Mehman N.SADIKHOV

ON EXISTENCE IN LARGE FOR ALMOST EVERYWHERE SOLUTION OF ONE-DIMENSIONAL MIXED PROBLEM FOR A CLASS OF KORTEWEG-DE VRIES-BURGERS TYPE NONLINEAR EQUATIONS

Abstract

This work presents a study of one-dimensional mixed problem with Riquier type homogenous boundary conditions for a class of Korteweg-de Vries-Burgers type semilinear differential equations. The concept of almost everywhere solution for the given mixed problem is introduced. The almost everywhere solution $u(t,x)$ of mixed problem under consideration is sought in the form of Fourier series

$$u(t,x)=\sum_{n=1}^{\infty}u_n(t)\sin nx \quad (0\le t\le T,\ 0\le x\le \pi).$$

After applying Fourier method, the problem of finding unknown Fourier coefficients $u_n(t)$ $(n=1,2,...)$ of sought almost everywhere solution $u(t,x)$ is reduced to solving some countable system of nonlinear integral equations. Then, the a priori estimate in $C\left([0,T];W_2^4(0,\pi)\right)$ is obtained for all the possible almost everywhere solutions of mixed problem under consideration, which, in turn, helps to prove existence in large theorem for almost everywhere solution.

This work is devoted to the study of existence in large for almost everywhere solution of the following one-dimensional mixed problem:

$$u_t(t,x)+\alpha u_{txxxx}(t,x)=F(t,x,u(t,x),u_x(t,x),\ u_{xx}(t,x),u_{xxx}(t,x),u_{xxxx}(t,x))$$

$$(0\le t\le T,\ 0\le x\le \pi), \tag{1}$$

$$u(0,x)=\varphi(x)\ (0\le x\le \pi), \tag{2}$$

$$u(t,0)=u(t,\pi)=u_{xx}(t,0)=u_{xx}(t,\pi)=0\ (0\le t\le T), \tag{3}$$

where $\alpha>0$ is a fixed constant; $0<T<+\infty$; F and φ are the given functions, and $u(t,x)$ is a sought function. We make a definition of an almost everywhere solution of problem (1)-(3) as follows:

Definition. *We define an almost everywhere solution of problem (1)-(3) as a function $u(t,x)$ with the following properties:*

a) $u(t,x),u_x(t,x),u_{xx}(t,x),\ u_{xxx}(t,x),\ u_t(t,x),u_{tx}(t,x),\ u_{txx}(t,x),\ u_{txxx}(t,x)\in C\left([0,T]\times[0,\pi]\right);\ \ u_{xxxx}(t,x),u_{txxxx}(t,x)\in C([0,T];L_2(0,\pi))$;

b) equation (1) is satisfied almost everywhere in $(0,T)\times(0,\pi)$;

c) all the conditions (2) and (3) are satisfied in ordinary sense.

Note that in [1] and [2], existence in small theorem and uniqueness in large theorem for almost everywhere solution of problem (1)-(3) are proved. And in this work, using results of [1], we prove by means of a priori estimates method existence in large theorem for almost everywhere solution of problem (1)-(3).

As the system $\{\sin nx\}_{n=1}^{\infty}$ forms a basis in the space $L_2(0,\pi)$, then it is obvious that every almost everywhere solution of problem (1)-(3) has the following form:

$$u(t,x)=\sum_{n=1}^{\infty}u_n(t)\sin nx, \tag{4}$$

where

$$u_n(t)=\frac{2}{\pi}\int_0^{\pi}u(t,x)\sin nxdx\ (n=1,2,...;t\in[0,T]). \tag{5}$$

