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CRACK WITH BONDINGS BETWEEN FACES IN RIB REINFORCED

Abstract

Interaction of reinforcing rib on growth of crack with bondings between faces is investigated.

The equilibrium boundary value problem of crack with bondings between faces in rib reinforced plate in action of environmental tensile load is reduced to non-linear singular integrodifferential equation.

The normal forces in bondings is found from solution of this equation. The condition of limit equilibrium of crack with end zone is formulated subject to criterion of limit draft of bond.

Statement of problem.

We consider unbounded isotropic plate weakened by the rectilinear crack with $2l$ in length.

In the paper the model of crack is considered in the presence of domains in which the faces of crack interact.

Assume that these domains join to apex of cracks and their sizes are preunknown, can be congruent with size of crack. Interaction of crack faces in end domain is modeled by introduction bonding between crack faces having the given deformation curve.

The physical nature of such cohesive forces and size of end zones in which the interaction of crack faces is realized, depend on the form of material. Crack faces are free from external forces. The transversal stiffening rib at the points $z = \pm L \pm iy_0$ is fastened to the plate. The homogeneous tensile stress $\sigma_y^{\infty} = \sigma_0$ acts at infinity. Action of fastened reinforcing rib in calculation scheme is replaced by four concentrated force is unknown and is to be defined in the process of solution of the problem. We select a part of crack in length d (end zone) adjoint to its apex ($\lambda \leq |x| \leq l;\ y = 0;\ d = l - \lambda$) in which crack faces interact, so that this interaction keeps crack opening.

For the mathematical description of interaction of crack faces we assume that in end zones between crack faces the cracks have bonding (cohesive forces), where law deformation which is given. In general case it is nonlinear deformation law [1-3].

By action of environmental loads in bondings between cracks the force $q(x)$ having only normal component because of symmetry of problems with respect to abscissa axis, will arise. Since end zones are small as against the rest parts of reinforced plate, we can mentally delete it replacing it by section whose surfaces interact among themselves by some law corresponding to action of remote material.

Consequently, the normal stress $q(x)$ will be applied to crack faces in end zones. The quantity of these stresses is previously unknown and is to be defined in the process of solution of boundary-value problem of fracture mechanics.

The boundary conditions in the considered problem have the following form

$$\begin{aligned} &\sigma_y - i\tau_{xy} = 0 \quad at \quad y = 0, \quad |x| \leq \lambda \\ &\sigma_y - i\tau_{xy} = q(x) \quad at \quad y = 0, \quad \lambda < |x| \leq l \end{aligned} \tag{1}$$

The basic relation's of the posed problem must be complemented by equations connecting the crack opening displacement and binding force in bonding.

This equation, without losing generality, in the considered problem can be represented in the following form

$$\upsilon\left(x_1\right)=C\left(x_1,q\right)q\left(x_1\right),\quad \upsilon\left(x_1\right)=V^{+}\left(x_1,0\right)-V^{-}\left(x_1,0\right), \tag{2}$$

where $\upsilon\left(x_1\right)$ is crack opening in end zone, x_1 is affix of points of crack faces in end zone; we can consider the fucntion $C\left(x_1,q\right)$ as effective compliance bond depending on tension of bondings.

On the basis of Kolosov-Muskheleshvili formulae [4] and boundary conditions on crack faces, the problem is reduced to determination of two analytical functions $\Phi\left(z\right)$ and $\Psi\left(z\right)$ from the boundary conditions

$$\Phi\left(t\right)+\overline{\Phi\left(t\right)}+\bar{t}\Phi\left(t\right)+\Psi\left(t\right)=\begin{cases}0 \quad at \quad |x|\le\lambda\\ q\left(x\right) \quad at \quad \lambda<|x|\le l\end{cases} \tag{3}$$

Solution of boundary value problem.

We seek solution of boundary value problem (3) in the following form

$$\begin{aligned}\varphi\left(z\right)&=\varphi_0\left(z\right)+\varphi_1\left(z\right);\\ \psi\left(z\right)&=\psi_0\left(z\right)+\psi_1\left(z\right);\end{aligned} \tag{4}$$

where $\Phi\left(z\right)=\varphi'\left(z\right)$, $\Psi\left(z\right)=\psi'\left(z\right)$, $\varphi_0\left(z\right),\psi_0\left(z\right)$ define the stress and strain fields of intack reinforced plate. In this case, as $\varphi_0\left(z\right)$ and $\psi_0\left(z\right)$ it follows to take [4]

