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AXISYMMETRIC WAVE PROPAGATION IN THE TWISTED CIRCULAR CYLINDER EMBEDDED IN A COMPRESSIBLE ELASTIC MEDIUM

Abstract

Within the scope of the piecewise homogeneous body model with the use of three-dimensional theory of elastic wave propagation in the initially stressed body the problem on the axisymmetric wave propagation in the initially twisted circular cylinder surrounded by the infinite elastic medium is considered. The mathematical formulation of the problem is presented and the solution method of that is proposed and developed. The numerical results are obtained and discussed. In particular, it is established that as a result of the initial twisting of the circular cylinder in the infinite elastic medium a new wave modes arise.

1. Introduction

Investigations of elastodynamic problems have a great significance in almost all branches of the natural sciences and modern engineering. Interesting and urgent problems which apply to the non-linear dynamical effects in the elastic medium are the elastodynamic problems for initially stressed bodies. It should be noted that initial stresses occur in structural elements during their manufacture and assemble, in the Earth's crust under the action of geostatic and geodynamic forces, in composite materials, etc.

At present, the mathematical modeling of elastodynamic problems the initially stressed bodies is made through utilization of the linearized theory of elastodynamics for initially stressed bodies which uses the linearization principle from the general non-linear theory of elasticity or its simplified modifications Note that linearized equations can be employed for investigations of all kinds of problems for initially deformed bodies if certain conditions are satisfied. In this case the so-called three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB) is preferred. Note that the equations and relations of TLTEWISB coincide with the corresponding ones for the linear theory of elastic waves in the absence of initial stresses.

Almost all investigations carried out by employing TLTEWISB refer to the influence of initial stresses on the speed and on the dispersion of various types of waves. The detail analyses of these investigations were made in the monograph [6]. The reviewer of the investigations carried out in the recent years has been made in the paper [1,2]. Analyses of these references shows that up to now in the mentioned investigations, except [4], it was assumed that the initial stresses in the body considered are caused by axial stretching or compression. But, in the paper [4] it was assumed that the initial stresses in the circular cylinder are caused by an initial twisting of this cylinder and the oscillation of this cylinder is studied. It should be noted that the investigations carried out in the paper [4] can also be considered as the attempt for generalization of the pioneer work [5] in this field.

Thus, taking the foregoing brief discussion into account, in the present paper the attempt is made for investigation of the influence of the initial twisting of the circular cylinder embedded with an infinite elastic body on the axisymmetric wave propagation in that. The investigation is carried out within the scope of the piecewise homogeneous body model by the use of the TLTEWISB. Note that the study of

the longitudinal and torsional wave propagation in the pre-stressed (pre-compressed) system "cylinder + surrounding infinite medium" has been studied in the papers [3] and [7], respectively.

2. Formulation of the problem

Consider the system "cylinder + surrounding infinite medium" schematically illustrated in Fig. 1. Associate with the cylinder the cylindrical system of coordinates $Or\theta z$ and the position of the points of the constituents we determine with the Lagrange coordinates in this coordinate system. The values related solid circular cylinder (with radius R) and surrounding infinite medium we denote by the upper indices (2) and (1) respectively. Assume that the cylinder is twisted before its compounding with containing medium and has a constant twist υ for the per unit length. According to this assumption, the cylinder has the initial stress $\sigma_{z\theta}^{(2),0} = \mu^{(2)}\upsilon r$, where $\mu^{(2)}$ is a shear modulus, r is a radial coordinate in the cylindrical system of coordinate $Or\theta z$, the Oz axis of which is directed along the central line of the cylinder.

Fig.1.

Thus, within the foregoing assumptions, we investigate the axisymmetric wave propagation directed along the cylinder. In this case the equation of motion is.

