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INVESTIGATIONS OF EIGEN-OSCILLATIONS OF A MEDIUM FILLED CYLINDRICAL SHELL REINFORCED BY TWO CROSS SYSTEMS OF RIBS

Abstract

The paper is devoted to the research of eigen-oscillations of a medium filled cylindrical shell stiffened by a cross system of ribs. Asymptotic analysis of frequencies and oscillations forms of the considered system in conducted on the smallness of the shell thickness, character of variability of stress state of a shell and medium, smallness of relation of elasticity modulus of medium and shell.

This paper is devoted to investigation of eigen frequencies of vibrations of circular closed cylindrical shell, hinge-supported along the edges, reinforced by a regular system of longitudinal and cross ribs with a compact filler. A shell is modeled as a constructively-orthotropic, and its motion is described by a system of equations in permutations.

Complemanting the motion equations of a shell and a filler, by contact conditions there has been obtained a frequency equation for eigen frequencies of vibrations of a shell, reinforced by medium filled longitudinal and cross ribs.

Influences of physical and mechanical parameters of ribs and medium on eigen oscillation frequency of the considered construction, are investigated.

In the given paper a shell is modelled as constructively-orthotropic and by [1] its motion is described by the following system of equations:

$$\left[\left(1+\gamma_c^{(1)}\right)\frac{\partial^2}{\partial\xi^2}+\frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2}\right]u+\frac{1+\nu}{2}\frac{\partial^2\vartheta}{\partial\xi\partial\theta}-$$

$$-\left(\nu\frac{\partial}{\partial\xi}+\delta_c^{(1)}\frac{\partial^3}{\partial\xi^3}\right)w-\rho_1\frac{\partial^2 u}{\partial t_1^2}=-\frac{R^2\left(1-\nu^2\right)}{Eh}q_x$$

$$\frac{1+\nu}{2}\frac{\partial^2 u}{\partial\xi\partial\theta}+\left\{\frac{1-\nu}{2}\left(1+4a^2\right)\frac{\partial^2}{\partial\xi^2}+\left[1+\left(1-\frac{h_s}{R}\right)^2\gamma_s^{(2)}+a^2\right]\frac{\partial^2}{\partial\theta^2}\right\}\upsilon+$$

$$+\left\{-\left[1+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]\frac{\partial}{\partial\theta}+(2-\nu)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}+\right.$$

$$\left.+\left[a^2-\left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\right]\frac{\partial^3}{\partial\theta^3}\right\}w-\rho_2\frac{\partial^2\upsilon}{\partial t_1^2}=-\frac{R^2\left(1-\nu^2\right)}{Eh}q_y$$

$$-\left(\nu\frac{\partial u}{\partial\xi}+\delta_c^{(1)}\frac{\partial^3}{\partial\xi^3}\right)+\left\{-\left[1+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]\frac{\partial}{\partial\theta}+(2-\nu)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}+\right.$$

$$\left.+\left[a^2-\left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\right]\frac{\partial^3}{\partial\theta^3}\right\}\upsilon+$$

$$\left[1+\gamma_s^{(2)}+\eta_{s1}^{(2)}+2\left(\delta_s^{(2)}+\eta_{s1}^{(2)}\right)\frac{\partial^2}{\partial\theta^2}+a^2\Delta\Delta+\right.\qquad(1)$$

$$+\left(\eta_{s1}^{(2)}+\eta_{s2}^{(2)}\right)\frac{\partial^4}{\partial\theta^4}+\eta_c^{(1)}\frac{\partial^4}{\partial\xi^4}\bigg]w+\rho_3\frac{\partial^2 w}{\partial t_1^2}=\frac{R^2\left(1-\nu^2\right)}{Eh}q_z$$