$$\varphi_0\left(z\right)=-\frac{1}{2\pi\left(1+æ\right)h}\sum_{k=1}^{4}\left(X_k+iY_k\right)\ln\left(z-z_k\right)+\frac{\sigma_0}{4}z;$$

$$\psi_0\left(z\right)=\frac{æ}{2\pi\left(1+æ\right)h}\sum_{k=1}^{4}\left(X_k-iY_k\right)\ln\left(z-z_k\right)+ \tag{5}$$

$$+\frac{1}{2\pi\left(1+æ\right)h}\sum_{k=1}^{4}\frac{\bar{z}_k\left(iY_k\right)}{z-z_k}+\frac{\sigma_0}{2}z$$

Here $\left(X_k,iY_k\right)$ are pin forces applied at the points z_k

$$\left(z_1=L+iy_0;\quad z_2=L-iy_0;\quad z_3=-L+iy_0;\quad z_4=-L-iy_0;\quad X_k=0;\right.$$

$$\left.Y_1=-P;\quad Y_2=P;\quad X_3=-P;\quad Y_4=P\right)$$

æ is a Muskhelishvili constant.

For determination of the analytical functions $\Phi_1\left(z\right)$ and $\Omega_1\left(z\right)=z\Phi_1\left(z\right)+\Psi_1\left(z\right)$ on the basis of (4)-(5) we obtain the following boundary value problem

$$\begin{aligned}&as\quad y=0,\quad |x|<\lambda\quad \Phi_1\left(z\right)+\overline{\Phi_1\left(z\right)}+\Omega_1\left(z\right)=f\left(x\right)\\ &as\quad y=0,\quad \lambda\le|x|\le\lambda+d\quad \Phi_1\left(z\right)+\overline{\Phi_1\left(z\right)}+\Omega_1\left(z\right)=q\left(x\right)+f\left(x\right),\end{aligned} \tag{6}$$

where $f(x) = -\left[\Phi_0(x) + \overline{\Phi_0(x)} + x\Phi_0(x) + \Psi_0(x)\right]$.

It follows to seek the solution of boundary value problem (6) in a class of everywhere bounded functions. Now, we note that by virtue of conditions with respect to the axis x, the fucntion $f(x)$ is real, therefore on the basis of (6) on whole real axis $\operatorname{Im}\Omega_1(z) = 0$. Consequently, allowing for the condition at infinity, we obtain

$$\Omega_1(z) = 0$$

Thus, on the basis of (6) for the fucntion $\Phi_1(z)$ we obtain the Dirichlet problem

$$\begin{gathered} as \quad y = 0, \quad |x| < \lambda \quad \operatorname{Re}\Phi_1(z) = \frac{1}{2} f(x); \\ as \quad y = 0, \quad \lambda \le |x| \le l \quad \operatorname{Re}\Phi_1(z) = \frac{1}{2}(q(x) + f(x)); \end{gathered} \tag{7}$$

as $z \to \infty \quad \Phi_1(z) \to 0$.

The following conjugation problem corresponds to boundary value problem (7)

$$\begin{gathered} \Phi_1^+(x) + \Phi_1^-(x) = f(x) \qquad -\lambda < x < \lambda \\ \Phi_1^+(x) + \Phi_1^-(x) = f(x) + q(x) \qquad \lambda < |x| < \lambda \end{gathered} \tag{8}$$

It is required to find a solution of (8), satisfying the condition

$$\bar{\Phi}_1(z) = \Phi_1(z)$$

The corresponding homogeneous problem has the form

$$\Phi_1^+(x) + \Phi_1^-(x) = 0 \qquad -l < x < l \tag{9}$$

We take the fucntion

$$X(z) = \sqrt{z^2 - l^2}$$

as a partial solution of homogeneous problem (9), implying that branch for which the equality-

$$X^+(x) = -X(x) \qquad on \quad |x| \le l \tag{10}$$

holds. On the basis of relations (10) we rewrite conjugation problem (9) as

$$\frac{\Phi_1^+(x)}{X^+(x)} - \frac{\Phi_1^+(x)}{X^-(x)} = 0 \qquad on \quad -l \le x \le l \tag{11}$$

From boundary condition it follows that the solution of homogeneous problem vanishing at infinity is equal to zero.