In the next, after applying Fourier method, the finding of functions $u_n(t)$ $(n=1,2,...)$ is reduced to solving the following countable system of nonlinear integral equations:

$$u_n(t)=\varphi_n+\frac{2}{\pi}\cdot\frac{1}{1+\alpha n^4}\cdot\int_0^t\int_0^{\pi}\mathcal{F}(u(\tau,x))\sin nxdxd\tau\quad(n=1,2,...;t\in[0,T]) \tag{6}$$

where

$$\varphi_n\equiv\frac{2}{\pi}\int_0^{\pi}\varphi(x)\sin nxdx\ (n=1,2,...), \tag{7}$$

$$\mathcal{F}(u(t,x))\equiv F(t,x,u(t,x),u_x(t,x),u_{xx}(t,x),u_{xxx}(t,x),u_{xxxx}(t,x)). \tag{8}$$

Using the definition of almost everywhere solution of problem (1)-(3), it is easy to prove the following

Lemma. *If* $u(t,x)=\sum\limits_{n=1}^{\infty}u_n(t)\sin nx$ *is any almost everywhere solution of problem (1)-(3), then functions* $u_n(t)$ $(n=1,2,...)$ *satisfy the system (6).*

We denote by $B^{\alpha_0,\ldots,\alpha_l}_{\beta_0,\ldots,\beta_l,T}$ a totality of all the functions $u(t,x)$ of the form (4) considered in $[0,T]\times[0,\pi]$ for which all the functions $u_n(t)\in C^{(l)}([0,T])$ and

$$J_T(u)\equiv\sum_{i=0}^{l}\left\{\sum_{n=1}^{\infty}\left(n^{\alpha_i}\cdot\max_{0\le t\le T}\left|u_n^{(i)}(t)\right|\right)^{\beta_i}\right\}^{\frac{1}{\beta_i}}<+\infty, \tag{9}$$

where $l\ge0$ is an integer, $\alpha_i\ge0(i=\overline{0,l})$, $1\le\beta_i\le2$ $(i=\overline{0,l})$. We define the norm in this set as $\|u\|=J_T(u)$. It is known (see [4] or [5]) that all these spaces are Banach spaces.

Throughout this paper we will use the following notations for functions $u(t,x)\in B^{\alpha_0,\ldots,\alpha_l}_{\beta_0,\ldots,\beta_l,T}$:

$$\|u\|_{B^{\alpha_0,\ldots,\alpha_l}_{\beta_0,\ldots,\beta_l,t}}\equiv\sum_{i=0}^{l}\left\{\sum_{n=1}^{\infty}\left(n^{\alpha_i}\cdot\max_{0\le\tau\le t}\left|u_n^{(i)}(\tau)\right|\right)^{\beta_i}\right\}^{\frac{1}{\beta_i}}\ (0\le t\le T). \tag{10}$$

In [1], by combining the generalized contracted mappings principle and Schauder's fixed point principle the following existence in small theorem (that is, true for sufficiently small values of T) for almost everywhere solution of problem (1)-(3) is proved:

Theorem 1. *Let*

1. $\varphi(x) \in C^{(3)}([0,\pi]), \varphi^{(4)}(x) \in L_2(0,\pi)$ *and* $\varphi(0) = \varphi(\pi) = \varphi''(0) = \varphi''(\pi) = 0$.

2. $F(t,x,u_1,...,u_5) \in C\left([0,T] \times [0,\pi] \times (-\infty,\infty)^5\right)$.

3. $\forall R > 0$ *in* $[0,T] \times [0,\pi] \times [-R,R]^4 \times (-\infty,\infty)$

$$|F(t,x,u_1,...,u_4,u_5) - F(t,x,u_1,...,u_4,\tilde{u}_5)| \leq C_R \cdot |u_5 - \tilde{u}_5|,$$

where $C_R > 0$ *is a constant.*

Then there exists in small an almost everywhere solution of problem (1)-(3).

Remark 1. As seen from the proof of Theorem 1 (available in [1]), to prove the existence in large for almost everywhere solution of problem (1)-(3) under the conditions of Theorem 1, it suffices to show that all the possible almost everywhere solutions of problem (1)-(3) belonging to $B^4_{2,T}$ are a priori bounded in $B^4_{2,T}$. With this aim, we prove the following two theorems of a priori boundedness (in a certain sense) of almost everywhere solutions of problem (1)-(3).