where $\rho_1=1+\bar\rho_c\bar\gamma_c^{(1)}$, $\rho_2=1+\bar\rho_s\bar\gamma_s^{(2)}$, $\rho_3=1+\bar\rho_c\bar\gamma_c^{(1)}+\bar\rho_s\bar\gamma_s^{(2)}$, $\xi=\frac{x}{R}$, $\theta=\frac{y}{R}$, $\delta_s^{(2)}=\frac{h_s}{R}\bar\gamma_s^2$, $\gamma_s^{(2)}=\frac{E_s\left(1-\nu^2\right)}{E}\bar\gamma_s^{(2)}$, $\gamma_c^{(1)}=\frac{E_c}{E}\left(1-\nu^2\right)\bar\gamma_c^{(1)}$, $\bar\gamma_c^{(1)}=\frac{E_cK}{2\pi Rh}$, $\bar\rho_c=\frac{\rho_c}{\rho_0}$, $t_1=\omega_0 t$, $\Delta=\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\theta^2}$, $\bar\rho_s=\frac{\rho_s}{\rho_0}$, $\eta_{s2}^{(2)}=\frac{E_s\left(1-\nu^2\right)}{E}\bar\eta_s^{(2)}$, $\eta_{s1}^{(2)}=\frac{E_sJ_{xs}\left(1-\nu^2\right)\left(1+k_2\right)}{EL_1R^2h}$, $\bar\eta_s^{(2)}=\left(\frac{h_s}{R}\right)^2\bar\gamma_s^{(2)}$, $a^2=\frac{h^2}{12R^2}$, $\delta_c^{(1)}=\frac{\eta_c}{R}\bar\gamma_c^{(1)}$, $\omega_1=\frac{\omega}{\omega_0}$, $\omega_1=\sqrt{\frac{\left(1-\nu^2\right)\rho_0R^2\omega^2}{E}}$, $\bar\gamma_s^{(2)}=\frac{F_s}{L_1h}\left(1+k_2\right)$, $\omega_0=\sqrt{\frac{E}{\left(1-\nu^2\right)\rho_0R^2}}$, $\eta_c^{(1)}=\frac{E_c\left(J_{yc}+h^2F_c\right)k_1}{2\pi R^3hE}\left(1-\nu^2\right)$, ρ_c, ρ_s are densities of materials of longitudinal and cross ribs, respectively, ρ_0 is a density of a shell material, F_c, F_s are areas of cross sections of longitudinal and cross ribs, respectively, R is a radius of a mean shell surface, h is a shell thickness, k_1, k_2 is quantity of longitudinal and cross ribs, respectively, E_c, E_s, E are modules of elasticity of longitudinal and cross ribs and shell, respectively, v is a Poisson coefficient of shell material, J_{yc}, J_{xs} is a moment of inertia of the cross section of a rib, respectively, with respect to the axes oz and ox, u, ϑ, w are components of the permutations of the mean shell surface, t is time, ω is desired frequency, q_x, q_0, q_z are components of pressure vector on shell from medium .

Taking the medium as linear-elastic, we'll describe its movements by Lame equation in permutations [2]:

$$a_e^2 qraddi\nu\vec s-a_t^2 rotrot\vec s+\omega^2\vec s=0 \tag{2}$$

Here, $\vec s\left(s_x,s_\theta,s_r\right)$ is a vector of shell permutation; $a_t=\sqrt{\frac{\lambda+2\mu}{\rho}}$, $a_e=\sqrt{\frac{\mu}{\rho}}$ are speeds of longitudinal and transverse waves propagation in medium. Here λ, μ are Lame coefficients for the medium. To the vector equation (2) it is necessary to add the contact conditions. Suppose, that contact between shell and medium is sliding, i.e. at $r=R$

$$w=s_z \tag{3}$$

$$q_x=-\sigma_{rx}=0,\quad q_0=-\sigma_{r0}=0,\quad q_z=-\sigma_{rr} \tag{4}$$

Components $\sigma_{rx},\sigma_{r\theta},\sigma_{rr}$ of the stresses tensor are defined in the following way [2]:

$$\sigma_{rx}=G_s\left(\frac{\partial S_x}{\partial r}+\frac{\partial S_r}{\partial x}\right);\quad \sigma_{r\theta}=G_s\left(\frac{\partial S_x}{r\partial\theta}+\frac{\partial S_\theta}{\partial x}\right)$$

$$\sigma_{rr}=2G_s\left(\frac{\partial S_r}{\partial r}+\frac{\nu_s\Delta_1}{1-2\nu_s}\right);$$

$$\Delta_1=\frac{\partial S_x}{\partial x}+\frac{\partial S_\theta}{r\partial\theta}+\frac{\partial S_r}{\partial r}+\frac{S_r}{r};\quad G_s=\frac{\tilde E_s}{2\left(1+\nu_s\right)} \tag{5}$$

Adding contact conditions (3) and (4) to the equations of shell movement (1) and medium (2), we come to the problem on eigen oscillations of reinforced medium filled shell. In other words, problem on eigen oscillations of reinforced cylindrical shell with the medium, is led to the joint integration of equations of shell and medium theory in case of fulfillment of the indicated conditions on their contact surface.

We'll search permutations of a shell in the form:

$$
\begin{array}{l}
u = u_0 \sin\chi\xi \cos n\theta \sin\omega_1 t_1, \\
\vartheta = \vartheta_0 \cos\chi\xi \sin n\theta \sin\omega_1 t_1, \\
w = w_0 \cos\chi\xi \cos n\theta \sin\omega_1 t_1.
\end{array}
\tag{6}
$$

Here u_0, ϑ_0, w_0 are the unknown constants; $\chi = \dfrac{m\pi R}{L}$ $(m = 1, 2, ...)$, m,n are wave numbers in the longitudinal and transverse lines, respectively; L is a shell length. We consider two ways of the solution of shell motion equation: a) filler's inertial effect upon vibration process is irrelevant; b) we cannot disregard influence of the filler motion inertia on vibration process.