We represent homogeneous conjugation problem (8) in the following form

$$\frac{\Phi_1^+(x)}{X^+(x)} - \frac{\Phi_1^-(x)}{X^-(x)} = \frac{F(x)}{X^+(x)} \qquad on \quad -l \le x \le l \tag{12}$$

Denote by

$$\Phi_*(z) = \Phi_1(z)/X(z); \quad F_*(x) = F(x)/X^+(x),$$

then boundary condition (12) has the form

$$\Phi_*^+ (z) - \Phi_*^- (z) = F_* (x) \quad on \quad -l < x < l$$

Here

$$F_* (x) = \frac{f (x)}{\sqrt{x^2 - l^2}} \quad as \quad |x| < \lambda \tag{13}$$

$$F_* (x) = \frac{q (x) + f (x)}{\sqrt{x^2 - l^2}} \quad as \quad \lambda \leq x \leq l$$

The desired solution of problem (8) is written as

$$\Phi_1 (z) = \frac{\sqrt{z^2 - l^2}}{2\pi i} \int_{-l}^{l} \frac{F_* (x) \, dx}{x - z} \tag{14}$$

According to behaviour of the fucntion $\Phi_1 (z)$ at infinity the solvability condition of boundary value problem has the form

$$\int_{-l}^{l} \frac{F_* (x) \, dx}{\sqrt{l^2 - x^2}} + \int_{-l}^{-\lambda} \frac{q (x) \, dx}{\sqrt{l^2 - x^2}} + \int_{\lambda}^{l} \frac{q (x) \, dx}{\sqrt{l^2 - x^2}} = 0 \tag{15}$$

This relation serves for determination the size of end zone. Now we compute the integrals in (14)-(15). We represent integral (14) in the following form

$$\Phi_1 (z) == \frac{\sqrt{z^2 - l^2}}{2\pi i} \left\{ \int_{-l}^{l} \frac{f (x) \, dx}{\sqrt{x^2 - l^2} (x - z)} + \right.$$

$$\left. + \int_{-l}^{-\lambda} \frac{q (x) \, dx}{\sqrt{x^2 - l^2} (x - z)} + \int_{\lambda}^{l} \frac{q (x) \, dx}{\sqrt{x^2 - l^2} (x - z)} \right\} \tag{16}$$

For computation of the first integral in braces we use calculus of residues (see formula (3) §70 [4]).

We shall have

$$\int_{-l}^{l} \frac{f (x) \, dx}{\sqrt{x^2 - l^2} (x - z)} =$$

$$= \pi i \left\{ \frac{f (z)}{\sqrt{z^2 - l^2}} - G_\infty (z) - G_1 (z) - G_2 (z) - G_3 (z) - G_4 (z) \right\} \tag{17}$$

Here $G_\infty (z), G_1 (z), G_2 (z), G_3 (z), G_4 (z)$ are leading parts of the functions $f (z) / \sqrt{z^2 - l^2}$ at the points $z = \infty, z_1, z_2, z_3, z_4$, respectively,

$$f (x) = -\frac{P y_0}{\pi h \left[y_0^2 + (x - L)^2 \right]} \left[\frac{3 + \nu}{2} - (1 + \nu) \frac{(x - L)^2}{y_0^2 + (x - L)^2} \right] -$$

$$-\frac{Py_0}{\pi h\left[y_0^2+(x-L)^2\right]}\left[\frac{3+\nu}{2}-(1+\nu)\frac{(x+L)^2}{y_0^2+(x+L)^2}\right]+\sigma_0 \qquad (18)$$

After defining the leading parts of the fucntion $f(z)/\sqrt{z^2-l^2}$ in the poles z_1, z_2, z_3, z_4 and $z=\infty$ the solution of the Dirichlet problem for the fucntion $\Phi_1(z)$ we rewrite as

$$\Phi_1(z)=-\frac{Py_0}{2\pi h\left[y_0^2+(z-L)^2\right]}\left[\frac{3+\nu}{2}-\frac{(1+\nu)(z-L)^2}{y_0^2+(z-L)^2}\right]-$$

$$-\frac{Py_0}{2\pi h\left[y_0^2+(z+L)^2\right]}\left[\frac{3+\nu}{2}-(1+\nu)\frac{(z+L)^2}{y_0^2+(z+L)^2}\right]+\frac{\sigma_0}{2}+$$

$$-\frac{Py_0}{2\pi h}\left\{\frac{\sqrt{z^2-l^2}}{\sqrt{z_1^2-l^2}}\left[\frac{-i}{2y_0(z-z_1)}-\frac{1}{4}\frac{(1+\nu)z_1\left(z_1^2-l^2\right)}{(z-z_1)}-\frac{(1+\nu)}{4(z-z_1)^2}\right]+\right.$$

