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STATEMENT OF A CONICAL SHELL FLUTTER PROBLEM

Abstract

In the proposed work it is considered a case of aeroelastic vibrations of a truncated conical shell, constituting a part of a right circular cone, streamlined by supersonic gas flow.

Introduction. A problem on aeroelastic vibrations of a slanting shell or a shell of revolution streamlined by supersonic gas flow is considered in the paper [1].

Expressions for the pressure of aerodynamic interaction between flow and oscillating shell are obtained in a general form. It is considered a partial case when a slanting shell occupies a part of the surface of a thin profile. It is shown that "dynamical" part of pressure consists of two constituents: the first of them is the well-known piston theory, but with a coefficient depending on the flow velocity in a sufficiently complicated way; the second one makes sense of contractive normal stress in median surface of a shell and obviously may exert noticeable effect on the character of vibrations and critical velocity of a flutter. The results of calculations of a plate flutter occupying a part of a surface of a thin wedge confirms this deduction [2].

In the present work we consider a case of aeroelastic vibrations of a truncated conical shell constituting a part of a right circular cone streamlined by supersocin gas flow that is important in applications.

1^0. Relations of gas dynamics. Let's consider a thin circular cone streamlined by a supersonic flow. Origin of a rectangular system of coordinates is located on a vertex, the axis x is directed along the velocity vector. In undeformable state an equation of a generator $z_1 = kz$, $k = tg\alpha$ α is angle of half-opening of a cone. Denote by $w(x,t)$ deflections of a shell (it ossupies a part $[x_1, x_2]$ of a cone, we first consider an axially symmetric case). On the part $[x_1, x_2]$ of the shell we have

$$z = kx - w(x,t) \tag{1}$$

Assume $(w(x,t)/kx) << 1$.

According to the law of plane sections, state of gas in the field between schock wave (Sh.W) and body is determined from the solution of a plane problem on a piston which moves by the law

$$z(t) = kvt - w(vt,t) \tag{2}$$

where v is stream velocity.

Solution of the streamline problem is sought by the expansion in small parameter

$$\frac{\rho^0}{\rho^*}=\frac{\gamma-1}{\gamma+1}\left[1+\frac{2a_0^2}{(\gamma-1)\,D^2}\right]\equiv\varepsilon a\,(D)$$

here ρ^0 is gas density before ShW, ρ^* - after ShW, D is velocity of propagation of ShW, a_0 is sound velocity in undisturbed flow, γ is polytropic exponent $\left(p/p^0=\left(\rho/\rho^0\right)^{\gamma}\right)$.

Introduce Lagrangian coordinates t and z, such that $dz=\rho^0 r^{\mu-1}dr$, r is the distance of particles from the axis at initial time. The desired functions: distance of particles from the axis $\xi=\xi\,(t,z)$, pressure $p=p\,(t,z)$, $\rho=\rho\,(t,z)$.

Equations of motion, conservation of mass, energy

$$\frac{\partial^2\xi}{\partial t^2}=-\xi^{\mu-1}\frac{\partial p}{\partial z};\quad \frac{\partial\xi}{\partial z}=\frac{1}{\rho\xi^{\mu-1}};\ \frac{\partial}{\partial t}\left(\frac{p}{\rho^{\gamma}}\right)=0 \tag{3}$$

Conditions on shock wave $z=z^*$

$$p^*=\frac{2}{\gamma+1}\rho^0D^2-\varepsilon p^0;\quad \rho^*=\frac{\rho^0}{\varepsilon a\,(D)}; \tag{4}$$

Conditions on piston (2)

$$z=0,\quad \xi\,(t,0)=kvt-w\,(vt,t) \tag{5}$$

Here p^0 is the pressure in unperturbed flow.