Theorem 2. *Let the right side of equation (1) be as follows:*

$$F(t,x,u,u_x,u_{xx},u_{xxx},u_{xxxx}) = f(t,x,u,u_x,u_{xx},u_{xxx},u_{xxxx})+$$

$$+f_0(t,x,u)\cdot u+f_1(t,u)\cdot u_x+(f_2(t,x,u,u_x,u_{xx},u_{xxx}))_x+(f_3(t,x,u,u_x,u_{xx}))_{xx}, \quad (11)$$

where

a) $f(t,x,u_1,...,u_5) \in C\left([0,T] \times [0,\pi] \times (-\infty,\infty)^5\right)$, *and in* $[0,T] \times [0,\pi] \times (-\infty,\infty)^5$

$$f(t,x,u_1,...,u_5) \cdot u_1 \leq C \cdot (1 + u_1^2 + u_2^2 + u_3^2); \quad (12)$$

b) $f_0(t,x,u) \in C([0,T] \times [0,\pi] \times (-\infty,\infty))$, *and in* $[0,T] \times [0,\pi] \times (-\infty,\infty)$

$$f_0(t,x,u) \leq C; \quad (13)$$

c)

$$f_1(t,u) \in C([0,T] \times (-\infty,\infty)); \quad (14)$$

d) $f_2(t,x,u_1,...,u_4), f_{2,\xi_i}(t,\xi_0,\xi_1,...,\xi_4)(i = \overline{0,4}) \in C\left([0,T] \times [0,\pi] \times (-\infty,\infty)^4\right)$, *and in* $[0,T] \times [0,\pi] \times (-\infty,\infty)^4$

$$-f_2(t,x,u_1,...,u_4) \cdot u_2 \leq C \cdot (1 + u_1^2 + u_2^2 + u_3^2); \quad (15)$$

e) $f_3(t,x,u_1,u_2,u_3), f_{3,\xi_i}(t,\xi_0,\xi_1,\xi_2,\xi_3)(i = \overline{0,3})$, $f_{3,\xi_i\xi_j}(t,\xi_0,\xi_1,\xi_2,\xi_3)(i,j = \overline{0,3}) \in C\left([0,T] \times [0,\pi] \times (-\infty,\infty)^3\right)$, *and in* $[0,T] \times [0,\pi] \times (-\infty,\infty)^3$

$$f_3(t,x,u_1,u_2,u_3) \cdot u_3 \leq C \cdot (1 + u_1^2 + u_2^2 + u_3^2); \quad (16)$$

besides,

$$f_3(t,0,0,u_2,0) = f_3(t,\pi,0,u_2,0) = 0 \ \ \forall t \in [0,T], u_2 \in (-\infty,\infty), \quad (17)$$

where $C > 0$ *is a constant.*

Then the following a priori estimate holds for all the possible almost everywhere solutions $u(t,x)$ *of problem (1)-(3):*

$$\int_0^\pi u_{xx}^2(t,x)dx \leq C_0 \quad \forall t \in [0,T]. \quad (18)$$

Proof. Let $u(t,x)$ be any almost everywhere solution of problem (1)-(3). Then, according to the definition of almost everywhere solution of problem (1)-(3), the equation (1) is satisfied almost everywhere in $(0,T)\times(0,\pi)$. On multiplying both sides of equation (1) by the function $2u(t,x)$, integrating the obtained equality over $[0,t]\times[0,\pi]$ and using relation (11), we get $\forall t\in[0,T]$:

$$2\int_0^t\int_0^\pi u_\tau(\tau,x)\cdot u(\tau,x)dxd\tau+2\alpha\int_0^t\int_0^\pi u_{\tau xxxx}(\tau,x)\cdot u_\tau(\tau,x)dxd\tau=$$

$$=2\int_0^t\int_0^\pi f(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x),u_{xxx}(\tau,x),u_{xxxx}(\tau,x))\cdot u(\tau,x)dxd\tau+$$

$$+2\int_0^t\int_0^\pi f_0(\tau,x,u(\tau,x))\cdot u^2(\tau,x)dxd\tau+2\int_0^t\int_0^\pi f_1(\tau,u(\tau,x))\cdot u_x(\tau,x)\cdot u(\tau,x)dxd\tau+$$

$$+2\int_0^t\int_0^\pi (f_2(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x),u_{xxx}(\tau,x)))_x\cdot u(\tau,x)dxd\tau+$$

$$+2\int_0^t\int_0^\pi (f_3(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x)))_{xx}\cdot u(\tau,x)dxd\tau. \quad (19)$$

