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ON AN INITIAL BOUNDARY-VALUE PROBLEM FOR OPERATOR-DIFFERENTIAL EQUATIONS OF THIRD ORDER IN HILBERT SPACE

Abstract

In the paper we obtain the conditions on coefficients of operator-differential equation of third order with discontinuous coefficients, that provide regular solvability of constructed initial-boundary problem in abstract Hilbert space. These conditions are expressed only by the properties of operator coefficients of operator-differential equation. Further, the estimations of intermediate derivatives are obtained.

In a separable Hilbert space let's consider the initial boundary-value problem

$$\frac{d^3u(t)}{dt^3} + \rho(t) A^3(t) + \sum_{j=0}^{3} A_{3-j} u^{(j)} = f(t), \ t \in R_+ = (0, \infty) \tag{1}$$

$$u''(0) = 0, \tag{2}$$

where the operator coefficients satisfy the following conditions:

1) A is a normal inverse operator whose spectrum is contained in corner sector

$$S_\varepsilon = \{\lambda : |\arg \lambda| \le \varepsilon\}, \ 0 \le \varepsilon < \frac{\pi}{6};$$

2) The operators $B_j = A_j A^{-j} \ \left(j = \overline{0,3}\right)$ are bounded in H;

3) $\rho(t) = \alpha^3$ at $t \in (0,1)$ and $\rho(t) = \beta^3$ at $t \in (1, \infty), \alpha > 0, \beta > 0, (\alpha \ne \beta)$. Here $f(t) \in L_2(R_+; H)$, and $u(t) \in W_2^3(R_+, H)$, where (see [1])

$$L_2(R_+; H) = \left\{ f : \|f\|_{L_2(R_+;H)} = \left(\int_0^\infty \|f(t)\|^2 dt \right)^{1/2} < \infty \right\},$$

$$W_2^3(R_+; H) = \left\{ u : A^3 u, \frac{d^3u}{dt^3} \in L_2(R_+; H), \|u\|_{W_2^3(R_+;H)} = \right.$$

$$= \left\{ \left(\left\| \frac{d^3u}{dt^3} \right\|^2_{L_2(R_+;H)} + \left\| A^3 u \right\|^2_{L_2(R_+;H)} \right)^{1/2} \right. .$$

Denote by $W_2^3(R_+; H; 2) = \left\{ u : u \in W_2^3(R_+; H), u''(0) = 0 \right\}$.

At fulfillment of condition 1) the operator A is represented in the form $A = UC$, where U is a unitary, and C is a positive definite operator, moreover $UCx = CUx, \ x \in D\left(A^2\right)$.

Denote by H_α a Hilbert space with scalar derivative $(x, y)_\alpha = (C^\alpha x, C^\alpha y), \alpha \ge 0$.

Definition 1. *If at $f(t) \in L_2(R_+;H)$ there exists a vector-function $u(t) \in W_2^3(R_+;H)$ which satisfies equation (1) almost everywhere in R_+, then we'll call $u(t)$ a regular solution of equation (1).*

If regular solution $u(t)$ satisfies the boundary condition (2) in the sense

$$\lim_{t\to 0}\left\|u''(t)\right\|_{1/2}=0$$

then we'll call $u(t)$ a regular solution of problem (1)-(2).

Definition 2. *If at any $f(t) \in L_2(R_+;H)$ there exists a regular solution of problem (1)-(2), which satisfies inequality*

$$\|u\|_{W_2^3(R_+;H)} \le const \cdot \|f\|_{L_2(R_+;H)},$$

we'll call the problem (1)-(2) regular-solvable.

In the present paper the conditions of regular solvability of problem (1)-(2) are found.

Denote by

$$P_0u=-\frac{d^3u}{dt^3}+\rho(t)A^3u,\ P_1u=\sum_{j=0}^{3}A_{3-j}u^{(j)},$$

where $u \in W_2^3(R_+;H;2)$.

It holds

Theorem 1. *Let the condition 1) be fulfilled. Then the operator P_0 maps the space $W_2^3(R_+;H;2)$ isomorphically onto $L_2(R_+;H)$.*

Proof. At first we show that the equation $P_0u=0$ has only a zero solution from the space $W_2^3(R_+;H;2)$.