We seek for the solution of system (3) by the expansion in ε:

$$\xi=\xi_0+\varepsilon\xi_1+...;\ p=p_0+\varepsilon p_1+...;\quad \rho=\varepsilon^{-1}\rho_0+\rho_1+...$$

Putting it into (3) we get systems for the zero and first approximations and integrate them. The zero approximation

$$\xi_0=\xi_0\,(t)\,;\quad p_0=p\,(t)-z\xi_0^{1-\mu}\frac{\partial^2\xi_0}{\partial t^2};\quad \rho_0=\frac{p_0^{1/\gamma}}{v_0\,(z)}; \tag{6}$$

the first approximation

$$\xi_1=\frac{1}{\xi_0^{\mu-1}}\int\limits_{z^*}^{z}v_0\,(z)\,p_0^{-1/\gamma}dz+\xi_1^*\,(t)$$

$$p_1=(\mu-1)\,\frac{\partial^2\xi_0}{\partial t^2}\frac{1}{\xi_0^{\mu}}\int\limits_{z^*}^{z}\xi_1dz-\frac{1}{\xi_0^{\mu-1}}\int\limits_{z^*}^{z}\frac{\partial^2\xi_1}{\partial t^2}dz+p_1^*\,(t) \tag{7}$$

$$\frac{p_1}{p_0}-\gamma\frac{\rho_1}{\rho_0}=v_1\,(z)$$

here $\xi_0\,(t)\,,p\,(t)\,,v_0\,(z)\,,\xi_1^*\,(t)\,,p_1^*\,(t)\,,v_1\,(z)$ are the unknown functions defined from boundary conditions.

Let $\xi_0\,(t)$ be a ShW motion law, then there will be $z^*=\rho^0\xi_0^{\mu}\,(t)\,/\mu$.

Then, from (4) we have: for $z = z^* = \rho^0 \xi_0^{\mu}(t)/\mu$ there should be

$$\xi_0 = \xi_0(t), \quad p_0 = \frac{1}{\gamma+1}\rho^0 \dot{\xi}_0^2 \qquad \rho_0 = \rho^0 / a\left(\dot{\xi}\right) \tag{8}$$

$$\xi_1 = 0, \quad p_1 = -p^0, \quad \rho_1 = 0.$$

It is convenient to pass from z to $\tau : z = \rho^0 \xi_0^{\mu}(\tau)/\mu$, then $z^* = \rho^0 \xi_0^{\mu}(t)/\mu$. Finally for p_0, p_1, ξ_1 we get

$$p_0 = \frac{2}{\gamma+1}\rho^0 \dot{\xi}_0^2 + \frac{1}{\mu}\rho^0 \xi_0 \ddot{\xi} - \ddot{\xi}_0 \xi_0^{1-\mu} z$$

$$p_1 = -(\mu-1)\frac{\rho^0 \ddot{\xi}_0}{\xi_0^{\mu}} \int_{\tau}^{t} \xi_1(t,\zeta)\, \xi_0^{\mu-1}(\zeta)\, \dot{\xi}_0(\zeta)\, d\zeta +$$

$$+\frac{\rho^0}{\xi_0^{\mu-1}} \int_{\tau}^{t} \frac{\partial^2 \xi_1}{\partial t^2} \xi_0^{\mu-1}(\zeta)\, \dot{\xi}_0(\zeta)\, d\zeta - p^0; \tag{9}$$

$$\xi_1 = \xi_1(t,\tau) = -\frac{1}{\xi_0^{\mu-1}} \int_{\tau}^{t} a\left(\dot{\xi}(\zeta)\right) \psi(t,\tau)\, \dot{\xi}_0^{1+\frac{2}{\gamma}}(\zeta)\, d\zeta,$$

$$\psi(t,\zeta) = \left[\dot{\xi}_0^2(t) + \frac{1}{\mu}\xi_0(t)\, \ddot{\xi}_0(t)\left(1 - \frac{\xi_0^{\mu}(\zeta)}{\xi_0^{\mu}(t)}\right)\right]^{-\frac{1}{\gamma}}.$$