Next, as $u(t,0)=u(t,\pi)$ $(0\le t\le T)$, then $\forall t\in[0,T]$ $\exists\xi=\xi_t\in(0,\pi)$ that $u_x(t,\xi_t)=0$. Then it is obvious that $\forall t\in[0,T]$ and $x\in[0,\pi]$:

$$u_x(t,x)=\int_{\xi_t}^x u_{\xi\xi}(t,\xi)d\xi,\ |u_x(t,x)|\le\int_0^\pi|u_{\xi\xi}(t,\xi)|\,d\xi=\int_0^\pi|u_{xx}(t,x)|\,dx,$$

$$u_x^2(t,x)\le\pi\cdot\int_0^\pi u_{xx}^2(t,x)dx,\ \int_0^\pi u_x^2(t,x)dx\le\pi^2\cdot\int_0^\pi u_{xx}^2(t,x)dx; \quad (20)$$

$$u(t,x)=\int_0^x u_\xi(t,\xi)d\xi,\ |u(t,x)|\le\int_0^x|u_\xi(t,\xi)|\,d\xi\le\int_0^\pi|u_x(t,x)|\,dx,$$

$$u^2(t,x)\le\pi\cdot\int_0^\pi u_x^2(t,x)dx\le\pi\cdot\pi^2\int_0^\pi u_{xx}^2(t,x)dx=\pi^3\cdot\int_0^\pi u_{xx}^2(t,x)dx, \quad (21)$$

$$\int_0^\pi u^2(t,x)dx\le\pi^4\cdot\int_0^\pi u_{xx}^2(t,x)dx. \quad (22)$$

Besides, using conditions (2) and (3), $\forall t\in[0,T]$ we have:

$$2\int_0^t\int_0^\pi u_\tau(\tau,x)\cdot u(\tau,x)dxd\tau=\int_0^\pi\left\{2\int_0^t u(\tau,x)\cdot u_\tau(\tau,x)d\tau\right\}dx=$$

$$= \int_0^\pi \left\{ \int_0^t \frac{\partial}{\partial \tau} [u^2(\tau, x)] d\tau \right\} dx =$$

$$= \int_0^\pi \left\{ u^2(t, x) - u^2(0, x) \right\} dx = \int_0^\pi u^2(t, x) dx - \int_0^\pi \varphi^2(x) dx, \tag{23}$$

$$2 \int_0^t \int_0^\pi u_{\tau xxxx}(\tau, x) \cdot u(\tau, x) dx d\tau = 2 \int_0^t \left\{ \int_0^\pi u_{\tau xxxx}(\tau, x) \cdot u(\tau, x) dx \right\} d\tau =$$

$$= 2 \int_0^t \left\{ u_{\tau xxx}(\tau, x) \cdot u(\tau, x) |_{x=0}^{x=\pi} - \int_0^\pi u_{\tau xxx}(\tau, x) \cdot u_x(\tau, x) dx \right\} d\tau =$$

$$= -2 \int_0^t \left\{ \int_0^\pi u_{\tau xxx}(\tau, x) \cdot u_x(\tau, x) dx \right\} d\tau =$$

$$= -2 \int_0^t \left\{ u_{\tau xx}(\tau, x) \cdot u_x(\tau, x) |_{x=0}^{x=\pi} - \int_0^\pi u_{\tau xx}(\tau, x) \cdot u_{xx}(\tau, x) dx \right\} d\tau =$$

$$= 2 \int_0^t \int_0^\pi u_{\tau xx}(\tau, x) \cdot u_{xx}(\tau, x) dx d\tau = \int_0^\pi \left\{ \int_0^t \frac{\partial}{\partial \tau} \left[u_{xx}^2(\tau, x) \right] d\tau \right\} dx =$$

$$= \int_0^\pi \left\{ u_{xx}^2(t, x) - u_{xx}^2(0, x) \right\} dx = \int_0^\pi u_{xx}^2(t, x) dx - \int_0^\pi \left(\varphi''(x) \right)^2 dx. \tag{24}$$