$$+\frac{\sqrt{z^2-l^2}}{\sqrt{z_2^2-l^2}}\left[\frac{i}{2y_0(z-z_2)}-\frac{1}{4}\frac{(1+\nu)z_2\left(z_2^2-l^2\right)}{(z-z_2)}-\frac{(1+\nu)}{4(z-z_2)^2}\right]+$$

$$+\frac{\sqrt{z^2-l^2}}{\sqrt{z_3^2-l^2}}\left[\frac{-i}{2y_0(z-z_3)}-\frac{1}{4}\frac{(1+\nu)z_3\left(z_3^2-l^2\right)}{(z-z_3)}-\frac{(1+\nu)}{4(z-z_3)^2}\right]+$$

$$\left.+\frac{\sqrt{z^2-l^2}}{\sqrt{z_4^2-l^2}}\left[\frac{i}{2y_0(z-z_4)}-\frac{1}{4}\frac{(1+\nu)z_4\left(z_4^2-l^2\right)}{(z-z_4)}-\frac{(1+\nu)}{4(z-z_4)^2}\right]\right\}+I_1; \qquad (19)$$

$$I_1=\int_{-l}^{-\lambda}\frac{q(x)\,dx}{\sqrt{x^2-l^2}\,(x-z)}+\int_{\lambda}^{l}\frac{q(x)\,dx}{\sqrt{x^2-l^2}\,(x-z)};$$

Here under the functions $\sqrt{l^2-z^2}$ it is implied that branch which at large $|z|$ has the form $\sqrt{l^2-z^2}=i\sqrt{z^2-l^2}=iz\left(1-l^2/2z^2+...\right)$.

The solvability condition of boundary value problem (15) which serves for defining the length of end zone, after integration gets the form:

$$\sigma_0\pi-\frac{Py_0}{h}\left[\frac{(1+æ)L}{4A\sqrt{A-B+2L^2}}+\frac{3y_0^2(7-æ)L^3}{A^2(A+B-2L^2)\sqrt{A-B+2L^2}}+\right.$$

$$\left.+\frac{(7-æ)L\left(2B-A-4L^2\right)}{2A^2\sqrt{A-B+2L^2}}+\frac{y_0^2(7-æ)L(B+A)\left(2B-A-4L^2\right)}{4A^3(A+B-2L^2)\sqrt{A-B+2L^2}}\right]=$$

$$=-\int_{-l}^{-\lambda}\frac{q(x)\,dx}{\sqrt{l^2-x^2}}-\int_{\lambda}^{l}\frac{q(x)\,dx}{\sqrt{l^2-x^2}}. \qquad (20)$$

Here $B=y_0^2+l^2+L^2;\quad A=\sqrt{B^2-4L^2l^2}$.

Determination of quantity P.

For determination of quantity of the force P we use Hooke's law. According to this law the desired quantity of the pin force P acting on every rivet from reinforcing rib is equal to

$$P=\frac{E_S F}{2y_0}\Delta\upsilon$$

Her E_s is the Young modulus of material of reinforcing rib, F is cross-section area of rib, $2y_0$ is distance between rivets, $\Delta\upsilon$ is mutual displacement of rivets that equals the lengthening of a rib.

Denote by r a radius of rivet. Following the work [5] naturally we assume that mutual displacement of the points $z=+L+i(y_0-r)$, $z=+L-i(y_0-r)$ in the considered problem is equal to indicated mutual displacement of rivets $\Delta\upsilon$. The noted additional compatibility condition allows to find effectively a solution of the posed problem.

For this it is necessary mutual displacement of the points

$$z=L+i(y_0-r)\quad and\quad z=+L-i(y_0-r)$$

in the considered problem. Using the Kolosov-Muskhelishvili relations [4] and (15), (16) and (18) after fulfilment of elementary though some bulky calculations mutual displacement of rivet $\Delta\upsilon$ of reference points, we find in the following form:

$$\Delta\upsilon=\Delta\upsilon_1+\Delta\upsilon_2$$

$$\Delta\upsilon_1=\frac{P}{2\pi\mu(1+æ)h}\left\{æ\ln\frac{r^2(4L^2+r^2)}{(r-2y_0)^2\left[4L^2+(2y_0-r)^2\right]}+\right. \tag{21}$$

$$\left.+\frac{32y_0L^2(y_0-r)}{(4L^2+r^2)\left[4L^2+(2y_0-r)^2\right]}\right\}+\frac{\sigma_0}{4\mu}(2æ+1)(y_0-r);$$

$$\Delta\upsilon_2=\frac{1}{\pi\mu}\int_0^l\frac{f(t)F(t)\,dt}{\sqrt{l^2-t^2}}+\frac{1}{\pi\mu}\int_\lambda^l\frac{q(x)F(t)\,dt}{\sqrt{l^2-t^2}}$$