This solution was expressed by $\xi_0(t)$; this function is found from the piston condition: for $\tau = 0$ $(z = 0)$ there should be

$$\xi(t) = \xi_0(t) + \varepsilon \xi_1(t) = z(t) = kvt - w(vt,t), \tag{10}$$

Functional $\xi_1(t)$ is essentially non-linear, therefore (10) is solved by the sequential approximations method. Procedure of the method, estimation and reasons in favour of convergence is in the paper [1], and we don't cite it here. We finally get (addends with ε at the first degree were retained)

$$\xi_0(t) = Dt - (1 + \varepsilon a(D)/\mu)\, w(vt,t) + \frac{\varepsilon}{2\mu^2\gamma} a(D)\, \ddot{w}(vt,t)\, t^2 -$$

$$-\frac{2\varepsilon}{\gamma}\left[(1-\gamma)\, a(D) + \gamma\right] t^{1-\mu} \int_0^t \tau^{\mu-1} \dot{w}(v\tau,\tau)\, d\tau. \tag{11}$$

2^0. Definition of interaction pressure. In the case of conical shell in the plane $x = vt$ we have a plane problem on extension of a cylindric piston, therefore $\mu = 2$. We have from (11)

$$\xi_0(t) = Dt - \left(1 + 2\varepsilon + \frac{\varepsilon}{2} a(D)\right) w(vt,t) + \frac{\varepsilon}{\gamma} a(D)\, \dot{w}(vt,t)\, t +$$

$$+\frac{\varepsilon}{8\gamma} a(D)\, \ddot{w}(v,t)\, t^2 - \frac{2\varepsilon}{t} \int_0^t w(v\zeta,\zeta)\, d\zeta$$

$$\xi_1(t,\tau)=\frac{Da(D)}{2}\left(\frac{\xi^2}{t}-t\right)-\frac{a(D)}{\gamma}\dot{w}(vt,t)\left(1-\frac{\xi^2}{t}\right)+\frac{2}{\gamma t}\int\limits_{\tau}^{t}w(vs,s)\,ds-$$

$$-w(vt,t)\left[a(D)\frac{\xi^2}{t^2}-2(1+a(D))\right]-\frac{a(D)}{8\gamma}\ddot{w}(vt,t)\left(t^2-2\tau^2+\frac{\tau^4}{t^2}\right).$$

By passing to the problem on streamline of a cone in the Euler system of coordinates connected with fixed body, it should be accepted:

$$\dot{w}=\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial x};\quad t=v/x$$

substitute $\xi_0(t)$ and $\xi_1(t,\tau)$ into (9) and carry out estimations similar to one in [1]; for the pressure to pass to the surface of a shell we'll get

$\Delta p=\left(p+\varepsilon p_1-p^0\right)_{\tau=0}=q_0(x)+q_1(x,t);$

here $q_0(x)$ is a quasistatic constituent, $q_1(x,t)$ is a dynamic one.

$$q_0(x)=\frac{2\rho^0D^2}{\gamma+1}\left(1+\varepsilon\frac{a(D)}{4}-\frac{\gamma p^0}{2\rho^0D^2}\right)-$$

$$-\frac{4\rho^0Dv}{\gamma+1}\left(1+\frac{3\varepsilon}{4}-\varepsilon\frac{11a(D)}{8\gamma}\right)\frac{\partial w_0}{\partial x}-$$

$$-\frac{\rho^0Dvx}{2}\left(1-\varepsilon\frac{3a(D)}{2\gamma(\gamma+1)}\right)\frac{\partial^2w_0}{\partial x^2},\tag{12}$$

$$q_1(x,t)=-\frac{4\rho^0D}{\gamma+1}\left(1+\frac{3\varepsilon}{4}-\varepsilon\frac{11a(D)}{8\gamma}\right)\left(\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial x}\right)-$$

$$-\frac{\rho^0Dvx}{2}\left(1-\varepsilon\frac{3a(D)}{2\gamma(\gamma+1)}\right)\frac{\partial^2w}{\partial x^2}.\tag{13}$$

Velocity of shock wave D is determined from quadratic equation $\varepsilon Da(D)+2vtg\alpha=2D$; after introdusing denotation $Mtg\beta=z$, $Mtg\alpha=z_0$ this equation takes the form $(3+\gamma)z^2-2(\gamma+1)z_0z-2=0$.

