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ON THE STABILITY OF ORTHOTROPIC ELASTICOPLASTIC PLATES WITH REGARD TO CROSS SHEARS

Abstract

The problem on solvability of orthotropic elasticoplastic rectangle plates with regard to deformations of cross shears is investigated in the paper. All main correlations and the system of stability equations of the considered plate are received in general form. The characteristic equation for definition of load parameter were received at articulate fixation of plate's borders. The numerical calculations were carried out at concrete values of parameters.

Let's consider the rectangular plate of length a, width b and thickness h, made of orthotropic elasticoplastic material, which is under influence of combined loads applied to its mean plane. The coordinate system was chosen in the following way: the axes OX and OY are situated in mean plane and directed from border angle on borders of a plate, the axis OZ is directed perpendicularly to them.

The state equations of the type of deformation theory of plasticity [3] are used for describing elasticoplastic properties of material of a plate: In this case the relation between components of stress (σ_{ij}) and deformations (ε_{ij}) are represented in the form:

$$\sigma_{11}=\frac{E_c}{\alpha-\beta^2}\left(\alpha\varepsilon_{11}+\beta\varepsilon_{22}\right),\qquad \sigma_{22}=\frac{E_c}{\alpha-\beta^2}\left(\varepsilon_{22}+\beta\varepsilon_{11}\right),$$

$$\sigma_{12}=2\frac{E_c}{\gamma}\varepsilon_{12},\qquad \sigma_{13}=G\varepsilon_{13},\qquad \sigma_{23}=G\varepsilon_{23}. \tag{1}$$

Here α,β,γ are the characteristics of anisotropy; $E_c=\dfrac{\sigma_n}{\varepsilon_n}$ is the intersecting module of a diagram $\sigma_n=\Phi\left(\varepsilon_n\right);$ σ_n and ε_n are stress and deformation intensities respectively, G is the shear modulus.

The relation between stress and deformations are defined from (1):

$$\begin{array}{c}\delta\sigma_{11}=a_{11}\delta\varepsilon_{11}+a_{12}\delta\varepsilon_{22}+a_{13}\delta\varepsilon_{12},\\ \delta\sigma_{22}=a_{21}\delta\varepsilon_{11}+a_{22}\delta\varepsilon_{22}+a_{23}\delta\varepsilon_{12},\\ \delta\sigma_{12}=a_{31}\delta\varepsilon_{11}+a_{32}\delta\varepsilon_{22}+a_{33}\delta\varepsilon_{12},\\ \delta\sigma_{13}=a_{55}\delta\varepsilon_{13},\qquad \delta\sigma_{23}=a_{44}\delta\varepsilon_{23}.\end{array} \tag{2}$$

Here the coefficients a_{ij} are the known functions of subcritical condition parameters.

Let's assume analogously to the paper [2], that tangential stresses σ_{13} and σ_{23} change by the law:

$$\sigma_{13}=f\left(z\right)\varphi\left(x,y\right),\quad \sigma_{23}=f\left(z\right)\psi\left(x,y\right), \tag{3}$$

where $\varphi,\ \psi$ are the sought functions of the coordinates x,y: the function $f\left(z\right)$ characterizes the changes in σ_{13} and σ_{23} on thickness of a plate.

For variation of deformation we accept the next expressions [2]:

$$\begin{array}{c}\delta\varepsilon_{11}=\dfrac{\partial u}{\partial x}-z\dfrac{\partial^2 w}{\partial x^2}+\dfrac{J_0}{a_{55}}\dfrac{\partial\varphi}{\partial x},\qquad \delta\varepsilon_{22}=\dfrac{\partial\vartheta}{\partial y}-z\dfrac{\partial^2 w}{\partial x^2}+\dfrac{J_0}{a_{55}}\dfrac{\partial\varphi}{\partial x},\\ \delta\varepsilon_{12}=\dfrac{\partial u}{\partial y}+\dfrac{\partial\vartheta}{\partial x}-2z\dfrac{\partial^2 w}{\partial x\partial y}+J_0\left(\dfrac{1}{a_{55}}\dfrac{\partial\varphi}{\partial y}+\dfrac{1}{a_{44}}\dfrac{\partial\psi}{\partial x}\right),\end{array} \tag{4}$$

where $J_0 = \int\limits_0^z f(z)\,dz$. Here u, ν, w are tangential transfer and deflection of corresponding to the mean plane point of a plate.