Next, using conditions (12), (13), (3), (15), (16), (17) and estimates (22), (20), we get $\forall t \in [0, T]$:

$$\int_0^t \int_0^\pi f(\tau, x, u(\tau, x), u_x(\tau, x), u_{xx}(\tau, x), u_{xxx}(\tau, x), u_{xxxx}(\tau, x)) \cdot u(\tau, x) dx d\tau \leq$$

$$\leq C \cdot \int_0^t \int_0^\pi \left\{ 1 + u^2(\tau, x) + u_x^2(\tau, x) + u_{xx}^2(\tau, x) \right\} dx \leq$$

$$\leq C \cdot \pi T + C \cdot (\pi^4 + \pi^2 + 1) \cdot \int_0^t \left\{ \int_0^\pi u_{xx}^2(\tau, x) dx \right\} d\tau; \tag{25}$$

$$\int_0^t \int_0^\pi f_0(\tau, x, u(\tau, x)) \cdot u^2(\tau, x) dx d\tau \leq C \cdot \int_0^t \int_0^\pi u^2(\tau, x) dx d\tau \leq$$

$$\leq C \cdot \pi^4 \cdot \int_0^t \left\{ \int_0^\pi u_{xx}^2(\tau, x) dx \right\} d\tau; \tag{26}$$

$$\int_0^t\int_0^\pi f_1(\tau,u(\tau,x))\cdot u_x(\tau,x)\cdot u(\tau,x)dxd\tau=\int_0^t\left\{\int_0^\pi\frac{\partial}{\partial x}\left[\int_0^{u(\tau,x)}f_1(\tau,\xi)\cdot\xi d\xi\right]dx\right\}d\tau=$$

$$=\int_0^t\left\{\int_0^{u(\tau,\pi)}f_1(\tau,\xi)\cdot\xi d\xi-\int_0^{u(\tau,0)}f_1(\tau,\xi)\cdot\xi d\xi\right\}d\tau=0; \qquad (27)$$

$$\int_0^t\int_0^\pi\left(f_2(\tau,\ x,u(\tau,x),u_x(\tau,x),\ u_{xx}(\tau,x),u_{xxx}(\tau,x))\right)_x\cdot u(\tau,x)dxd\tau=$$

$$=\int_0^t\{\,f_2(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x),u_{xxx}(\tau,x))\cdot u(\tau,x)|_{x=0}^{x=\pi}-$$

$$-\int_0^\pi f_2(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x),u_{xxx}(\tau,x))\cdot u_x(\tau,x)dx\right\}d\tau=$$

$$=-\int_0^t\left\{\int_0^\pi f_2(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x),u_{xxx}(\tau,x))\cdot u_x(\tau,x)dx\right\}d\tau\le$$

$$\le C\cdot\int_0^t\left\{\int_0^\pi\left[1+u^2(\tau,x)+u_x^2(\tau,x)+\ u_{xx}^2(\tau,x)\right]dx\right\}d\tau\le$$

$$\le C\cdot\pi T+C\cdot(\pi^4+\pi^2+1)\cdot\int_0^t\left\{\int_0^\pi u_{xx}^2(\tau,x)dx\right\}d\tau; \qquad (28)$$

$$\int_0^t\int_0^\pi\left(f_3(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x))\right)_{xx}\cdot u(\tau,x)dxd\tau=$$

$$=\int_0^t\left\{\left(f_3(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x))\right)_x\cdot u(\tau,x)|_{x=0}^{x=\pi}-\right.$$