State of a shell is described by the equations of technical theory in a mixed form. Since $\Delta p=q_0+q_1$, we represent deflections and efforts functions in the sum of the basic (quasistatistical) and perturbed (dynamic) states; $w=w_0(x)+w_1(x,t)$; $F=F_0(x)+F_1(x,t)$.

Let's linearize the basic system, introduce dimensionless coordinates and parameters and make estimations in the pressure function q_0; we get a basic state equation

$$\frac{tg\alpha}{12(1-v^2)}\frac{h^2}{r_2^2}\Delta^2\dot{w}_0-\frac{1}{s}\frac{s\partial^2F_0}{\partial s^2}=q_0^*;\tag{14}$$

$$tg\alpha\Delta^2F_0+\frac{1}{s}\frac{\partial^2w_0}{\partial s^2}=0,$$

boundary conditions of hinge support

$$s=s_1,\quad s=1:w_0=0,\quad\frac{\partial^2w_0}{\partial s^2}+\frac{v}{s}\frac{\partial w_0}{\partial s}=0\tag{15}$$

$$\frac{\partial F_0}{\partial s} = 0, \quad \frac{\partial^2 F_0}{\partial s^2} = 0,$$

here s is a dimensionless coordinate

$$q_0^* = B_1 \left(1 + \frac{\varepsilon}{4} a^* (z) - \frac{1}{2z^2} \right);$$

$$B_1 = \frac{2\gamma}{\gamma + 1} \frac{p_0}{E} \frac{r_2^2}{h^2} z^2 tg\alpha; \quad a^* (z) = 1 + \frac{2}{(\gamma - 1) z^2}$$

The solution of the system in perturbations is sought in the class of functions $w = W(s) \cos n\varphi \exp(\omega t)$; $F = \Phi(s) \cos n\varphi \exp(\omega t)$. For $W(s)$, $\Phi(s)$ we get the system

$$tg\alpha \Delta_n^2 \Phi + \frac{1}{s} W'' = 0,$$

$$\frac{tg\alpha}{12(1 - v^2)} \frac{h^2}{r_2^2} \Delta_n^2 W - \frac{1}{s} \Phi'' - tg\alpha \frac{h}{r_2} F_0' \frac{1}{s} W'' -$$

$$-tg\alpha \frac{h}{r_2} F_0'' \left(\frac{1}{s} W' - \frac{n^2}{s^2 \sin^2 \alpha} W \right) + A_3 s W'' + A_2 W'' = \lambda W \qquad (16)$$

here $\Delta_n = \partial^2/\partial s^2 - (\partial/\partial s)/s - n^2/\sin^2\alpha$; $A_4\Omega^2 + A_1\Omega + \lambda = 0$, $\Omega = r_2\omega/c_0$, $c_0^2 = E/\rho$,

ρ is density of shell's material; parameters A_i in a sufficiently complicated way depend on $z = Mtg\beta$. Boundary conditions of a hinge support

$$s = s_1, \ s = 1 : W = 0, \ W'' + \frac{1}{s} W' = 0$$

$$\Phi' - \frac{n^2}{\sin^2 \alpha} \Phi = 0; \quad \Phi'' = 0 \qquad (17)$$

Statement of the flutter problem is traditional; in a complex plane λ it is constructed a stability parabola $A_4 (Jm\lambda)^2 = A_1^2 \operatorname{Re} \lambda$ that separates the domain of stable ($\operatorname{Re}\Omega < 0$) and unstable ($\operatorname{Re}\Omega > 0$) vibrations ; λ located interior to a parabola responds to stable vibrations. As is known, eigen-value problem (16), (17) has a discrete spectrum, therefore, in fact, the problem is stated as follows; to find the eigen value that by increasing M will first come to stability parabola.

Remark 1. For $M \leq M_{kp}$ the basic state should be statically stable;

Remark 2. Critical velocity depends on n : $M_{kp} = M_{kp}(n)$; $M_{kp}(n_{kp}) = \min\limits_n M_{kp}(n)$ is assumed to be truth critical velocity of a flutter.

References

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