The variations of moments and crosscutting efforts are calculated by the formulae:

$$\delta M_{ij} = \int_{-h/2}^{h/2} \delta\sigma_{ij} z dz, \quad \delta N_i = \int_{-h/2}^{h/2} \delta\sigma_{13} dz, \quad (i = 1, 2). \tag{5}$$

Allowing for (2)-(4) from (5) we will get:

$$\delta M_{11} = -\frac{h^3}{12}\left(a_{11}\frac{\partial^2 w}{\partial x^2} + a_{12}\frac{\partial^2 w}{\partial y^2} + a_{13}\frac{\partial^2 w}{\partial x \partial y}\right) +$$
$$+\frac{J_1}{a_{55}}\left[a_{11}\frac{\partial \varphi}{\partial x} + a_{12}\frac{\partial \psi}{\partial y} + a_{13}\left(\frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}\right)\right],$$
$$\delta M_{22} = -\frac{h^3}{12}\left(a_{21}\frac{\partial^2 w}{\partial x^2} + a_{22}\frac{\partial^2 w}{\partial y^2} + a_{23}\frac{\partial^2 w}{\partial x \partial y}\right) +$$
$$+\frac{J_1}{a_{55}}\left[a_{21}\frac{\partial \varphi}{\partial x} + a_{22}\frac{\partial \psi}{\partial y} + a_{23}\left(\frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}\right)\right],$$
$$\delta M_{12} = -\frac{h^3}{12}\left(a_{31}\frac{\partial^2 w}{\partial x^2} + a_{32}\frac{\partial^2 w}{\partial y^2} + a_{33}\frac{\partial^2 w}{\partial x \partial y}\right) +$$
$$+\frac{J_1}{a_{55}}\left[a_{31}\frac{\partial \varphi}{\partial x} + a_{32}\frac{\partial \psi}{\partial y} + a_{33}\left(\frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}\right)\right],$$
$$\delta N_1 = J_2\varphi, \quad \delta N_2 = J_2\psi, J_1 = \int_{-h/2}^{h/2} z \cdot J_0(z)\,dz, \quad J_2 = \int_{-h/2}^{h/2} f(z)\,dz. \tag{6}$$

The flexion equations of the considered plate are the followings:

$$\frac{\partial \delta M_{11}}{\partial x} + \frac{\partial \delta M_{12}}{\partial y} = \delta N_1, \quad \frac{\partial \delta M_{12}}{\partial x} + \frac{\partial \delta M_{22}}{\partial y} = \delta N_2,$$
$$\frac{\partial \delta N_1}{\partial x} + \frac{\partial \delta M_2}{\partial y} + T_{11}\frac{\partial^2 w}{\partial x^2} + T_{22}\frac{\partial^2 w}{\partial y^2} + 2T_{12}\frac{\partial^2 w}{\partial x \partial y} = 0. \tag{7}$$

Allowing for (6) the system (7) is transformed to the form:

$$\frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} = \frac{h}{J_2}\left(p\frac{\partial^2 w}{\partial x^2} + q\frac{\partial^2 w}{\partial y^2} + 2\tau\frac{\partial^2 w}{\partial x \partial y}\right),$$
$$a_{11}\frac{\partial^3 w}{\partial x^3} + (a_{12} + 2a_{33})\frac{\partial^3 w}{\partial x \partial y^2} + (2a_{13} + a_{31})\frac{\partial^3 w}{\partial x^2 \partial y} + a_{32}\frac{\partial^3 w}{\partial y^3} -$$
$$-\frac{12}{h^2}\frac{J_1}{a_{55}}\left\{a_{11}\frac{\partial^2 \varphi}{\partial x^2} + (a_{32} + a_{13})\frac{\partial^2 \varphi}{\partial x \partial y} + a_{33}\frac{\partial^2 \varphi}{\partial y^2} + a_{13}\frac{\partial^2 \psi}{\partial x^2} +\right.$$
$$\left.+ (a_{33} + a_{12})\frac{\partial^2 \psi}{\partial x \partial y} + a_{32}\frac{\partial^2 \psi}{\partial y^2}\right\} + \frac{12}{h^3}J_2\psi = 0,$$
$$a_{31}\frac{\partial^3 w}{\partial x^3} + (2a_{33} + a_{21})\frac{\partial^3 w}{\partial x^2 \partial y} + (a_{32} + 2a_{23})\frac{\partial^3 w}{\partial x \partial y^2} + a_{22}\frac{\partial^3 w}{\partial y^3} -$$

$$-\frac{12}{h^3}\frac{J_1}{a_{55}}\left\{a_{31}\frac{\partial^2\varphi}{\partial x^2}+(a_{33}+a_{21})\frac{\partial^2\varphi}{\partial x\partial y}+a_{23}\frac{\partial^2\varphi}{\partial y^2}+a_{33}\frac{\partial^2\psi}{\partial x^2}+\right.$$

$$\left.+(a_{32}+a_{23})\frac{\partial^2\psi}{\partial x\partial y}+a_{22}\frac{\partial^2\psi}{\partial y^2}\right\}+\frac{12}{h^3}J_2\psi=0. \tag{8}$$

Here it's accepted, that $T_{11}=-ph,\ T_{22}=-qh,\ T_{12}=-\tau h.$

So, the system of stability of orthotropic elasticoplastic plates with regard to cross shear are obtained in the form (8).

