MECHANICS

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STABILITY OF AN ORTHOTROPIC RECTANGULAR PLATE OF VARIABLE THICKNESS

Abstract

A problem on stability of orthotropic rectangular plate of variable thickness compressed in one direction is considered. Equilibrium equation and strain compatibility condition are derived for a general case. Detailed analysis is carried out for cylindric form of stability loss when the thickness along the length changes by the linear law.

As is known, rectangular plates made of different materials are the most popularized structural elements used in many fields of construction of engineering complexes, by designing and erecting different assignment bridges, in modern machine-building and in a number of other fields. In many cases designers have to work with variable thickness plates made of different materials. Lately in connection with the fact that synthetically made materials (not only synthetic materials) find wide application in engineering practice, construction of new stability analysis methods are required.

A plate of variable thickness made of orthotropic materials [1] is widely used among these materials.

In the case when during use a plate is under the action of compressive loads, stability analysis is of great importance.

In this paper we investigate a problem on stability of a rectangular plate of thickness $h = 2h_0 f(x)$ compressed along the long side by uniformly-distributed load of intensity P. The coordinate system is chosen as follows: the axes x and y are located on a median plane, the axis z is perpendicular to them. The function $f(x)$ with its derivatives is a continuous function

$$2h_0 = \frac{\bar{h} + \underline{h}}{\bar{h} - \underline{h}},$$

here $2\bar{h}$ and $2\underline{h}$ are the greatest and least values of plate thickness, respectively. Relation between stresses $\sigma_1, \sigma_2, \sigma_0$ and strains $\varepsilon_1, \varepsilon_2, \varepsilon_0$ is of the form [2]:

$$\sigma_1 = \frac{E_1}{1-\nu_1\nu_2}\left(\varepsilon_1 + \nu_2\varepsilon_2\right), \ \sigma_2 = \frac{E_2}{1-\nu_1\nu_2}\left(\varepsilon_2 + \nu_1\varepsilon_1\right), \quad \sigma_0 = G\varepsilon_0. \tag{1}$$

E_1, E_2 and ν_1, ν_2 are elasticity modules and Poisson coefficients, G is a shear modulus.

$$\varepsilon_1 = e_1 + \chi_1 z, \quad \varepsilon_2 = e_2 + \chi_2 z, \quad \varepsilon_0 = e_0 + \chi_0 z,$$

$$\sigma_1 = \frac{E_1}{1-\nu_1\nu_2}\left[\left(e_1 + \nu_2 e_2\right) + \left(\chi_1 + \nu_2\chi_2\right) z\right],$$

$$\sigma_2 = \frac{E_2}{1-\nu_1\nu_2}\left[\left(e_2 + \nu_1 e_1\right) + \left(\chi_2 + \nu_1\chi_1\right) z\right],$$

$$\sigma_0 = G\left(e_0 + \chi_0 z\right). \tag{2}$$

Allowing for (2) in relations $T_i = \int\limits_{-h_0 f(x)}^{h_0 f(x)} \sigma_i dz$, $M_i = \int\limits_{-h_0 f(x)}^{h_0 f(x)} \sigma_i z dz$ we get:

$$T_1 = \frac{2E_1 h_0}{1-\nu_1\nu_2}\left(e_1 + \nu_2 e_2\right) f\left(x\right),$$

$$T_2 = \frac{2E_2 h_0}{1-\nu_1\nu_2}\left(e_2 + \nu_1 e_1\right) f\left(x\right), \tag{3}$$

$$T_0 = 2Gh_0 e_0 f\left(x\right).$$

$$M_1 = \frac{2h_0^3 E_1}{3\left(1-\nu_1\nu_2\right)}\left(\chi_1 + \nu_2\chi_2\right) f^3\left(x\right),$$

$$M_2 = \frac{2h_0^3 E_2}{3\left(1-\nu_1\nu_2\right)}\left(\chi_2 + \nu_1\chi_1\right) f^3\left(x\right), \tag{4}$$

$$M_0 = \frac{2}{3} h_0^3 G\chi_0 f^3\left(x\right).$$

Here e_1, e_2, e_0 are strains in median surface; χ_1 and χ_2 are curvatures, χ_0 is a median surface torsion.

$$\chi_1 = -\frac{\partial^2\omega}{\partial x^2}, \quad \chi_2 = -\frac{\partial^2\omega}{\partial y^2}, \quad \chi_0 = -\frac{\partial^2\omega}{\partial x\partial y}, \tag{5}$$

where ω is a deflection.