Really, the equation $P_0u=0$ has a solution from the space $W_2^3(R_+;H)$ in the form

$$u_0(t)=\begin{cases}\xi_1(t)=e^{\alpha\omega_1 tA}\varphi_1+e^{\alpha\omega_2(t-1)A}\varphi_2+e^{\alpha\omega_3(t-1)A}\varphi_3, & t\in(0,1)\\ \xi_2(t)=e^{\beta\omega_1(t-1)A}\varphi_4, & t\in(1,\infty)\end{cases} \tag{3}$$

where $\varphi_j \in H_{5/2}\left(j=\overline{1,4}\right)$, $\omega_1=-1$, $\omega_2=\frac{1}{2}-i\frac{\sqrt{3}}{2}$, $\omega_3=\frac{1}{2}+i\frac{\sqrt{3}}{2}$.

From the condition $u_0(t) \in W_2^3(R_+;H;2)$ it follows that $u_0(t)=0$. Further from the results of the paper [4] it follows that the equation

$$P_0u=\frac{d^3u}{dt^3}+\rho(t)A^3u=f(t) \tag{4}$$

has a regular solution $u_1(t)$ from the space $W_2^3(R;H)$. Then the general solution of equation (4) from the space $W_2^3(R;H)$ will have the following form:

$$u(t)=\tilde{u}_1(t)+u_0(t),$$

where $\tilde{u}_1(t)$ is a contraction $u_1(t)$ on $[0,\infty)$, and $u_0(t)$ is determined from (3).

Using the belongness $u(t) \in W_2^3(R_+;H;2)$ it is easy to determine the vectors $\varphi_1,\varphi_2,\varphi_3,\varphi_4 \in H_{5/2}$ and find the solution of the equation $P_0u=f$. Since P_0 is

a continuous operator from $W_2^3(R_+;H;2)$ on $L_2(R_+;H)$, then from the Banach theorem on inverse operator it follows the truth of the assertion of the theorem.

Lemma. *Let the condition 1) be fulfilled. Then at any* $u(t)\in W_2^3(R_+;H;2)$ *it holds the inequality*

$$\left\|\rho^{-1/2}P_0u\right\|^2_{L_2(R_+;H)}\geq\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}+\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}+$$

$$+\cos 3\varepsilon\left\|u'(0)\right\|^2_{3/2}-2\sin 3\varepsilon\cdot\left\|\rho^{1/2}A^3u\right\|_{L_2(R_+;H)}\cdot\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}. \qquad (5)$$

Proof. Multiplying the both hand sides of equality $P_0u=\frac{d^3u}{dt^3}+\rho(t)A^3u$ by $\rho^{-1/2}(t)$we find

$$\left\|\rho^{-1/2}\frac{d^3u}{dt^3}+\rho^{1/2}A^3u\right\|^2_{L_2\left(R_{+;H}\right)}=\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}+\left\|\rho^{1/2}A^3u\right\|_{L_2(R_+;H)}+$$

$$+2\operatorname{Re}\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}=\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}. \qquad (6)$$

Allowing for $u\in W_2^3(R_+;H;2)$ after integrating by parts we obtain:

$$\left(\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}=\left(A^{*1/2}u'(0),A^{3/2}u'(0)\right)-\left(A^{*3}u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}$$

or

$$2\operatorname{Re}\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}=\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}+\left(A^3u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}=$$

$$=\left(A^{*1/2}u'(0),A^{3/2}u'(0)\right)+\left(\left(A^3-A^{*3}\right)u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}\geq$$

$$\geq\cos 3\varepsilon\left\|u'(0)\right\|^2_{3/2}-2\sin 3\varepsilon\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}\cdot\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}. \qquad (7)$$

Allowing for (7) in equality (6) we complete the proof of the lemma.

Theorem 2. *Let the condition 1) be fulfilled. Then at any* $u\in W_2^3(R_+;H;2)$ *it holds the following estimations:*

$$\left\|A^{3-j}u^{(j)}\right\|_{L_2(R_+;H)}\leq C_j(\varepsilon;\alpha;\beta)\left\|P_0u\right\|_{L_2(R_+;H)},\quad j=\overline{0,3}, \qquad (8)$$

where

$$C_0(\varepsilon,\alpha,\beta)=\frac{1}{\cos 3\varepsilon}\cdot\frac{1}{\min\left(\alpha^3;\beta^3\right)},\ C_1(\varepsilon;\alpha;\beta)=$$

$$=\frac{2}{3^{1/2}}(1-\sin 3\varepsilon)^{-1/2}\frac{\max\left(\alpha^{1/2};\beta^{1/2}\right)}{\min\left(\alpha^{5/2};\beta^{5/2}\right)},$$

$$C_2(\varepsilon,\alpha,\beta)=\frac{2^{2/3}}{3^{1/2}}(1-\sin 3\varepsilon)^{-1/2}\frac{\max(\alpha;\beta)}{\min\left(\alpha^2;\beta^2\right)},$$

$$C_3(\varepsilon,\alpha,\beta)=\frac{1}{\cos 3\varepsilon}\frac{\max\left(\alpha^{3/2};\beta^{3/2}\right)}{\min\left(\alpha^{3/2};\beta^{3/2}\right)},$$

