Susanbar A. GIYASBEYLI

THE ASYMPTOTIC ANALYSIS OF EIGENFREQUENCIES OF OSCILLATIONS OF THE CYLINDRICAL ENVELOPES, STRENGTHENED BY TRANSVERSE RIBS AND FILLED WITH FLUID

Abstract

In the present work the asymptotic analysis of eigenfrequencies of oscillations of the envelope, strengthened by transverse ribs, filled with fluid has been done. The motion of envelope is described on base of the theory of constructively-orthotropic envelopes. The partial equations are obtained and with the help of asymptotic method the simple formulae for getting eigenfrequencies of the considered constructions have been deduced.

In the modern constructions and devices the ribbed cylindrical envelopes, filled with fluid are found. The given work is devoted to definition of the eigenfrequencies of oscillations of such constructions.

It is supposed that the envelope is simply supported on butt-ends. The theory of constructively-orthotropic envelopes is used for description of its motion. The fluid is designed as ideal and compressible.

On the base of asymptotic method the simplified formulae for calculation of eigenfrequencies of oscillations of constructively-orthotropic envelopes, filled with fluid are obtained.

The simultaneous equations of constructively-orthotropic envelope's motion, under the action of hydrodynamic load, according to [1] has the form:

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2}\right)u+\frac{1+\nu}{2}\frac{\partial^2\vartheta}{\partial\xi\partial\theta}-\nu\frac{\partial}{\partial\xi}w-\rho_1\frac{\partial^2 u}{\partial t_1^2}=0$$

$$\frac{1+\nu}{2}\frac{\partial^2 u}{\partial\xi\partial\theta}+\left\{\frac{1-\nu}{2}\left(1+\eta a^2\right)\frac{\partial^2}{\partial\xi^2}+\left[1+\left(1-\frac{h_s}{r}\right)^2\gamma_s^2+a^2\right]\frac{\partial^2}{\partial\theta^2}\right\}\vartheta+$$

$$+\left\{-\left[1+\left(1-\frac{h_s}{}\right.\right.\right.$$

where $\rho_1 = 1,\ \rho_2 = 1 + \bar{\rho}_s \bar{\gamma}_s = \rho_3,\ \ \bar{\gamma}_s^2 = \dfrac{F_s}{L_1 h}(1 + k_1)$ (L_1 is the length of envelope, F_s is an area of cross-section of ribs, k_1 is the number of transverse ribs), $\bar{\rho}_s = \dfrac{\rho_s}{\rho_0}$ ($\rho_0,\ \rho_s$ is the density of materials of envelope and ribs, respectively)

$$\delta_s^2 = \frac{h_s}{R}\bar{\gamma}_s^2,\ \ \eta_{s2}^2 = \frac{E_s\left(1-\nu^2\right)}{E}\eta_s^{-2},\ \ \eta_{s1}^2 = \frac{E_s J_{xs}\left(1-\nu^2\right)(1+k_1)}{E L_1 R^2 h},\ \ \bar{\eta}_s^2 = \left(\frac{h_s}{R}\right)^2 \bar{\gamma}_s^2,$$

$\delta_s^2 = \dfrac{h_s}{R}\bar{\gamma}_s^2,\ \ \gamma_s^2 = \dfrac{E_s\left(1-\nu^2\right)}{E}\bar{\gamma}_s^2$, $E,\ \nu$ are elasticity modulus and Poisson coefficient of the material of envelope, respectively, $R,\ h$ are radius and thickness of envelope, E_s is an elasticity modulus of the ribs material, $a^2 = \dfrac{h^2}{12R^2},\ \ \Delta = \dfrac{\partial^2}{\partial\xi^2} + \dfrac{\partial^2}{\partial\theta^2}$, J_{xs} is a moment of inertia of cross-section of the rib with regard to the axis OX, $\theta = \dfrac{y}{R}$, u, ϑ, w are components of displacements of middle surface of envelope, $t = \omega_0 t$, $\omega = \sqrt{\dfrac{E}{\left(1-\nu^2\right)\rho_0 R^2}}$, q_z is a pressure of fluid on envelope.

The linearized wave equation describing propagation of small perturbations in the ideal compressible fluid, has the form [2]:

$$\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0, \tag{2}$$

where Φ is potential, and c is a speed of sound distribution in fluid.

Equation of envelope motion (1) and fluid (2) is complemented by the contact conditions, which have the form:

$$\vartheta_r = \frac{\partial w}{\partial t},\quad q_z = -P. \tag{3}$$

The hydrodynamic pressure P and radial speed ϑ in fluid are defined by the following way [2]:

$$P = -\rho\frac{\partial\Phi}{\partial t},\ \ \vartheta_r = \frac{\partial\Phi}{\partial r} \tag{4}$$

where ρ is density of fluid.

Supplementing by contact conditions (3) the motion equation of envelope and fluid, we come to problem on eigen oscillations of supported envelope, filled with fluid. So, the problem on eigen oscillations of supported cylindrical envelope with fluid is reduced to joined integrating of the equations of the theory of envelopes and fluid at execution of indicated conditions on the surface of their contact.