$$\begin{gathered}\frac{\partial \sigma_{rr}^{(n)}}{\partial r}+\frac{\partial \sigma_{zr}^{(n)}}{\partial z}+\frac{1}{r}\left(\sigma_{rr}^{(n)}-\sigma_{\theta\theta}^{(n)}\right)-\sigma_{z\theta}^{(n),0}\delta_2^n\frac{2}{r}\frac{\partial u_\theta^{(n)}}{\partial z}=\rho^{(n)}\frac{\partial^2 u_r^{(n)}}{\partial t^2},\\ \frac{\partial \sigma_{r\theta}^{(n)}}{\partial r}+\frac{2}{r}\sigma_{\theta\theta}^{(n)}+\frac{\partial \sigma_{z\theta}^{(n)}}{\partial z}+\frac{2}{r}\sigma_{z\theta}^{(n)}\delta_2^n\frac{\partial u_r^{(n)}}{\partial z}=\rho^{(n)}\frac{\partial^2 u_\theta^{(n)}}{\partial t^2},\\ \frac{\partial \sigma_{rz}^{(n)}}{\partial r}+\frac{1}{r}\sigma_{rz}^{(n)}+\frac{\partial \sigma_{zz}^{(n)}}{\partial z}=\rho^{(n)}\frac{\partial^2 u_z^{(n)}}{\partial t^2},\quad n=1,2.\end{gathered}\tag{1}$$

It is assumed that the materials of the cylinder and surrounding medium are

homogeneous and isotropic. The elasticity relations for these materials are

$$\sigma_{rr}^{(n)} = \lambda^{(n)}\left(\varepsilon_{rr}^{(n)} + \varepsilon_{\theta\theta}^{(n)} + \varepsilon_{zz}^{(n)}\right) + 2\mu^{(n)}\varepsilon_{rr}^{(n)},$$

$$\sigma_{\theta\theta}^{(n)} = \lambda^{(n)}\left(\varepsilon_{rr}^{(n)} + \varepsilon_{\theta\theta}^{(n)} + \varepsilon_{zz}^{(n)}\right) + 2\mu^{(n)}\varepsilon_{\theta\theta}^{(n)},$$

$$\sigma_{zz}^{(n)} = \lambda^{(n)}\left(\varepsilon_{rr}^{(n)} + \varepsilon_{\theta\theta}^{(n)} + \varepsilon_{zz}^{(n)}\right) + 2\mu^{(n)}\varepsilon_{zz}^{(n)}, \quad \sigma_{r\theta}^{(n)} = 2\mu^{(n)}\varepsilon_{r\theta}^{(n)}, \tag{2}$$

$$\sigma_{rz}^{(n)} = 2\mu^{(n)}\varepsilon_{rz}^{(n)}, \quad \sigma_{r\theta}^{(n)} = 2\mu^{(n)}\varepsilon_{z\theta}^{(n)},$$

where

$$\varepsilon_{rr}^{(n)} = \frac{\partial u_r^{(n)}}{\partial r}, \ \varepsilon_{\theta\theta}^{(n)} = \frac{u_r^{(n)}}{r}, \ \varepsilon_{zz}^{(n)} = \frac{\partial u_z^{(n)}}{\partial z}, \ \varepsilon_{r\theta}^{(n)} = \frac{1}{2}\left(\frac{\partial u_\theta^{(n)}}{\partial r} - \frac{u_\theta^{(n)}}{r}\right),$$

$$\varepsilon_{rz}^{(n)} = \frac{1}{2}\left(\frac{\partial u_z^{(n)}}{\partial r} + \frac{\partial u_r^{(n)}}{\partial z}\right), \quad \varepsilon_{r\theta}^{(n)} = \frac{1}{2}\frac{\partial u_\theta^{(n)}}{\partial z}. \tag{3}$$

In equations (1) – (3) the conventional notation is used.

It is assumed that on the interface contact surface $r = R$ the following complete contact conditions are satisfied.

$$\sigma_{rr}^{(2)}\,|_{r=R} = \sigma_{rr}^{(1)}\,|_{r=R}\,, \quad \left(\sigma_{r\theta}^{(2)} - \sigma_{z\theta}^{(2),0}\frac{\partial u_r^{(2)}}{\partial z}\right)\Bigg|_{r=R},$$
$$\left(\sigma_{rz}^{(2)} - \sigma_{z\theta}^{(2),0}\frac{u_\theta^{(2)}}{r}\right)\Bigg|_{r=R} = \sigma_{rz}^{(1)}\,|_{r=R}\,, \tag{4}$$

$$u_r^{(2)}\,|_{r=R} = u_r^{(1)}\,|_{r=R}\,, \ u_\theta^{(2)}\,|_{r=R} = u_\theta^{(1)}\,|_{r=R}\,, \ u_z^{(2)}\,|_{r=R} = u_z^{(1)}\,|_{r=R}\,.$$