Proof. From inequality (5) it follows that

$$\left\|\rho^{-1/2}P_0u\right\|^2_{L_2(R_+;H)}\geq\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}-$$

$$-2\sin 3\varepsilon\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}\cdot\left\|\rho^{1/2}A^3u\right\|_{L_2(R_+;H)}+$$

$$+\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}\geq\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}-\sin^2 3\varepsilon\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}-$$

$$-\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}+\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}=$$

$$=\cos^2 3\varepsilon\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}. \tag{9}$$

Analogously we have

$$\left\|\rho^{-1/2}P_0u\right\|^2_{L_2(R_+;H)}\geq\cos^2 3\varepsilon\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}. \tag{10}$$

From inequality (9) it follows that

$$\left\|A^3u\right\|^2_{L_2(R_+;H)}\leq\max_t\rho^{-1}(t)\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}\leq$$

$$\leq\max_t\rho^{-1}(t)\cdot\frac{1}{\cos^2 3\varepsilon}\left\|\rho^{-1/2}P_0u\right\|^2_{L_2(R_+;H)}\leq$$

$$\leq\max_t\rho^{-2}(t)\cdot\frac{1}{\cos^2 3\varepsilon}\left\|P_0u\right\|^2.$$

Whence we obtain that

$$\left\|A^3u\right\|^2_{L_2(R_+;H)}\leq\frac{1}{\cos 3\varepsilon}\cdot\frac{1}{\min\left(\alpha^3,\beta^3\right)}\left\|P_0u\right\|_{L_2(R_+;H)}. \tag{11}$$

Analogously we obtain that

$$\left\|\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}\leq\frac{1}{\cos 3\varepsilon}\frac{\max\left(\alpha^{3/2};\beta^{3/2}\right)}{\min\left(\alpha^{3/2};\beta^{3/2}\right)}\left\|P_0u\right\|_{L_2(R_+;H)}. \tag{12}$$

Thus, inequality (8) at $j=0$ and $j=3$ is true. Now let's prove inequality (8) in other cases.

At $u \in W_2^3(R_+;H;2)$ after integrating by part we obtain that

$$\left\|A\frac{d^2u}{dt^2}\right\|^2_{L_2(R_+;H)} = \left\|C\frac{d^2u}{dt^2}\right\|_{L_2(R_+;H)} = \int\limits_0^\infty \left(C\frac{d^2u}{dt^2}, C\frac{d^2u}{dt^2}\right)dt =$$

$$= -\int\limits_0^\infty \left(C^2\frac{d^2u}{dt^2}, C\frac{d^3u}{dt^3}\right) \le \left\|C^2\frac{d^2u}{dt^2}\right\|_{L_2(R_+;H)} \cdot \left\|\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)} =$$

$$= \left\|A^2\frac{du}{dt}\right\|_{L_2(R_+;H)} \cdot \left\|\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}. \tag{13}$$

Further from the results of the paper [3] it follows that

$$\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} = \left\|C^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \le$$

$$\le 2\left\|C\frac{d^2u}{dt^2}\right\|_{L_2(R_+;H)} \cdot \left\|C^3u\right\|_{L_2(R_+;H)} =$$

$$= 2\left\|A\frac{d^2u}{dt^2}\right\|_{L_2(R_+;H)} \cdot \left\|A^3u\right\|_{L_2(R_+;H)}. \tag{14}$$

Allowing for (13) in (14) we obtain:

$$\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \le 2\left\|A^2\frac{du}{dt}\right\|^{1/2}_{L_2(R_+;H)} \cdot \left\|A^3u\right\|_{L_2(R_+;H)}$$

or

$$\left\|A^2\frac{du}{dt}\right\|^{3/2}_{L_2(R_+;H)} \le 2\left\|\frac{d^3u}{dt^3}\right\|^{1/2}_{L_2(R_+;H)} \cdot \left\|A^3u\right\|_{L_2(R_+;H)}.$$

