MECHANICS

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ON INFLUENCE OF WALL LAYER OF SEDIMENT ON THE MOTION OF SUSPENSION IN PIPE

Abstract

We generalize Poiseuille formula in a pipe on whose walls sediment layer with different rheological properties is generated when inhomogeneous liquid flows. The relation between the values of mean in section concentration of particles $< \alpha >$ and mean consumption concentration α_f by the quantity α_0, used in constructing the profile α and dimensionless velocity of flows, is established.

One dimensional method for describing the process of liquid and suspension mixture pumping in pipe has a great application in engineering practice. Therewith the mixture flow is considered with velocity, temperature, pressure and density constant in section. Change of parameter data may occur only in one direction along the pipeline [1,2]. However, change of mixture flow on radius of pipeline may influence on the character and intensity of flow. Very often occurrence of relative velocity of phases reduces to particles deposition (depending on relation of liquid and material densities of hard or high molecular combinations) on the walls of a pipe in gravity field. This phenomenon reports itself on hydraulic characteristics of flows and may reduce to partial or total plugging of channels and also to catastrophic pressure build-up.

Lateral displacement effect of single spherical particles of overdiluted suspension in laminar flows in capillaries was first investigated in [3]. Afterwards this effect called the Segre-Silberberg effect was theoretically and experimentally investigated in a great number of papers. Some of them are in the references of [4,5]. Discovery and investigation of this effect stimulated to study lateral forces acting on a small partiecle in inhomogeneous one-dimensional flow.

Sedimentary layer of hard particles on the pipe walls, unlike the mixture itself may behave as rheological, viscoelastic, viscoplastic medium and decreasing the pipe radius it creates additional hydraulic resistance. Therefore a problem on simulation of processes of mixture flow in a pipeline must include the properties of a layer arising in deposition of hard and high molecular combinations on the pipe wall [6].

1. Let's consider the motion of mono-dispersible suspension of small particles in a pipe of radius R_0. Assume that stationary distribution of concentration is achieved as the result of the fact that convective migration flow of particles in lateral direction stipulated by volume force is compensated by contrariwise directed diffusion flow in inhomogeneous field of concentration. Here thermo dynamical force is determined from the classic condition that a convective flow of particles generated by it were exactly equal to diffusive one [9]. Gravity and buoyancy forces don't reduce to appearance of phase sliding in equidense suspensions, therefore, they may be included into effective pressure by a standard way. If sliding velocity is considerably less by modulus than mean velocities of the both phases, that is typical for the considered suspensions, then we can write the equation of pulse preservation of suspension in the whole provided phase incompressibility, on the form [6-11]

$$\mu_0 \frac{1}{r}\frac{d}{dr}\left(rM(\alpha)\frac{d\upsilon}{dr}\right) = \frac{dP}{dz}, \quad \frac{dP}{dr} = 0; \quad P \neq f(r), \tag{1.1}$$

where r, z are longitudinal and lateral coordinates, α is volume concentration of dispersed phase, $M(\alpha)$ is an increasing function of α, $M(0) = 1$, υ is the mean velocity of liquid, P is mean pressure, μ_0 is dynamical viscosity of single liquid.

Only Stock's viscous force F_s and Faxen force F_f [9,10] act on the particle from the side of liquid in longitudinal direction

$$F_s = \frac{9\alpha\mu_0}{2a^2}M(\alpha)(\upsilon - w), \qquad F_f = \frac{3\alpha\mu_0}{4}\frac{1}{r}\frac{d}{dr}\left(rM(\alpha)\frac{d\upsilon}{dr}\right), \tag{1.2}$$

where w is average velocity of particles, a is its spherical radius. In longitudinal direction these forces compensate each other. Then for the velocity of sliding we get:

$$\upsilon - w = -\frac{a^2}{6M(\alpha)}\frac{1}{r}\frac{d}{dr}\left(rM(\alpha)\frac{d\upsilon}{dr}\right). \tag{1.3}$$

We determine the lift acting on all particles in the unit volume in the same way using the expression for this force obtained in [8] for a unit particle, we have

$$F_M = \frac{3\cdot 6,46\alpha\rho_f}{4\pi a}\left[\nu_0 M(\alpha)\left|\frac{d\upsilon}{dr}\right|\right]^{1/2}(u-\upsilon)\, sign\left(\frac{d\upsilon}{dr}\right), \quad \nu_0 = \mu_0/\rho_f. \tag{1.4}$$

