 & G_{yx}^{(1)} = 1.23\times 10^6 psi \\ (II) & E_x^{(2)} = 1.6\times 10^5 psi & Z_y^{(2)} = 3.52\times 10^2 psi \\ & \nu_{yx}^{(1)} = 0.32 & G_{yx}^{(2)} = 2.9\times 10^5 psi. \end{array}$$

Let $2h = H$.

The numerical analysis shows (fig. 2)

- if the crack is in the medium denoted by the index I, then by increasing of l/h the correction function $\Psi_{**}\left(l\right)$ increases.

- If the cracj ij in the medium denoted by the index 2, then by increasing of l/h the correction function $\Psi_{**}\left(l\right)$ decreases.

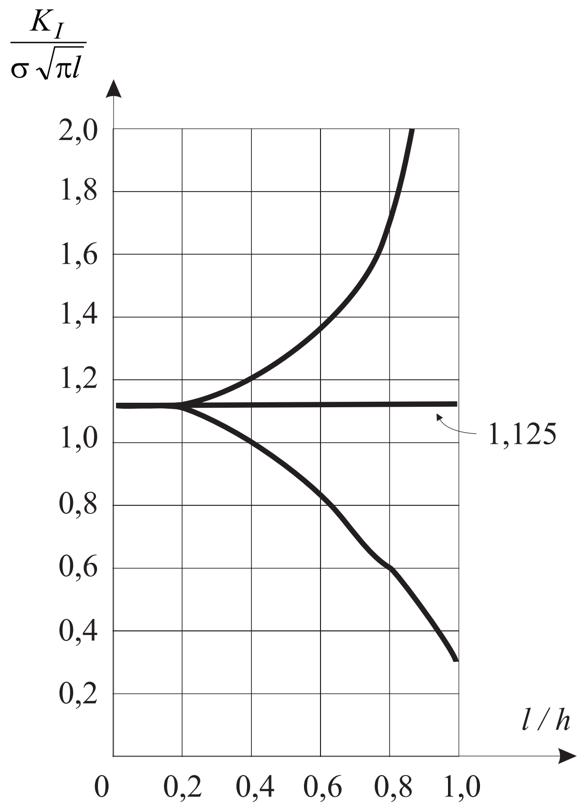

Fig.2.

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