In case a)

$$
\begin{gathered}
S_x = \left[\left(-kr\frac{\partial I_n(kr)}{\partial r} - 4(1-\nu_s)\,kI_n(kr)\right)A_s + kI_n(kr)\,B_s\right] \times \\
\times \cos n\theta \sin\chi\xi \sin\omega_1 t_1 \\
S_\theta = \left[-\frac{n}{r} I_n(kr)\,B_s - \frac{\partial I_n(kr)}{\partial r} C_s\right] \sin n\theta \cos\chi\xi \sin\omega_1 t_1 \\
S_r = \left[-k^2 r I_n(kr)\,B_s - \frac{\partial I_n(kr)}{\partial r} B_s + \frac{n}{r} I_n(kr)\,C_s\right] \cos n\theta \cos\chi\xi \sin\omega_1 t_1
\end{gathered}
\tag{7}
$$

In case b)

$$
\begin{gathered}
S_x = \left[kI_n(\gamma_e r)\,\tilde{A}_s - \frac{\gamma_t^2}{\mu_t} k_n(\gamma_e r)\,\tilde{C}_s\right] \cos n\theta \sin\chi\xi \sin\omega_1 t_1 \\
S_\theta = \left[-\frac{n}{r} I_n(\gamma_e r)\,\tilde{A}_s - \frac{nk}{r\mu_t} I_n(\gamma_t r)\,\tilde{C}_s - \frac{1}{n}\frac{\partial I_n(\gamma_t r)}{\partial r}\tilde{B}_s\right] \times \\
\times \sin n\theta \cos\chi\xi \sin\omega_1 t_1 \\
S_r = \left[\frac{\partial I_n(\gamma_e r)}{\partial r}\tilde{A}_s - \frac{k}{\mu_t}\frac{\partial I_n(\gamma_t r)}{\partial r}\tilde{C}_s + \frac{nk}{r} I_n(\gamma_t r)\,\tilde{B}_s\right] \cos n\theta \cos\chi\xi \sin\omega_1 t_1
\end{gathered}
\tag{8}
$$

Here $\gamma_e^2 = k^2 - \mu_e^2$, $\gamma_t^2 = k^2 - \mu_t^2$, $\mu_e = \dfrac{\omega}{a_e}$, $\mu_1 = \dfrac{\omega}{a_t}$, I_n is modified n-th order Bessel function of first kind.

After substitution of (7) and (8) in (1), taking (5) into account, the problem is reduced to homogeneous system of linear algebraic equations of third order, whose nontrivial solution is possible only if its determinant equals zero. In this case equation with respect to ω_1 is transcendental, since it contains Bessel function. We can write it in the form:

In case a)

$$
\det \|a_{ij}\| = 0 \qquad (i, j = 1, 2, 3) \tag{9}
$$

In case (b)

$$
\det \|b_{ij}\| = 0 \qquad (i, j = 1, 2, 3) \tag{10}
$$

Coefficients $a_{ij}(\omega_1)$ and $b_{ij}(\omega_1)$ $(i=1,2,3)$ are of bulky form, therefore we don't write out them. Just note, that they depend on the above mentioned geometrical and physical parameters, characterizing the system.

Equations (9) and (10) with respect to $\omega_1^2=\lambda$ have the form:

$$\lambda^3-\alpha_1\lambda^2+\alpha_2\lambda-\alpha_3=0 \tag{11}$$

Here

$$\alpha_1=\rho_1^{-1}\left[a_{11}+a_{22}\frac{\rho_1}{\rho_2}+\left(a_{33}-q_z^{(0)}\right)\frac{\rho_1}{\rho_3}\right];$$

$$\alpha_2=\rho_1^{-1}\rho_2^{-1}\left[a_{11}a_{22}-\frac{(1+\nu)^2}{4}n^2\chi^2+\frac{\rho_1}{\rho_3}\left(a_{22}a_{33}-a_{22}q_z^{(0)}-a_{32}^2\right)+\right.$$

$$\left.+\frac{\rho_1}{\rho_3}\left(a_{11}a_{33}-a_{11}q_z^{(0)}+\delta_c^{(1)^2}\chi^6-\nu^2\chi^2\right)\right];$$