Hence we have ($\delta > 0$):

$$\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \le 2^{4/3}\left(\left\|\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}\right)^{1/3} \cdot \left(\left\|A^3u\right\|^2_{L_2(R_+;H)}\right)^{2/3} \le$$

$$\le 2^{4/3}\left(\max_t \rho(t)\right)^{1/3} \cdot \left(\max_t \rho^{-1}(t)\right)^{2/3} \cdot$$

$$\cdot \left(\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}\right)^{1/3} \left(\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}\right)^{2/3} \le$$

$$\le 2^{4/3}\frac{\max(\alpha;\beta)}{\min\left(\alpha^2;\beta^2\right)}\left(\delta\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^{1/3}_{L_2(R_+;H)}\right)^{1/3} \cdot \left(\frac{1}{\delta^{1/2}}\left\|\rho^{1/2}A^3u\right\|^2\right)^{2/3}_{L_2(R_+;H)} \le$$

$$\le 2^{4/3}\left(\frac{1}{3}\delta\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^{1/3}_{L_2(R_+;H)} + \frac{2}{3}\frac{1}{\delta^{1/2}}\left\|\rho^{1/2}A^3u\right\|^2\right)^2_{L_2(R_+;H)}. \tag{15}$$

Choosing $\frac{1}{3}\delta = \frac{2}{3\delta^{1/2}}$, i.e. $\delta = 2^{2/3}$, from (15) we obtain:

$$\left\| A^2 \frac{du}{dt} \right\|^2_{L_2(R_+;H)} \leq \frac{2^2}{3} \cdot \frac{\max(\alpha;\beta)}{\min\left(\alpha^2;\beta^2\right)} \cdot$$

$$\cdot \left(\left\| \rho^{-1/2} \frac{d^3u}{dt^3} \right\|^2_{L_2(R_+;H)} + \left\| \rho^{1/2} A^3 u \right\|^2_{L_2(R_+;H)} \right).$$

Allowing for the lemma in this inequality we obtain that

$$\left\| A^2 \frac{du}{dt} \right\|^2_{L_2(R_+;H)} \leq \frac{2^2}{3} \cdot \frac{\max(\alpha;\beta)}{\min\left(\alpha^2;\beta^2\right)} \cdot \frac{1}{1-\sin 3\varepsilon} \left\| \rho^{-1/2} P_0 u \right\|^2_{L_2(R_+;H)} \leq$$

$$\leq \frac{2^2}{3} \frac{\max(\alpha;\beta)}{\min\left(\alpha^5;\beta^5\right)} \cdot \frac{1}{1-\sin 3\varepsilon} \left\| \rho^{-1/2} P_0 u \right\|^2_{L_2(R_+;H)} \leq$$

$$\leq \frac{2^2}{3} \frac{\max(\alpha;\beta)}{\min\left(\alpha^5;\beta^5\right)} \cdot \frac{1}{1-\sin 3\varepsilon} \left\| P_0 u \right\|^2_{L_2(R_+;H)},$$

i.e.,

$$\left\| A \frac{du}{dt} \right\|_{L_2(R_+;H)} \leq \frac{2^2}{3^{1/2}} \frac{\max\left(\alpha^{1/2};\beta^{1/2}\right)}{\min\left(\alpha^{5/2};\beta^{5/2}\right)} \cdot \frac{1}{(1-\sin 3\varepsilon)^{1/2}} \left\| P_0 u \right\|^2_{L_2(R_+;H)}. \tag{16}$$

Further, analogously we obtain:

$$\left\| A^2 \frac{d^2u}{dt^2} \right\|^2_{L_2(R_+;H)} \leq 2^{1/2} \left\| A^2 \frac{d^2u}{dt^2} \right\|^{1/2}_{L_2(R_+;H)} \cdot \left\| A^3 u \right\|^{1/2}_{L_2(R_+;H)} \cdot \left\| \frac{d^3u}{dt^3} \right\|_{L_2(R_+;H)}$$

or

$$\left\| A \frac{d^2u}{dt^2} \right\|_{L_2(R_+;H)} \leq 2^{1/3} \left\| A^3 u \right\|^{1/3}_{L_2(R_+;H)} \cdot \left\| \frac{d^3u}{dt^3} \right\|_{L_2(R_+;H)} \leq$$

$$\leq 2^{1/3} \left(\max_t \rho^{-1/2}(t) \right)^{1/3} \cdot \left(\max_t \rho^{1/2}(t) \right)^{2/3} \cdot$$

$$\cdot \left\| \rho^{1/2} A^3 u \right\|^{1/3}_{L_2(R_+;H)} \cdot \left\| \rho^{-1/2} \frac{d^3u}{dt^3} \right\|_{L_2(R_+;H)}.$$