$$-f_3(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x))\cdot u_x(\tau,x)|_{x=0}^{x=\pi}+$$

$$\left.+\int_0^\pi f_3(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x))\cdot u_{xx}(\tau,x)dx\right\}d\tau=$$

$$=\int_0^t\int_0^\pi f_3(\tau,x,u(\tau,x),u_x(\tau,x),u_{xx}(\tau,x))\cdot u_{xx}(\tau,x)dxd\tau\le$$

$$\le C\cdot\int_0^t\int_0^\pi\left\{1+u^2(\tau,x)+u_x^2(\tau,x)+u_{xx}^2(\tau,x)\right\}dxd\tau\le$$

$$\leq C\cdot \pi T + C\cdot (\pi^4+\pi^2+1)\cdot \int\limits_0^t \left\{ \int\limits_0^\pi u_{xx}^2(\tau,x)dx \right\} d\tau. \qquad (29)$$

Now, using relations (23), (24) and estimates (25)-(29), from (19) we obtain that $\forall t\in[0,T]$:

$$\int\limits_0^\pi u^2(t,x)dx + \alpha \int\limits_0^\pi u_{xx}^2(t,x)dx \leq \int\limits_0^\pi \varphi^2(x)dx + \alpha \int\limits_0^\pi (\varphi''(x))^2 dx + 6\pi T\cdot C +$$

$$+2(3+3\pi^2+4\pi^4)\cdot C\cdot \int\limits_0^t \left\{ \int\limits_0^\pi u_{xx}^2(\tau,x)dx \right\} d\tau,$$

consequently,

$$\int\limits_0^\pi u_{xx}^2(t,x)dx \leq \frac{1}{\alpha}\cdot \left\{ \int\limits_0^\pi \varphi^2(x)dx + \alpha \int\limits_0^\pi \left(\varphi''(x)\right)^2 dx + 6\pi T\cdot C \right\} +$$

$$+\frac{2}{\alpha}(3+3\pi^2+4\pi^4)\cdot C\cdot \int\limits_0^t \left\{ \int\limits_0^\pi u_{xx}^2(\tau,x)dx \right\} d\tau. \qquad (30)$$

Applying Bellman's inequality, from (30) we obtain that the a priori estimate (18) is true. Theorem is now proved.

Corollary 1. *Under the conditions of Theorem 2, by virtue of a priori estimate (18) and estimates (21) and (20), the following a priori estimates hold for all the possible almost everywhere solutions* $u(t,x)$ *of problem (1)-(3):*

$$\|u(t,x)\|_{C(Q_T)} \leq R_0, \ \ \|u_x(t,x)\|_{C(Q_T)} \leq R_0, \qquad (31)$$

where $Q_T \equiv [0,T]\times[0,\pi]$ *and* $R_0>0$ *is a constant independent of* u.

Theorem 3. *Let*

1. All the conditions of Theorem 2 be satisfied;

2. $\forall R>0$ *in* $[0,T]\times[0,\pi]\times[-R,R]^2\times(-\infty,\infty)^3$

$$|F(t,x,u_1,...,u_5)| \leq C_R\cdot\left(1+u_3^2+|u_3|\cdot|u_4|+|u_4|+|u_5|\right), \qquad (32)$$

where $C_R>0$ *is a constant.*

Then for all the possible almost everywhere solutions $u(t,x)$ *of problem (1)-(3), belonging to* $B_{2,T}^4$*, the following a priori estimate holds:*

$$\|u(t,x)\|_{B_{2,T}^4} \leq C_0. \qquad (33)$$

Proof. Let $u(t,x)$ be any almost everywhere solution of problem (1)-(3) belonging to the space $B_{2,T}^4$. By virtue of condition 1 of this theorem and according to Theorem 2, it follows the trueness of a priori estimate (18) for all the possible almost everywhere solutions of problem (1)-(3). Moreover, it is true for all the possible almost everywhere solutions $u(t,x)$ of problem (1)-(3) belonging to the space

$B^4_{2,T}$. And from this estimate, as noted above in Corollary 1, it follows the trueness of a priori estimates (31).