$$\alpha_3=\rho_1^{-1}\rho_2^{-1}\rho_3^{-1}\times$$

$$\times\left[-a_{22}\delta_c^{(1)}\chi^6+\left(\nu a_{22}-\nu(1+\nu)na_{32}+\frac{(1+\nu)^2n^2}{4}\left(a_{33}-q_z^{(0)}\right)\chi^2\right)-\right.$$

$$\left.-a_{11}a_{22}a_{33}+a_{11}a_{32}^2+a_{11}a_{22}q_z^{(0)}\right].\ \chi=kR.$$

$$a_{11}=\left(1+\gamma_c^{(1)}\right)\chi^2+\frac{1-\nu}{2}n^2;\quad a_{22}=\frac{1-\nu}{2}\chi^2+\left(1+\gamma_s^{(2)}\right)n^2;$$

$$a_{23}=n\left[1+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]+(2-\nu)a^2\chi^2n+\left[a^2-\left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\right]n^3;$$

$$a_{33}=1+\gamma_s^{(2)}+\eta_{s1}^{(2)}-2\left(\delta_s^{(2)}+\eta_{s1}^{(2)}\right)n^2+a^2\left(n^2+\chi^2\right)^2+n^4\left(\eta_{s1}^{(2)}+\eta_{s2}^{(2)}\right)+\eta_c^{(1)}\chi^4.$$

In α_i $(i=1,2,3)$ $q_z^{(0)}$ is a stress amplitude of q_z:

$$q_z=q_z^{(0)}w_0\cos n\theta\sin kx\sin\omega_1t_1$$

To find $q_z^{(0)}$ we apply formulae for logarithmic derivatives of Bessel's function I_n for large n $(x<<n)$ [2]:

$$\frac{I_n'(x)}{I_n(x)}\approx\frac{n}{x}+\frac{x}{2n} \tag{12}$$

Using formulae (9) for $q_z^{(0)}$ we get:

In case a)

$$q_z^{(0)}=-\tilde{\chi}(1-\nu)nE_s^*;\quad \tilde{\chi}=\frac{1-\nu^2}{2(1+\nu_s)};\quad E_s^*=\frac{\tilde{E}}{Eh_*},\quad h_*=\frac{h}{R} \tag{13}$$

where $\tilde{E}_s$ is a modulus of elasticity of a filler material $\tilde{E}_s/E<<1$.

In case b)

$$q_z^{(0)}\approx-\tilde{\chi}(1-\nu)nE_s^*-2\frac{\rho_s^*}{n}\lambda \tag{14}$$

Note, that limitation with respect to the index and argument of Bessel function gives the following inequality for a parameter of vibration frequency of system λ:

$$0 < \lambda << \chi^2 E_s^* / \rho_s^* \tag{15}$$

For the following analysis it is necessary to select parameters in equation (11), which highly influence on coefficients $a_{ij}(\omega_1)$ and $b_{ij}(\omega_1)$ $(i = 1, 2, 3)$. Such parameters for the considered constructively-orthotropic shell are: dimensionless flexural rigidity of longitudinal and transversively-reinforced ribs $\eta_c^{(1)}$,$\delta_c^{(1)}$,$\eta_{s2}^{(2)}$,$\delta_s^{(2)}$ and a^2,n.

Besides, we suppose, that

$$E_s^* \sim h_*^{\alpha} \quad (\alpha \geq 0), \quad n >> 1, \quad \chi \sim 0(1).$$

In case a) the roots of equation (11), satisfying inequality (15) have the form:

$$\lambda \approx -\frac{\tilde{\alpha}_3}{\tilde{\alpha}_2} \tag{16}$$

Expressions for $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$ come from α_2 and α_3, respectively, by substitution of $q_z^{(0)}$ by its approximate expression from (13). If in (11) we substitute $q_z^{(0)}$ by its approximate expression from (14), then we receive parameters of vibration of frequency of the considered system, corresponding to case b). Expressions for $q_z^{(0)}$ from (14) show, that taking into account influence of inertia of filler on vibration process of the considered system leads to reduction of oscillation frequency value in comparison with oscillation frequency of a system, when inertia of filler force on oscillation process is small little. In case b) for λ we get:

$$\lambda \approx \frac{\tilde{\alpha}_3}{\tilde{\alpha}_2 + 2\rho_1^{-1}\rho_2^{-2}\rho_3^{-3}\left(a_{11}a_{22} - \frac{(1+\nu)^2}{4}n^2\chi^2\right)\frac{\rho_s^*}{n}} \tag{17}$$

Note, that expressions (16) and (17) for $\rho_s = E_s = 0$ coincide with the result obtained in [13], and for $\rho_c = E_c = 0$ coincides with the result obtained in [4], respectively.

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