This force calls migration of particles in longitudinal direction of flow and as a result, the inhomogeneity stipulates initiation of thermodynamical force [9]

$$F_T = -\frac{3\alpha}{4\pi a^3}\left(\frac{\partial\pi}{\partial\alpha}\right)_{P,T}\frac{d\alpha}{dr}, \tag{1.5}$$

where π is a chemical potential of particles, and differentiation (1.5) is conducted under constant pressure and temperature. Then, preservation of transverse component of dispersed phase pulse gives

$$\left(\frac{\partial\pi}{\partial\alpha}\right)_{P,T}\frac{d\alpha}{dr} = 6,46\alpha^2\rho_f\left[\upsilon_0 M(\alpha)\left|\frac{d\upsilon}{dr}\right|\right]^{1/2}(u-w)\, sign\left(\frac{d\upsilon}{dr}\right). \tag{1.6}$$

For the given π and P the equations (1.1), (1.3) and (1.6) is a closed system of equations for the unknown functions υ, α and w. Substituting (1.3) into (1.6), we get

$$\begin{gathered}\left(\frac{\partial\pi}{\partial\alpha}\right)_{P,T}\frac{d\alpha}{dr} = -\frac{6,46\alpha^4\rho_f}{6M(\alpha)}\upsilon_0 M(\alpha)\left|\frac{d\upsilon}{dr}\right|^{1/2}\times \\ \times\frac{1}{r}\frac{d}{dr}\left(rM(\alpha)\frac{d\upsilon}{dr}\right) sign\left(\frac{d\upsilon}{dr}\right).\end{gathered} \tag{1.7}$$

For the equations (1.1) and (1.7) the boundary conditions on υ and α follow from the symmetry condition of flow on the boundary of suspension and sedimentary layer generated on the walls.

A) Determine stationary equation of motion of suspension by the transverse alive section of a pipe: $S = \pi R^2;\ R = R_0 - h;\ h << R_0$ (h is thickness of a layer)

$$-\frac{dP}{dZ} - \frac{2}{R} M(\alpha)\tau(R) = 0. \tag{1.8}$$

Determine tangent stress on the boundary of suspension and layer by the Darcy-Weissbach formula [6,10]

$$\tau(R) = \frac{\lambda}{8}\rho_f \upsilon_1^2(R), \tag{1.9}$$

where λ is a resistance coefficient for friction head loss.

It follows from (1.8) and (1.9) that

$$\upsilon_1(R) = \sqrt{-\frac{4R}{\lambda M(\alpha)\rho_f}\frac{dP}{dZ}}. \tag{1.10}$$

B) If suspension deposition in wall layers are deformed as viseoplastic liquid, then for its motion $\Delta P > \Delta P_0$ is necessary

$$\tau_n = \tau_0 + \mu' \frac{d\upsilon_1}{dr}, \quad \tau_n > \tau_0, \tag{1.11}$$

where τ_n is friction stress, τ_0 is limit stress, μ' is dynamical coefficient of structural viscosity, ΔP_0 is dynamical pressure drop in a pipe.

On the other hand, stationary flow of viscous liquid in a pipe is determined by the Poiseuille formula and therefore $\tau_n = \Delta P \cdot r/2l$, l is the length of a pipe.

From the equality of these expressions and adhesion conditions on the walls we have

$$\upsilon_1 = \frac{\Delta P}{4\mu' l}(R_0^2 - r_1^2) - \frac{\tau_0}{\mu'}(R_0 - r_1), \quad R \le r_0 \le R_0, \quad \tau_n > \tau_0. \tag{1.12}$$

For $\Delta P \le \Delta P_0$, $(d\upsilon_1/dr) \equiv 0$ a suspension deposition layer behaves as a solid and a radius of flow area equals

$$R = \frac{2l\tau_0}{\Delta P}; \quad \tau \le \tau_0. \tag{1.13}$$

In the case A) for constant values of volume concentration ($\alpha = \alpha_0$) and pressure drop ($\partial P/\partial Z = -\Delta P/l$), $\Delta P = P_1 - P_2$ the solution of the problem (1.1) and (1.10) is of the form

$$\upsilon = \frac{\Delta P}{4\mu_0 l M(\alpha_0)}(R^2 - r^2) + \sqrt{\frac{4R\Delta P}{\lambda M(\alpha_0)\rho_f}}, \quad 0 \le r \le R. \tag{1.14}$$

It is seen from the expression (1.14) for the velocity that υ increases according to decrease of resistance coefficient λ and vice versa, υ decreases according to increase of λ. As $\lambda \to \infty$ this formula passes to Pouiselle formula, the velocity and also suspension consumption decreases in connection with contraction of alive section of a pipe: $\pi R^2 < \pi R_0^2$.