Next, as proved in [2] (see estimate (39)), $\forall t \in [0,T]$ we have:

$$\|u\|^2_{B^4_{2,t}} \leq a_0 + \frac{\pi T}{\alpha^2} + \int_0^t \int_0^\pi \{\mathcal{F}(u(\tau,x))\}^2 \, dx d\tau, \tag{34}$$

where $a_0 \equiv 2 \sum_{n=1}^{\infty} (n^4 \cdot \varphi_n)^2, \alpha > 0$ is a number appearing in equation (1), and φ_n $(n = 1, 2, ...)$ and $\mathcal{F}$ are defined by relations (7) and (8).

Now, using a priori estimates (31) and condition (32) with $R = R_0$, we obtain that $\forall \tau \in [0,T]$ and $x \in [0,\pi]$:

$$|\mathcal{F}(u(\tau,x))| = |F(\tau, x, u(\tau,x), u_x(\tau,x), u_{xx}(\tau,x), u_{xxx}(\tau,x), u_{xxxx}(\tau,x))| \leq$$

$$\leq C_{R_0} \cdot \left\{1 + u^2_{xx}(\tau,x) + |u_{xx}(\tau,x)| \cdot |u_{xxx}(\tau,x)| + |u_{xxx}(\tau,x)| + |u_{xxxx}(\tau,x)|\right\},$$

$$\int_0^\pi \{\mathcal{F}(u(\tau,x))\}^2 \, dx \leq 5C^2_{R_0} \cdot \pi + 5C^2_{R_0} \cdot \left\{ \int_0^\pi u^4_{xx}(\tau,x)dx + \int_0^\pi u^2_{xx}(\tau,x) \times \right.$$

$$\left. \times u^2_{xxx}(\tau,x)dx + \int_0^\pi u^2_{xxx}(\tau,x)dx + \int_0^\pi u^2_{xxxx}(\tau,x)dx \right\}. \tag{35}$$

Next, in view of the structure of space $B^4_{2,T}, \forall \tau \in [0,T]$ we have:

$$\|u_{xx}(\tau,x)\|_{C([0,\pi])} \leq \|u\|_{B^2_{1.\tau}} \leq \|u\|_{B^3_{1.\tau}} \leq \frac{\pi}{\sqrt{6}} \cdot \|u\|_{B^4_{2.\tau}}, \tag{36}$$

$$\|u_{xxx}(\tau,x)\|_{C([0,\pi])} \leq \|u\|_{B^3_{1.\tau}} \leq \frac{\pi}{\sqrt{6}} \cdot \|u\|_{B^4_{2.\tau}}, \tag{37}$$

$$\int_0^\pi u^2_{xxxx}(\tau,x)dx \leq \frac{\pi}{2} \cdot \|u\|^2_{B^4_{2,\tau}}. \tag{38}$$

Now, using estimates (36), (37) and a priori estimate (18), we obtain that $\forall \tau \in [0,T]$:

$$\int_0^\pi u^4_{xx}(\tau,x)dx \leq \|u_{xx}(\tau,x)\|^2_{C([0,\pi])} \cdot \int_0^\pi u^2_{xx}(\tau,x)dx \leq \frac{\pi^2}{6} \cdot \|u\|^2_{B^4_{2.\tau}} \cdot C_0, \tag{39}$$

$$\int_0^\pi u^2_{xx}(\tau,x) \cdot u^2_{xxx}(\tau,x)dx \leq \|u_{xxx}(\tau,x)\|^2_{C([0,\pi])} \times$$

$$\times \int_0^\pi u^2_{xx}(\tau,x)dx \leq \frac{\pi^2}{6} \cdot \|u\|^2_{B^4_{2.\tau}} \cdot C_0, \tag{40}$$

$$\int_0^\pi u_{xxx}^2(\tau,x)dx \le \|u_{xxx}(\tau,x)\|_{C([0,\pi])}^2 \cdot \pi \le \frac{\pi^3}{6} \cdot \|u\|_{B_{2.\tau}^4}^2 . \tag{41}$$