Moreover, it is assumed that

$$\left|\sigma_{rr}^{(1)}\right|; \ \left|\sigma_{\theta\theta}^{(1)}\right|; \ \left|\sigma_{zz}^{(1)}\right|; \ \left|\sigma_{r\theta}^{(1)}\right|; \ \left|\sigma_{rz}^{(1)}\right|;$$

$$\left|\sigma_{r\theta}^{(1)}\right|; \ \left|u_r^{(1)}\right|; \ \left|u_\theta^{(1)}\right|; \ \left|u_z^{(1)}\right| \le M \ \text{ as } r \to \infty. \tag{5}$$

Thus, within the foregoing it is required to investigate the dispersion of the axisimmetric wave propagating along the Oz axis. With this we exhaust the formulation of the problem considered.

3. Method of solution

As we assume that the wave in the system considered is axisymmetric and propagates along the Oz axis, then we can present the displacements of the constituent as follows.

$$u_r^{(n)} = U^{(n)}(r)\cos(kz-\omega t), \ u_\theta^{(n)} = V^{(n)}(r)\cos(kz-\omega t), \ u_z^{(n)} = W^{(n)}(r)\cos(kz-\omega t). \tag{6}$$

From equations (1) – (3) we obtain the following equation for the unknown functions in (6).

$$\left(2+\alpha^{(n)}\right)\left[\frac{d^2U^{(n)}}{dr_1^2}+\frac{d}{dr_1}\left(\frac{U^{(n)}}{r_1}\right)\right]+\left(1+\alpha^{(n)}\right)\frac{dW^{(n)}}{dr_1}-$$

$$-U^{(n)}\upsilon^{(2)}\delta_2^nV^{(n)}=-\frac{c^2}{\left(c_2^{(n)}\right)^2}U^{(n)},$$

$$\frac{d}{dr_1}\left[\frac{dV^{(n)}}{dr_1}-\frac{V^{(n)}}{r_1}\right]+\frac{2}{r_1}\left[\frac{dV^{(n)}}{dr_1}-\frac{V^{(n)}}{r_1}\right]-V^{(n)}-\upsilon^{(2)}\delta_2^nU^{(n)}=-\frac{c^2}{\left(c_2^{(n)}\right)^2}V^{(n)},$$

$$\frac{d}{dr_1}\left[\frac{dW^{(n)}}{dr_1}-U^{(n)}\right]+\frac{1}{r_1}\left[\frac{dW^{(n)}}{dr_1}-U^{(n)}\right]-$$

$$-\alpha^{(n)}\left[\frac{dU^{(n)}}{dr_1}+\frac{U^{(n)}}{r_1}+W^{(n)}\right]-2W^{(n)}=-\frac{c^2}{\left(c_2^{(n)}\right)^2}W^{(n)}. \tag{7}$$

where $r_1=kr$.

The solution of the equation (7) we seek as follows.

$$U^{(n)}=A^{(n)}J_1(\chi^{(n)}r_1),\ V^{(n)}=B^{(n)}J_0(\chi^{(n)}r_1),\ W^{(n)}=C^{(n)}J_1(\chi^{(n)}r_1) \tag{8}$$

where $J_0(\chi^{(n)}r_1)$ are $J_1(\chi^{(n)}r_1)$ the Bessel functions of the first kind the zeroth and first order respectively. Substituting (8) into (7) and using the relations

$$J_0'(x)=-J_1(x),\ J_p'(x)+\frac{p}{x}J_p(x)=J_{p-1}(x) \tag{9}$$

the following equation is obtained from (7) for determination $\chi^{(n)}$.

$$A^{(n)}\left[(2+\alpha^{(n)})(-\chi^{(n)})^2+((s^{(n)})^2-1)\right]+$$

$$+C^{(n)}(1+\alpha^{(n)})(-\chi^{(n)})-B^{(n)}\upsilon^{(2)}2\delta_2^n=0,$$

$$-A^{(n)}\upsilon^{(2)}\delta_2^n+B^{(n)}\left[-(\chi^{(n)})^2+(s^{(n)})^2-1\right]=0,$$

$$-A^{(n)}(1+\alpha^{(n)})\chi^{(n)}+C^{(n)}\left[-(\chi^{(n)})^2-\alpha^{(n)}+(s^{(n)})^2-2\right]=0. \tag{10}$$