Using the expressions for moments and curvatures we'll get the following stability equation:

$$B_1 f^3\left(x\right)\frac{\partial^4\omega}{\partial x^4} + B_2 f^3\left(x\right)\frac{\partial^4\omega}{\partial x^2\partial y^2} + B_3 f^3\left(x\right)\frac{\partial^4\omega}{\partial y^4} +$$

$$+3f\left(x\right)\left[2\left(\frac{df\left(x\right)}{dx}\right)^2 + f\left(x\right)\frac{d^2 f\left(x\right)}{dx^2}\right]\left(B_1\frac{\partial^2\omega}{\partial x^2} + B_4\frac{\partial^2\omega}{\partial y^2}\right) +$$

$$+6B_1 f^2\left(x\right)\frac{df\left(x\right)}{dx}\frac{\partial^3\omega}{\partial x^3} + B_5 f^2\left(x\right)\frac{df\left(x\right)}{dx}\frac{\partial^3\omega}{\partial x\partial y^2} - \tag{6}$$

$$-\frac{\partial^2\omega}{\partial x^2}T_1 - 2T_0\frac{\partial^2\omega}{\partial x\partial y} - T_2\frac{\partial^2\omega}{\partial y^2} = 0.$$

Then we use strain compatibility equation:

$$\frac{\partial^2 e_1}{\partial y^2} + \frac{\partial^2 e_2}{\partial x^2} - \frac{\partial^2 e_0}{\partial x\partial y} = \left(\frac{\partial^2\omega}{\partial x\partial y}\right)^2 - \frac{\partial^2\omega}{\partial x^2}\frac{\partial^2\omega}{\partial y^2}. \tag{7}$$

Express the forces T_1, T_2 and T_0 by the force function φ as follows:

$$T_1 = \frac{\partial^2\varphi}{\partial y^2}; \quad T_2 = \frac{\partial^2\varphi}{\partial x^2}; \quad T_0 = -\frac{\partial^2\varphi}{\partial x\partial y}.$$

From (3) we find e_1, e_2, e_0 and put into (7), After some transformations strain compatibility equation will be of the form:

$$\frac{A_1}{f(x)}\frac{\partial^4\varphi}{\partial x^4}+\frac{A_2}{f(x)}\frac{\partial^4\varphi}{\partial x^2\partial y^2}+\frac{A_3}{f(x)}\frac{\partial^4\varphi}{\partial y^4}+$$

$$+\frac{1}{f^2(x)}\left[\frac{2}{f(x)}\left(\frac{df(x)}{dx}\right)^2-\frac{d^2f(x)}{dx^2}\right]\left(A_1\frac{\partial^2\varphi}{\partial x^2}-A_4\frac{\partial^4\varphi}{\partial y^2}\right)-$$

$$-\frac{2A_1}{f^2(x)}\frac{df(x)}{dx}\frac{\partial^3\varphi}{\partial x^3}+\frac{A_5}{f^2(x)}\frac{df(x)}{dx}\frac{\partial^3\varphi}{\partial x\partial y^2}=\left(\frac{\partial^2\omega}{\partial x\partial y}\right)^2-\frac{\partial^2\omega}{\partial x^2}\frac{\partial^2\omega}{\partial y^2}. \tag{8}$$

Here we introduce the denotation:

$$A_1=\frac{1}{2E_2h_0},\quad A_2=\frac{E_1-2G\nu_1}{2E_1Gh_0},\quad A_3=\frac{1}{2E_1h_0},$$

$$A_4=\frac{\nu_1}{2E_1h_0},\quad A_5=-A_2;$$

$$B_1=\frac{2E_1h_0^3}{3(1-\nu_1\nu_2)},\qquad B_2=\frac{4h_0^3\left[E_1\nu_2+G(1-\nu_1\nu_2)\right]}{3(1-\nu_1\nu_2)},$$

$$B_3=\frac{2E_2h_0^3}{3(1-\nu_1\nu_2)},\qquad B_4=\frac{2E_1\nu_2h_0^3}{3(1-\nu_1\nu_2)},\quad B_5=3B_2.$$