Here $F(t)=(æ+1)f_1(t)+2(y_0-r)f_2(t)$;

$$f_1(t)=D\sin\varphi+\sqrt{l^2-t^2}\ln\frac{D^2\cos^2\varphi+\left(D\sin\varphi-\sqrt{l^2-t^2}\right)^2}{D^2\cos^2\varphi+\left(D\sin\varphi+\sqrt{l^2-t^2}\right)^2};$$

$$f_2(t)=\frac{1}{d_1^2+d_2^2}D\left[L(d_0\cos\varphi-d_1\sin\varphi)-(y_0-r)(d_1\cos\varphi+d_0\sin\varphi)\right];$$

$$\varphi=\frac{1}{2}arctg\frac{d_1}{B};\quad d_0=t^2-L^2+(y_0-r)^2;\quad d_1=2L(y_0-r);$$

$$A_1=\sqrt{B^2+d_1^2};\ D=\sqrt{A_1}$$

The desired force P is determined by the following relations

$$P=\frac{E_sF}{8\mu y_0\left(1-a-b_*\right)}\left[\sigma_0\left(2æ+1\right)\left(y_0-r\right)+\right.$$

$$\left.+\frac{4\sigma_0}{\pi}\int\limits_0^l\frac{F\left(t\right)dt}{\sqrt{l^2-t^2}}+\frac{4}{\pi}\int\limits_\lambda^l\frac{F\left(t\right)q\left(t\right)dt}{\sqrt{l^2-t^2}}\right] \tag{22}$$

Here

$$a=\frac{E_SF}{4\pi\mu y_0\left(1+æ\right)h}\left\{æ\ln\frac{r^2\left(4L^2+r^2\right)}{\left(r-2y_0\right)^2\left[4L^2+\left(2y_0-r\right)^2\right]}+\right.$$

$$\left.+\frac{32y_0L^2\left(y_0-r\right)}{\left(4L^2+r^2\right)\left[4L^2+\left(2y_0-r\right)^2\right]}\right\}; \tag{23}$$

$$b_*=\frac{E_sF}{2\pi\mu y_0h}\int\limits_0^l\frac{f_0\left(t\right)F\left(t\right)dt}{\sqrt{l^2-t^2}}, \tag{24}$$

where the fucntion $f_0\left(t\right)$ is determined by the relation

$$\frac{P}{h}f_0\left(t\right)=f\left(t\right)-\sigma_0 \tag{25}$$

obtained relation (22) contains the unknown parameter λ characterizing the length of end zone and the unknown force $q\left(x\right)$ in bondings between crack faces.

Using the relation

$$2\mu\frac{\partial}{\partial x}\left(u+iv\right)=æ\Phi\left(z\right)-\overline{\Phi\left(z\right)}-z\overline{\Phi'\left(z\right)}-\overline{\Psi\left(z\right)}$$

and boundary values of the fucntion $\Phi_1\left(z\right)$ we obtain on the segment $|x|\leq l$ the following equality

$$\Phi_1^+\left(x\right)-\Phi_1^-\left(x\right)=\frac{2\mu}{1+æ}\left[\frac{\partial}{\partial x}\left(u^+-u^-\right)+i\frac{\partial}{\partial x}\left(v^+-v^-\right)\right] \tag{26}$$

Using the Sohotski-Plemel formula [6] and allowing for formula (16) we find

$$\Phi_1^+\left(x\right)-\Phi_1^-\left(x\right)=\frac{\sqrt{l^2-x^2}}{\pi i}\left[\int\limits_{-l}^l\frac{\left[f\left(t\right)+q\left(t\right)\right]dt}{\sqrt{l^2-t^2}\left(t-x\right)}\right] \tag{27}$$

We put the obtained expression (27) to the right hand side of equation (26) and allowing for (2) we obtain a nonlinear differential integral equation with respect to the unknown function $q\left(x\right)$

$$-\frac{\sqrt{l^2-x^2}}{\pi}\left[\int\limits_{-l}^l\frac{f\left(t\right)dt}{\sqrt{l^2-t^2}\left(t-x\right)}+\int\limits_{-l}^l\frac{q\left(t\right)dt}{\sqrt{l^2-t^2}\left(t-x\right)}\right]=$$

$$= \frac{2\mu}{1+æ}\frac{\partial}{\partial x}\left(C\left(x,q\right)q\left(x\right)\right) \tag{28}$$

Equation (28) represents nonlinear integral equation with the Cauchy kernel and can be solved only numerically. For its solving we can use the collocational scheme with approximation of the unknown fucntion.