Hence we have ($\delta > 0$)

$$\left\| A \frac{d^2u}{dt^2} \right\|^2_{L_2(R_+;H)} \leq 2^{2/3} \frac{\max\left(\alpha^2;\beta^2\right)}{\min(\alpha;\beta)} \left(\delta \left\| \rho^{1/2} A^3 u \right\|^2_{L_2(R_+;H)} \right) \cdot$$

$$\cdot \left(\frac{1}{\delta^{1/2}} \left\| \rho^{-1/2} \frac{d^3u}{dt^3} \right\|^2_{L_2(R_+;H)} \right)^{2/3} \leq 2^{2/3} \frac{\max\left(\alpha^2;\beta^2\right)}{\min(\alpha;\beta)} \cdot$$

$$\cdot \left(\frac{\delta}{3} \left\| \rho^{1/2} A^3 u \right\|^2_{L_2(R_+;H)} + \frac{2}{3\delta^{1/2}} \left\| \rho^{-1/2} \frac{d^3u}{dt^3} \right\|^2_{L_2(R_+;H)} \right).$$

Assuming $\frac{\delta}{3}=\frac{2}{3\delta^{1/2}}$, i.e., $\delta=2^{2/3}$ we have:

$$\left\|A\frac{d^2u}{dt^2}\right\|^2_{L_2(R_+;H)}\leq 2^{2/3}\frac{2^{2/3}}{3}\frac{\max\left(\alpha^2;\beta^2\right)}{\min\left(\alpha;\beta\right)}\left(\left\|\rho^{1/2}A^3u\right\|^2_{L_2(R_+;H)}+\right.$$

$$\left.+\left\|\rho^{-1/2}\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}\right)\leq\frac{2^{4/3}}{3}\frac{\max\left(\alpha^2;\beta^2\right)}{\min\left(\alpha;\beta\right)}\cdot$$

$$\cdot\frac{1}{1-\sin 3\varepsilon}\cdot\frac{1}{\min\left(\alpha^3;\beta^3\right)}\cdot\|P_0u\|^2_{L_2(R_+;H)}$$

i.e.,

$$\left\|A\frac{d^2u}{dt^2}\right\|^2_{L_2(R_+;H)}\leq\frac{2^{2/3}}{3^{1/2}}\frac{\max\left(\alpha;\beta\right)}{\min\left(\alpha^2;\beta^2\right)}\cdot$$

$$\cdot\frac{1}{\left(1-\sin 3\varepsilon\right)^{1/2}}\|P_0u\|^2_{L_2(R_+;H)}\,. \tag{17}$$

From inequality (11), (12), (16), (17) it follows the assertion of the theorem.

Theorem 3. *Let conditions 1)-3) be fulfilled and the inequality*

$$K\left(\varepsilon;\alpha;\beta\right)=\sum_{j=0}^{3}C_j\left(\varepsilon;\alpha;\beta\right)\|B_{3-j}\|<1,$$

hold, where the numbers $C_j\left(\varepsilon;\alpha;\beta\right)$ $\left(j=\overline{0,3}\right)$ *are determined from theorem 2. Then problem (1)-(2) is regularly solvable.*

Proof. Let's write the problem (1)-(2) in the form of equation $P_0u+P_1u=f$, where $f\in L_2\left(R_+;H\right),\ u\in W_2^3\left(R_+;H;2\right)$. By theorem 1 the operator P_0^{-1} exists and bounded. Denote by $u=P_0^{-1}\upsilon$. Then we obtain the equation $\upsilon+P_1P_0^{-1}\upsilon=f$ in the space $L_2\left(R_+;H\right)$.

Since by theorem 2,

$$\left\|P_1P_0^{-1}\upsilon\right\|_{L_2(R_+;H)}=\|P_1u\|_{L_2(R_+;H)}\leq\sum_{j=0}^{3}\left\|A_{3-j}u^{(j)}\right\|_{L_2(R_+;H)}\leq$$

$$\leq\sum_{j=0}^{3}\|B_{3-j}\|\,C_j\left(\varepsilon;\alpha;\beta\right)\|P_0u\|_{L_2(R_+;H)}=$$

$$=\left(\sum_{j=0}^{3}\|B_{3-j}\|\,C_j\left(\varepsilon;\alpha;\beta\right)\right)\|\upsilon\|_{L_2(R_+;H)}=K\left(\varepsilon;\alpha;\beta\right)\|\upsilon\|_{L_2(R_+;H)}$$

and $K\left(\varepsilon;\alpha;\beta\right)<1$,then operator $E+P_1P_0^{-1}$ is invertible in the space $L_2\left(R_+;H\right)$ and therefore $u=P_0^{-1}\left(E+P_1P_0^{-1}\right)^{-1}f$. Hence it follows that $\|u\|_{W_2^3(R_+;H)}\leq const\,\|f\|_{L_2(R_+;H)}$.

The theorem is proved.

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