According to the well-known procedure, we obtain that

$$(\chi_1^{(n)})^3+(\chi_1^{(n)})^2a_4^{(n)}+\chi_1^{(n)}a_2^{(n)}+a_0^{(n)}=0, \tag{11}$$

where

$$\chi_1^{(n)}=\left(\chi^{(n)}\right)^2,\ a_4^{(n)}=\frac{e_4^{(n)}+f_4^{(n)}}{e_6^{(n)}},\ a_2^{(n)}=\frac{e_2^{(n)}+f_2^{(n)}}{e_6^{(n)}},\ a_0^{(n)}=\frac{e_0^{(n)}+f_0^{(n)}}{e_6^{(n)}},$$

$$a_6^{(n)}=-(2+\alpha^{(n)}),\ e_4^{(n)}=(3+\alpha^{(n)})((s^{(n)})^2-1)+(2+\alpha^{(n)})(-\alpha^{(n)}+(s^{(n)})^2-2),$$

$$e_2^{(n)} = -((s^{(n)})^2 - 1)\left[(s^{(n)})^2 - 1 + (3+\alpha^{(n)})(-\alpha^{(n)} + (s^{(n)})^2 - 2)\right],$$

$$e_0^{(n)} = ((s^{(n)})^2 - 1)(-\alpha^{(n)} + (s^{(n)})^2 - 2),\ f_4^{(n)} = (1+\alpha^{(n)})^2,$$

$$f_2^{(n)} = -(1+\alpha^{(n)})^2((s^{(n)})^2 - 1) + 2(\upsilon_1^{(2)})^2\delta_2^n,\ s^{(n)} = c/c_2^{(n)},\ \upsilon_1^{(2)} = \upsilon^{(2)}R. \quad (12)$$

From (11) it can be written that

$$\chi_{11}^{(n)} = \frac{\varphi^{(n)}}{|\varphi^{(n)}|}2\sqrt{-P^{(n)}/3}\cos(\varphi^{(n)}/3) - a_4^{(n)}/3,\ \varphi^{(n)} = \arctan(2\sqrt{-D^{(n)}}/(-Q^{(n)})),$$

$$\chi_{12}^{(n)} = \frac{\varphi^{(n0}}{|\varphi^{(n)}|}\sqrt{-P^{(n)}/3}(-\cos(\varphi^{(n)}/3) - \sqrt{3}\sin(\varphi^{(n)}/3)) - a_4^{(n)}/3,$$

$$\chi_{13}^{(n)} = \frac{\varphi^{(n)}}{|\varphi^{(n)}|}\sqrt{-P^{(n)}/3}(-\cos(\varphi^{(n)}/3) + \sqrt{3}\sin(\varphi^{(n)}/3)) - a_4^{(n)}/3,$$

$$P^{(n)} = -(a_4^{(n)})^2/3 + a_2^{(n)},\ Q^{(n)} = 2(a_4^{(n)})^3/27 - a_2^{(n)}a_4^{(n)}/3 + a_0^{(n)},$$

$$D^{(n)} = (Q^{(n)})^2/4 + (P^{(n)})^3/27. \quad (13)$$

We will consider the cases where

$$\chi_{1i}^{(n)} > 0 \ \text{ or } \ \chi_{1i}^{(n)} < 0, \quad i = 1,2,3. \quad (14)$$

The cases for which $\chi_{1i}^{(n)}$ are complex numbers will be omitted in consideration. At the same time the cases where $\chi_{1i}^{(n)} = 0$ correspond the non dispersive wave velocity and therefore can also be neglected in consideration. Thus, the corresponding particular solution to the equation (7) is determined as follows.