In order to refrain the solution of integrodifferential equation we represent equation (28) in the following form

$$\frac{1+æ}{2\mu}\int\limits_{-l}^{x} Q\left(x\right)dx = C\left(x,q\right)q\left(x\right) \tag{29}$$

Here

$$Q\left(x\right) = -\frac{\sqrt{l^2-x^2}}{\pi}\left[\int\limits_{-l}^{l}\frac{f\left(t\right)dt}{\sqrt{l^2-t^2}\left(t-x\right)} + \int\limits_{-l}^{l}\frac{q\left(t\right)dt}{\sqrt{l^2-t^2}\left(t-x\right)}\right]$$

We divide the segment $[-l,l]$ into M node t_m $(m=1,2,...,M)$ and require the fulfilment of conditions (29) in nodes. As a result instead of equation (29) we obtain an algebraic system of M_1 equations for determination of approximated values $q\ (t_m)$ $(m=1,2,...,M_1)$

$$C_0 Q\left(t_1\right) = C\left(t_1\right)q\left(t_1\right)$$

$$C_0\left(Q\left(t_1\right)+Q\left(t_2\right)\right) = C\left(t_2\right)q\left(t_2\right)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \tag{30}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$C\sum_{m=1}^{M_1} Q\left(t_m\right) = C\left(t_{M_1}\right)q\left(t_{M_1}\right)$$

where $C_0 = \dfrac{1+æ}{2\mu}\dfrac{\pi l}{M}$; M_1 is a number of nodes belonging to the end zone of cracks.

Note that for obtaining the algebraic systems, all the integration integrals were led to one interval $[-1,1]$, and then with the integrals were changed by finite sum the help of Gauss type quadrature formula.

Even in special case of linearly elastic connections system (3) is nonlinear because of the unknown size of end zone. In this connection for solution of obtained system (30), (20), (22) in case of linear connections, the method of successive approximations is used, the essence of which is next one. We solve system (30), (22) for some value λ_* (for example, for $\lambda_* = l/3$) with respect to M_1 unknown $q_1^0, q_2^0, ..., q_{M_1}^0$.

The values λ_* and the found quantities $q_1^0, q_2^0, ..., q_{M_1}^0$ are substituted in (20), i.e. in unused system of equation (30), (22) and (20) in which the integrals are used by the sum with the help of Gauss type quadrature formula. The taken value of the parameter λ_* and the values $q_1^0, q_2^0, ..., q_{M_1}^0$ corresponding to this parameter will not, in general, satisfy equation (20) of the system. Therefore, selecting the values of the parameters λ_*, the computation is repeated as long as last equation (20) of the system will satisfy with the given accuracy.

In case of nonlinear law of deformation of bondings [7] for determination of faces in end zones the method similar to the method of elastic solutions [8] is used.

It is assumed that the law of deformation of interparticle bondings (cohesive forces) is linear at $\upsilon \leq \upsilon_*$.

The first step of iteration process of computations consists of solution of a system of equations (30), (20) and (22) for linearly-elastic interparticle bondings. The subsequent iterations are satisfied only in those cases if on a part of end zone it holds $\upsilon(x) > \upsilon$. For these iterations the system of equations in each approximation for quasibrittle bondings with effective compliance, variable along the end zone of the crack and depending on the forces in bondings obtained at the previous step calculation, is solved. Computation of effective compliance is performed similar to determination of cut modulus in the method of variable of elasticity parameters [9].

Sequence approximations method process is completed as soon as the forces along the end-zone obtained at two sequential iterations differ one from very little.

For determination of limiting-equilibrium state of crack apex, the condition of critical crack opening is used. It is assumed that break of bondings on border of end domain $x = x_0$ occurs by fulfilling the condition

$$\upsilon(x_0) = \delta_k \tag{31}$$

where δ_k is limiting length (stretching) of bonding.

The joint solution of system (30), (20), (22) and (31) enables to find critical environmental load. The dependence of residual strength of rib reinforced plate weakend by crack is established by computations.

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