The displacement of envelope we'll search in the form:

$$\begin{aligned} u &= u_0 \sin\chi\xi \cos n\theta \cos\omega_1 t_1, \\ \vartheta &= \vartheta_0 \cos\chi\xi \sin n\theta \cos\omega_1 t_1, \\ w &= w_0 \cos\chi\xi \cos n\theta \cos\omega_1 t_1. \end{aligned} \tag{5}$$

Here u_0, ϑ_0, w_0 are the unknown constants; $\chi = \dfrac{m\pi}{L}$ $(m = 1, 2, ...)$, m, n are wave numbers in lengthwise and circular directions, respectively, L is the length of envelope.

The potential of speeds Φ has the form:

$$\Phi = A\cos\chi\xi J_n(\gamma r)\cos n\theta\sin\omega_1 t_1, \tag{6}$$

where $\gamma^2 = -\chi^2 + \dfrac{\omega^2}{c^2}$, J_n are first kind n-th order Bessel's functions, A is an integration constant.

Using formulae (4) and contact conditions (3) for q_z it is possible to obtain:

$$q_z = \frac{\rho E\omega_1 J_n(\gamma R)}{(1-\nu^2)\rho_0\gamma J_n'(\gamma R)} w_0\cos\chi\xi\cos n\theta\cos\omega_1 t_1. \tag{7}$$

After substituting (5) in (1), taking into account (7), the problem is reduced to the third order homogeneous system of the linear algebraic equation, whose non-trivial solution is possible in case, when its determinant is equal to zero. In this case the equation with respect to ω_1 is transcendental, since it contains Bessel's fucntion. It can be written in the form:

$$a_1(\omega_1)\omega_1^6 + a_2(\omega_1)\omega_1^4 + a_3\omega_1^2 + a_4 = 0. \tag{8}$$

The coefficients $a_i(\omega_1)$ $(i = 1, 2, ..., 5)$ have cumbrous view, therefore we don't write out them here. Let us note, that they depend on above-stated geometrical and physical parameters, characterizing the system.

For next analysis in equation (8) it is necessary to distinguish the parameters, which substantially influence on the coefficients $a_i(\omega_1)$ $(i = 1, 2, ..., 4)$. Such parameters for the considered constructively-orthotropic envelope are: dimensionless bend rigidities of lengthwise supposed ribs, a^2, n, χ, and also dimensionless eccentricities of lengthwise ribs δ_c^1. Inasmuch as $\delta_c^1 \leq \sqrt{\eta_c^1}$, by analyzing the order of the coefficients $a_i(\omega_1)$ it is taken, that $\delta_c^1 \approx \sqrt{\eta_c^1}$. besides, for simplification of the partial equation it is taken the formulae for logarithmic derivatives of the Bessel's functions J_n for large n $(x << n)$ [2, 4]:

$$\frac{J_n'(x)}{J_n(x)} \approx \frac{n}{x} - \frac{x}{2n}. \tag{9}$$

Using formula (9), for ω_1^2 we'll obtain

$$\omega_1^2 = \frac{1}{n^2(\rho_2 n^2 + \rho_2 + \varphi_1)(1+\gamma_s^2)} \times$$
$$\left\{ \left[1+\gamma_s^2-\nu^2\right]\chi^4 + 2\nu n^2\left(n^2-1\right)\delta_s^2\chi^2 + n^4\left(n^2-1\right)^2\left[\eta_{s2}^2 + \right.\right.$$
$$\left.\left. + \left(a^2+\eta_{s1}^2\right)\left(1+\gamma_s^2\right)\right]\right\} \tag{10}$$

where $\varphi = \dfrac{\rho R}{\rho_0 h n^4}$.

We'll remark, that as $\rho = 0$ formula for ω_1^2 coincides with formula for ω_1^2, corresponding to oscillation of envelope without fluid [1]. From (10) it is clear, that account of influence of the fluid leads to decrease of the values of eigenfrequency of oscillations of supported envelope, in comparison with eigenfrequency of oscillations of supported envelope without fluid.

eferences

[1]. Amiro I.Ja., Zarutskii V.A. *The theory of rib envelopes.* Design methods of *envelopes.* Kiev: "Naukova Dumka", 1980, p.367. (Russian)

[2]. Yanke Ye., Emdeh F., Lesh F. *Special functions.* M.: "Nauka", 1977, p.342. (Russian)

[3]. Stejko A.V. *Bend oscillations of the cylindrical envelope with regard to elasticity of boundary transverse frames. Hydrodynamics and theory of elasticity.* 1974, issue 18, pp.142-147. (Russian)

[4]. Latifov F.S. *The oscillations of envelope with elastic and fluid medium.* Baku: "Elm", 1999, p.164. (Russian)

Susanbar A. Giyasbeyli
Baku State University.
23, Z.I.Khalilov str., AZ1148, Baku, Azerbaijan.
Tel.: (99412) 441 74 28 (off.)

Received January 29, 2004; Revised July 01, 2004.
Translated by Mamedova Sh.N.