$$U_i^{(2)} = J_1\left(\sqrt{\chi_{1i}^{(2)}}r_1\right),\ V_i^{(2)} = J_1\left(\sqrt{\chi_{1i}^{(2)}}r_1\right),\ W_i^{(2)} = J_0\left(\sqrt{\chi_{1i}^{(2)}}r_1\right)$$
$$U_i^{(1)} = Y_1\left(\sqrt{\chi_{1i}^{(1)}}r_1\right),\ V_i^{(1)} = Y_1\left(\sqrt{\chi_{1i}^{(1)}}r_1\right),\ W_i^{(1)} = Y_0\left(\sqrt{\chi_{1i}^{(1)}}r_1\right)$$
$$\text{for the case where } \chi_{1i}^{(n)} > 0,$$

$$U_i^{(2)} = I_1\left(\sqrt{\left|\chi_{1i}^{(2)}\right|}r_1\right),\ V_i^{(2)} = I_1\left(\sqrt{\left|\chi_{1i}^{(2)}\right|}r_1\right),\ W_i^{(2)} = I_0\left(\sqrt{\left|\chi_{1i}^{(2)}\right|}r_1\right)$$
$$U_i^{(1)} = K_1\left(\sqrt{\left|\chi_{1i}^{(1)}\right|}r_1\right),\ V_i^{(1)} = K_1\left(\sqrt{\left|\chi_{1i}^{(1)}\right|}r_1\right),\ W_i^{(1)} = K_0\left(\sqrt{\left|\chi_{1i}^{(1)}\right|}r_1\right)$$
$$\text{for the case where } \chi_{1i}^{(n)} < 0. \quad (15)$$

According to the foregoing procedures the general solution to the equation (7) is

$$U^{(n)} = \sum_i^3 A_i^{(n)}U_i^{(n)},\ V^{(n)} = \sum_i^3 B_i^{(n)}V_i^{(n)},\ W^{(n)} = \sum_i^3 C_i^{(n)}W_i^{(n)}, \quad (16)$$

where

$$\left.\begin{aligned} C_i^{(n)} &= \frac{\sqrt{\chi_{1i}^{(n)}}(1+\alpha^{(n)})}{-\chi_{1i}^{(n)}-\alpha^{(n)}+(s^{(n)})^2-2}A_i^{(n)} \\ B_i^{(n)} &= \frac{\upsilon_1^{(2)}\delta_2^n}{-\chi_{1i}^{(n)}+(s^{(n)})^2-1}A_i^{(n)} \end{aligned}\right\} \text{for the case where } \chi_{1i}^{(n)}>0,$$

$$\left.\begin{aligned} C_i^{(n)} &= \frac{\sqrt{\left|\chi_{1i}^{(n)}\right|}(1+\alpha^{(n)})}{\left|\chi_{1i}^{(n)}\right|-\alpha^{(n)}+(s^{(n)})^2-2}A_i^{(n)} \\ B_i^{(n)} &= \frac{\upsilon_1^{(2)}\delta_2^n}{\left|\chi_{1i}^{(n)}\right|+(s^{(n)})^2-1}A_i^{(n)} \end{aligned}\right\} \text{for the case where } \chi_{1i}^{(n)}<0. \qquad (17)$$

Using the foregoing general solution to the equation (7) and the relations (2), (3) we obtain the algebraic equations for the unknown constants $A_i^{(n)}$ from contact condition (4). Equating the determinant of the coefficients of the unknowns (denote it by α_{ij}) of the mentioned algebraic equation we attain the dispersion equation for the considered wave propagation. This equation can be written, formally, as follows.

$$\det\left\|\alpha_{ij}(s^{(n)},kR,\ \upsilon_1^{(2)}\right\|=0,\ i;\ j=1,2,3,4,5,6. \qquad (18)$$

For reducing the size of the paper the explicit expression of the coefficients α_{ij} are not shown here. It should be noted in the case where $\upsilon^{(2)}=0$ the equation can be presented aa

$$\left(\det\left\|\alpha_{1ij}(s^{(n)},kR)\right\|\right)\left(\det\left\|\alpha_{2\ell n}(s^{(n)},kR)\right\|\right)=0,\ i;\ j=1,2,3,4,\ \ \ell;n=1,2 \quad (19)$$

where the equations

$$\det\left\|\alpha_{1ij}(s^{(n)},kR)\right\|=0 \qquad (20)$$

and

$$\det\left\|\alpha_{2\ell n}(s^{(n)},kR)\right\|=0 \qquad (21)$$

are the dispersion equations for the axisymmetric longitudinal and torsional wave propagation in the considered body, respectively. Consequently, under existence the initial twisting in the cylinder the axisymmetric longitudinal and torsional waves in the considered body can not be propagate separately, i.e. these waves must propagate together. This fact was also noted in the paper [4]. However, in the paper [4] the concrete results were obtained for the influence of the initial twisting of the circular cylinder on the oscillation frequency of that only. The results on the influence of the mentioned twisting on the wave propagation velocity in the quantitative and qualitative sense are not given only in the paper [4], but also in the other related investigations. Consequently, the first attempt in the foregoing sense is made in the present paper and this attempt is examined for the system "initially twisted circular cylinder +surrounding infinite elastic medium". The mentioned attempt is realized by the numerical results given and discussed in the next section.