In the case B) under the above-indicated assumptions the solution of problem (1.1) and (1.19) has the form

$$v = \frac{\Delta P}{4\mu_0 l M(\alpha_0)}(R^2 - r^2) + \frac{\Delta P}{4\mu' l}(R_0^2 - R^2) - \frac{\tau_0}{\mu'}(R_0 - R),$$
$$0 \le r \le R,\ \tau > \tau_0. \tag{1.15}$$

Hence it follows that motion of viscoplastic wall layer made of small sedimentary particles may reduce to increase of suspension velocity on a central axis of a pipe. However, adhesion of this layer to the pipe wall contracts the alive section of flow and lowers its velocity determined by the Poiseuille formula.

2. Using the known formula for chemical potential of particles

$$\pi = const + kTF(\alpha) \tag{2.1}$$

$$F(\alpha) = In\alpha - \alpha + \frac{\alpha(8-5\alpha)}{(1-\alpha)^2}.$$

Introducing $\partial P/\partial Z = \Delta P$ and denoting $x = R$ it is convenient to pass to dimensional variables

$$\xi = x/R,\quad V = \mu_0\vartheta/(\Delta P R^2).$$

Then the equations (1.1) and (1.7) are reduced to the system for the unknown functions $V(\xi)$ and $\alpha(\xi)$

$$\frac{1}{\xi}\frac{d}{d\xi}\left(M(\alpha)\xi\frac{dV}{d\xi}\right) = -1$$

$$M\frac{dF}{d\alpha}\frac{d\alpha}{d\xi} = \Gamma\left|M\frac{dV}{d\xi}\right|^{1/2}\frac{1}{\xi}\left(M(\alpha)\xi\frac{dV}{d\xi}\right) \tag{2.2}$$

$$\Gamma = (6,46/6)a^4(\Delta P R)^{3/2}(\rho^{1/2}v_0 kT).$$

The condition for α is determined either by representation of mean in section concentration of particles $<\alpha>$ or mean consumption concentration α_f.

$$2\int_0^1 \alpha(\xi)\xi d\xi = <\alpha>$$

$$\int_0^1 \alpha(\xi)V(\xi)\xi d\xi \Big/ \left(\int_0^1 V(\xi)\xi d\xi\right) = \alpha_f. \tag{2.3}$$

The function $M(\alpha)$ is determined from numerical representations for the ratio μ/μ_0 and here [11]

$$M(\alpha) = \left(1 - \frac{5}{2}\alpha\right)^{-1}. \tag{2.4}$$

From the first equation in (2.2) with boundary condition $(dV/d\xi) = 0$ for $\xi = 0$.

$$M\frac{dV}{d\xi} = -\frac{\xi}{2} \tag{2.5}$$

Substituting (2.5) into (2.2) and integrating it from α_0 to α we get

$$I(\alpha)-I(\alpha_0)=\frac{2}{3}\frac{\Gamma}{\sqrt{2}}\xi^{3/2},\quad 0\le\xi\le 1,\tag{2.6}$$

for $\alpha_0<\alpha^*$ where α_0 is concentration on the plane or axis of flow symmetry $\xi=0$, α^* is concentration in dense packing state,

$$I(\alpha)=\int\limits_{\alpha}^{\alpha_{\cdot 0}} M(\alpha)\frac{dF}{dx}d\alpha.\tag{2.7}$$

In (2.7) allowing for (2.6) and (2.4) the function may be expressed by the knowns. It is easy to carry out numerical calculations for $I(\alpha)$ and in special case $M(\alpha)=(1-\alpha)^{-5/2}$ to integrate it.

It densely packed kernel of particles filling the domain $\alpha=\alpha^*$, $0\le\zeta\le\zeta^*$ is generated in the flow, then the flow in the domain $\zeta*\le\zeta\le 1$ will be of the form

$$I(\alpha)=\frac{2}{3}\frac{\Gamma}{\sqrt{2}}\left(\zeta^{3/2}-\zeta*^{3/2}\right).\tag{2.8}$$

For small α we approximatly have $F\approx In\alpha$, $M=1$ and it follows from (2.6) and (2.7) that

$$\alpha\approx\alpha_0\exp\left(-\frac{2}{3}\frac{\Gamma}{\sqrt{2}}\zeta^{3/2}\right).\tag{2.9}$$

Using the found function $\alpha(\zeta)$ in the equation (2.5) we find the profiles of dimensionless velocity of the flow

$$V(\zeta)=\frac{1}{2}\int\limits_{\zeta}^{1}\frac{\zeta d\zeta}{M(\alpha)},\quad \zeta*\le\zeta<1;\tag{2.10}$$

$$V(\zeta)=V(\zeta*)=V*,\quad 0\le\zeta<\zeta*.\tag{2.11}$$

The function $V(\zeta)$ corresponding to profiles of concentration (2.6) for $\alpha_0=$ $=0,3$ is represented in figures 1 and 2.

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