Then, by virtue of estimates (39)-(41) and (38), from (35) we obtain that $\forall \tau \in [0,T]$:

$$\int_0^\pi \{\mathcal{F}(u(\tau,x))\}^2 dx \le 5\pi \cdot C_{R_0}^2 + 5C_{R_0}^2 \cdot \left(2 \cdot \frac{\pi^2}{6} \cdot C_0 + \frac{\pi^3}{6} + \frac{\pi}{2}\right) \|u\|_{B_{2.\tau}^4}^2 =$$

$$= 5\pi \cdot C_{R_0}^2 + \frac{5\pi}{6} \cdot (2\pi \cdot C_0 + \pi^2 + 3) \cdot C_{R_0}^2 \cdot \|u\|_{B_{2.\tau}^4}^2 . \tag{42}$$

Thus, using estimate (42), from (34) we obtain that $\forall t \in [0,T]$:

$$\|u\|_{B_{2.t}^4}^2 \le a_0 + \frac{5\pi^2 T^2}{\alpha^2} \cdot C_{R_0}^2 + \frac{5\pi^2 T}{6\alpha^2} \cdot (2\pi \cdot C_0 + \pi^2 + 3) \cdot C_{R_0}^2 \cdot \int_0^t \|u\|_{B_{2.\tau}^4}^2 d\tau. \tag{43}$$

Applying Bellman's inequality, from (43) we obtain the trueness of a priori estimate (33). Theorem is now proved.

Thus, by virtue of Remark 1, from Theorems 1 and 3 it follows the trueness of the following existence in large theorem for almost everywhere solution of problem (1)-(3).

Theorem 4. *Let*

1. All the conditions of Theorem 1 be satisfied.

2. All the conditions of Theorem 2 be satisfied.

3. The condition 2 of Theorem 3 be satisfied.

Then there exists an almost everywhere solution of problem (1)-(3).

Remark 2. In conclusion, we note that (as mentioned in [6]) a special case of equation (1) with

$$F = \beta u_{xx} - (g(u))_x, \beta > 0, \tag{44}$$

is called Korteweg-de Vries-Burgers equation.

References

[1]. Khudaverdiyev K.I., Sadikhov M.N. *On the existence in small for almost everywhere solution of one-dimensional mixed problem for a class of Korteweg-de Vries-Burgers type nonlinear equations.* – Vestnik Bakinskogo Gosudarstvennogo Universiteta, seriya fiziko-matematicheskikh nauk, 2008, No 3 (Russian)

[2]. Sadykhov Mehman N. *Investigation of uniqueness of almost everywhere solution of one-dimensional mixed problem for a Korteweg-de Vries-Burgers type nonlinear equation.* Proceedings of Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan, V. XXVIII (XXXVI), 2008, pp. 103-112

[3]. Khudaverdiyev K.I., Sadikhov M.N. *Investigation of almost everywhere solution of one-dimensional mixed problem for a Korteweg-de Vries-Burgers type nonlinear equation.* – Materiali Respublikanskoy Nauchnoy Konferentsii, posvyashennoy

85-letnemu yubileyu G.A.Aliyeva, Izdatelstvo Bakinskogo Universiteta, Baku, 2008, pp.118-119 (Russian)

[4]. Khudaverdiyev K.I. *On the theory of multidimensional mixed problems for nonlinear hyperbolic equations.* – Dis... dokt.fiz.-mat.nauk, Baku, Azerbaijan State University, 1973, 319 pages (Russian)

[5]. Khudaverdiyev K.I. *Multidimensional mixed problem for nonlinear hyperbolic equations.* – Azerbaycan Dovlet Universitetinin Neshriyyati, 1987, 88 pages (Azeri)

[6]. Aassila Mohammed. *Decay of solutions of some nonlinear equations.* Port. math., 2003, 60, No 4, pp.389-409.

Mehman N. Sadikhov
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F. Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439 47 20 (off.).

Received June 25, 2008; Revised September 30, 2008.