In this case equilibrium equation (6) will take form ($\delta T_{ij}=0$):

$$B_1f^3(x)\frac{\partial^4\omega}{\partial x^4}+B_2f^3(x)\frac{\partial^4\omega}{\partial x^2\partial y^2}+B_3f^3(x)\frac{\partial^4\omega}{\partial y^4}+$$

$$+3f(x)\left[2\left(\frac{df(x)}{dx}\right)^2+f(x)\frac{d^2f(x)}{dx^2}\right]\left(B_1\frac{\partial^2\omega}{\partial x^2}+B_4\frac{\partial^2\omega}{\partial y^2}\right)+ \tag{9}$$

$$-6B_1f^2(x)\frac{df(x)}{dx}\frac{\partial^3\omega}{\partial x^3}+B_5f^2(x)\frac{df(x)}{dx}\frac{\partial^3\omega}{\partial x\partial y^2}+P\frac{\partial^2\omega}{\partial x^2}=0.$$

We'll search for the solution (9) in the following form:

$$\omega=C\sin\alpha x\sin\beta x, \tag{10}$$

here $\alpha=\dfrac{m\pi}{a}$, $\beta=\dfrac{n\pi}{b}$.

Putting (10) into (9) and using Bubnov-Galerkin method for $f(x)=1+\varepsilon xa^{-1}$ we get:

$$P=\left(B_1\alpha^2+B_2\beta^2+B_3\frac{\beta^4}{\alpha^2}\right)\left(1+\frac{3}{2}\varepsilon+\varepsilon^2+\frac{1}{4}\varepsilon^3\right)-8\varepsilon^2\left(B_1+B_2\frac{\beta^2}{\alpha^2}\right).$$

In the case of cylindrical form of bending loss of stability equation (9) will take the form:

$$B_1f^3(x)\frac{\partial^4\omega}{\partial x^4}+3B_1f(x)\left[2\left(\frac{df(x)}{dx}\right)^2+f(x)\frac{d^2f(x)}{dx^2}\right]\frac{\partial^2\omega}{\partial x^2}+$$

$$+6B_1 f^2(x)\frac{df(x)}{dx}\frac{\partial^3\omega}{\partial x^3}+P\frac{\partial^2\omega}{\partial x^2}=0. \tag{11}$$

Write the solution in the form of

$$\omega = C\sin\alpha x. \tag{12}$$

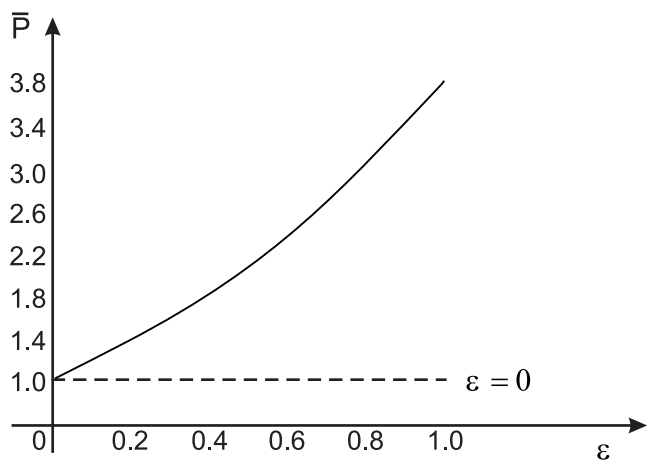
In this case, for $f(x)=1+\varepsilon x a^{-1}$ we'll get:

$$P=B_1\alpha^2\left(1+\frac{3}{2}\varepsilon+\varepsilon^2+\frac{1}{4}\varepsilon^3\right).$$

For concrete values of typical parameters the calculations are performed and the results are represented in the figure and table

$$\bar{P}=\frac{P}{B_1\alpha^2}=1+\frac{3}{2}\varepsilon+\varepsilon^2+\frac{1}{4}\varepsilon^3.$$

ε	$\bar{P}$
0	1.000
0.1	1.160
0.2	1.342
0.3	1.547
0.4	1.776
0.5	2.031
0.6	2.314
0.7	2.626
0.8	2.968
0.9	3.342
1.0	3.750



References

[1]. Lekhnitskii S.P. *Elasticity theory of anisotropic bodies.* M., Gostechizdat, 1960, 384 p. (Russian)

[2]. Volmir A.S. *Stability of elastic systems.* M., 1966, 879 p. (Russian)

[3]. Alfutov N.A. *Basis of stability analysis of elastic systems.* M., Mashinostroeniye, 1991, 332 p. (Russian)

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