If we consider general from of stability loss it uni-lateral compressed plate at articulated support on all contour, the solution of system (8) can be represented in the following form:

$$\begin{aligned} w&=f\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b},\\ \varphi&=c_\varphi\cdot\cos\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b},\\ \psi&=c_\psi\cdot\sin\tfrac{m\pi x}{a}\cdot\cos\frac{n\pi y}{b}, \end{aligned} \tag{9}$$

where $f,\ c_\varphi,\ c_\psi$ are constant coefficients, m and n are integers showing the number of half-waves along the corresponding sides of a plate.

Substituting (9) in (8) we'll get the system of linear algebraic equations with respect to coefficients $f,\ c_\varphi,\ c_\psi$.

Equaling to zero the determinant of this system, we'll get the characteristic equation for definition of parameter of load:

$$\frac{h}{J_2}P\left(\frac{m\pi}{a}\right)^2=\frac{A_1}{A_2}, \tag{10}$$

where

$$A_1=\frac{12}{h^3}J_2\left[(2a_{33}+a_{21})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)+a_{22}\left(\frac{n\pi}{b}\right)^3\right]\times$$

$$\times\left\langle\left(\frac{n\pi}{b}\right)\left\{1+\frac{1}{a_{55}}\frac{J_1}{J_2}\left[a_{11}\left(\frac{m\pi}{a}\right)^2+a_{33}\left(\frac{n\pi}{b}\right)^2\right]\right\}-\right.$$

$$\left.-\left(\frac{m\pi}{a}\right)\frac{1}{a_{55}}\frac{J_1}{J_2}(a_{33}+a_{12})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\right\rangle+\frac{12}{h^3}J_2\left[a_{11}\left(\frac{m\pi}{a}\right)^3+\right.$$

$$\left.+(a_{12}+2a_{33})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2\right]\times$$

$$\times\left\langle\left(\frac{m\pi}{a}\right)\left\{1+\frac{1}{a_{55}}\frac{J_1}{J_2}\left[a_{33}\left(\frac{m\pi}{a}\right)^2+a_{22}\left(\frac{n\pi}{b}\right)^2\right]\right\}-\right.$$

$$\left.-\left(\frac{n\pi}{b}\right)\frac{1}{a_{55}}\frac{J_1}{J_2}(a_{33}+a_{12})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\right\rangle,$$

$$A_2=\left(\frac{12}{h^3}J_2\right)^2\left\langle\left\{1+\frac{1}{a_{55}}\frac{J_1}{J_2}\left[a_{11}\left(\frac{m\pi}{a}\right)^2+a_{33}\left(\frac{n\pi}{b}\right)^2\right]\right\}\times\right.$$

$$\times\left\{1+\frac{1}{a_{55}}\frac{J_1}{J_2}\left[a_{33}\left(\frac{m\pi}{a}\right)^2+a_{22}\left(\frac{n\pi}{b}\right)^2\right]\right\}-$$

$$\left.-\frac{1}{a_{55}^2}\left(\frac{J_1}{J_2}\right)^2(a_{33}+a_{21})^2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\right\rangle.$$

For carrying out numerical calculations the function $f(z)$ was accepted in the following form:

$$f(z) = \frac{1}{2}\left(\frac{h^2}{4} - z^2\right).$$

In this case from (6) we'll get

$$J_1 = \frac{h^5}{120}, \qquad J_2 = \frac{h^3}{12}.$$

The results of numerical calculations are represented in fig.1. Here the solution of the similar problem ignoring cross shears [3] is denoted by a dotted line. Here σ_s is a yield point, λ is a coefficient of linear hardening of a plate's material. As we see from the obtained results for orthotropic material the disregard of cross shears may lead to essential errors by defining critical load.

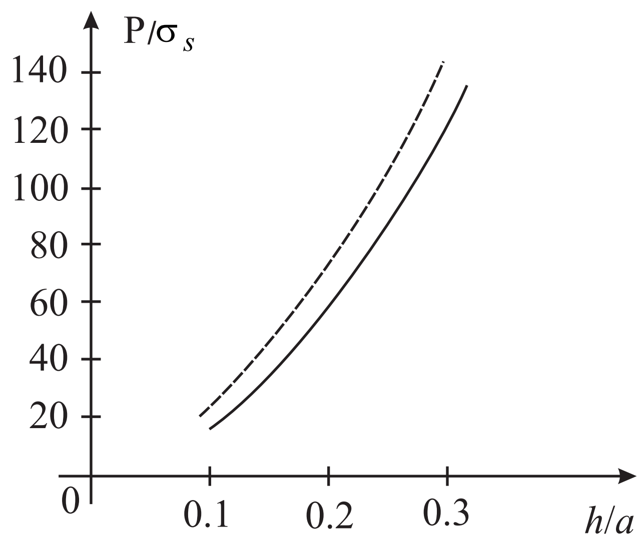

Fig.1
$\lambda = 0,90; \quad \alpha = 1,2; \quad \beta = 0,55; \quad \gamma = 3.$

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