4. Numerical results and discussions

In this section instead of upper indices (2) and (1) we will use the upper indices (f) and (m) respectively. Assume that $\nu^{(f)} = \nu^{(m)} = 0.3$ (where $\nu^{(f)}\left(\nu^{(m)}\right)$ is the Poisson's ratio for the cylinder (surrounding medium) material), $E^{(f)}/E^{(m)} = 5.0$ (where $E^{(f)}\left(E^{(m)}\right)$ is a modulus of elasticity of the cylinder (surrounding medium) material). First we consider the case where $\upsilon^{(f)} = 0.0$ and the dispersion curves attained from the solution to the equation (20) (for the axisymmetric longitudinal wave) and (21) (for the torsional wave). The dispersion curves for the first and second modes of these waves are given in Fig. 2. We denote the longitudinal (torsional) wave velocity through $c_L\,(c_T)$. It follows from these graphs that there exists such value of kR (denote it by $(kR)_{*I}$ and $(kR)_{*II}$ for the first and second modes respectively) for which the relations $c_L < c_T$ under $kR < (kR)_{*I}$ (for the first mode), $kR < (kR)_{*II}$ (for the second mode) and $c_L > c_T$ under $kR > (kR)_{*I}$ (for the first mode), $kR > (kR)_{*II}$ for the second mode satisfy. But in the case where $\upsilon^{(f)} > 0.0$ the first (second) mode of the axisymmetric wave propagation, approximately, consists of the part of the dispersion curve of the longitudinal (tortional) axisymmetric wave corresponding the case where $kR < (kR)_{*I}$ and the part of the dispersion curve of the torsional (longitudinal) wave propagation corresponding the case where $kR > (kR)_{*I}$. The similar statement occurs also for the third and fourth modes of the considered wave propagation. Consequently, the existence of the initial twisting in the circular cylinder changes qualitatively the axisymmetric wave propagation.

Note that within the scale used in Fig.2 the differences (in the quantitative sense) between the dispersion curves constructed under $\upsilon^{(2)} > 0$ and under $\upsilon^{(2)} = 0$ do not observed clearly. How this initial twisting changes the velocity of the wave propagation in the quantitative sense is shown in Fig. 3. Note that in this figure the behavior of the dispersion curves around the point indicated by a circle in Fig. 2 and corresponding to the intersection of the longitudinal and torsional waves' dispersion curves is illustrated. In Fig. 3 the dispersion curves are constructed for the various values of the parameter $\upsilon^{(2)}R$ which characterizes the initial twisting of the circular cylinder in the infinite elastic medium. The graphs given in Fig. 3 show how the dispersion curves separated from each other around the mentioned intersection point with $\upsilon^{(2)}R$ as well as how the initial twisting changes the wave propagation velocity in the system considered.

5. Conclusion

In the present paper within the scope of the piecewise homogeneous body model with the use of TLTEWISB the problem on the axisymmetric wave propagation in the initially twisted circular cylinder surrounded by the infinite elastic medium is considered. The mathematical formulation of the problem is presented and the solution method of that is proposed and developed. The numerical results are presented and discussed. According to these results it can be made the following concrete results:

- the initial twisting of the circular cylinder changes the quantitatively the wave propagation in the considered mechanical system;

- in the cases where, approximately, $kR > (kR)_{*I}$ and $kR > (kR)_{*II}$ ($kR < (kR)_{*I}$ and $kR < (kR)_{*II}$) the aforementioned twisting causes to decrease (to increase) the wave propagation velocity, where $(kR)_{*I}$ ($(kR)_{*II}$) corresponds to the intersection point of the dispersion curve in the firs (in the second) mode of the axisymmetric longitudinal and torsional wave propagation under $\upsilon^{(2)} = 0$.

The investigations related the question considered in the present paper will be continued in further works of the author.

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