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FREE VIBRATIONS OF LATERALLY STIFFENED MEDIUM-FILLED CYLINDRICAL SHELLS UNDER AXIAL COMPRESSION AND WITH REGARD TO FRICTION

Abstract

The present paper is devoted to investigation of free vibrations of medium-filled cylindrical shells stiffened by discretely distributed lateral systems of ribs under axial compression and with regard to friction between a shell ad medium. Influence of external medium parameters on eigen vibrations frequency parameter of the system is analyzed.

The paper is devoted to investigation of free vibrations of medium-filled cylindrical shells stiffened by discretely distributed annular ribs under axial compression and with regard to friction between contact surfaces of a shell and filler. Influence of external medium parameter on the eigen vibrations frequency parameter of the system is analyzed.

Experience of creation of optimal constructions, stored in different fields of machine building, aircraft industry, shipbuilding and etc. leads to large extension of structural materials. In its turn, this fact makes necessary more complete account of properties of materials and constructions for rational design and conduction of reliable strength analysis. For more complete description of bearing capacity of constructions, external actions of filler should be taken into account. One of such actions is its contact with elastic medium. As a matter of fact, the external action forces are surface forces and are stipulated by the contact between a shell and elastic filler. The contact is of complicated character and depends on different factors: mechanical parameters of a filler, surface of a shell and etc. One of the principal factors are friction forces stipulated by interaction of a shell and filler. The solution of such type problems represents mathematical difficulty. This difficultly deepens with regard to dynamical effects necessary in the problems of seismic stability, vibrations that are oftenly met in engineering. Therefore, elaboration of approximate method is required. One of the approximate methods is variational method. This is explained by the fact that it allows to get non-contradictory approximate theory of thin-shelled constructions of shells and bars type.

In the given paper, by means of the variational principle we investigate vibrations of a thin laterally stiffened cylindrical shell under dynamic interaction with a filler, under axial compression and with regard to friction in contact. Dependences of frequency of eigen vibrations on wave formation in peripheral direction with regard to friction in contact between a shell and filler, are constructed.

Note that the solutions described in references belong mainly to a strengthened mediumless cylindrical shell [1]. Vibrations of smooth cylindrical filled shells have been sufficiently studied in the papers [2,3]. Vibrations of elastic medium-filled cylindrical shells strengthened by longitudinal ribs are investigated in the paper [4].

The present paper is devoted to investigation of free vibrations of medium-filled cylindrical shells stiffened by discretely distributed lateral systems of ribs under axial compression and with regard to friction between a shell ad medium. Influence of external medium parameters on eigen vibrations frequency parameter of the system is analyzed.

The problem is solved by energetic method. Potential energy of the shell loaded by axial contractive forces is of the form [1]:

$$
\begin{aligned}
\ni= &\frac{Eh}{2(1-v^2)}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial u}{\partial \xi}+\frac{\partial v}{\partial \theta}-w\right)^2+2(1-\nu)\times\right.\\
&\left.\times\left[\frac{\partial u}{\partial \xi}\left(\frac{\partial v}{\partial \theta}-w\right)-\frac{1}{4}\left(\frac{\partial u}{\partial \theta}+\frac{\partial v}{\partial \xi}\right)^2\right]\right\}d\xi d\theta+\\
&+\frac{Eh^3}{24(1-\nu^2)R^2}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial^2 w}{\partial \xi^2}+\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial v}{\partial \theta}\right)^2-2(1-\nu)\times\right.\\
&\left.\times\left[\frac{\partial^2 w}{\partial \xi^2}\left(\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial v}{\partial \theta}\right)-\left(\frac{\partial^2 w}{\partial \xi\partial \theta}+\frac{\partial v}{\partial \xi}\right)^2\right]\right\}d\xi d\theta+\\
&+\frac{E_h}{2R}\sum_{j=1}^{k_1}\int_0^{2\pi}\left[F_h\left(\frac{\partial v}{\partial \theta}-w-\frac{h_h}{R}\frac{\partial^2 w}{\partial \theta^2}\right)^2+\frac{I_{xh}}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2}+w\right)^2+\right.\\
&\left.+\frac{G_h}{R^2E_h}I_{kp.h}\left(\frac{\partial^2 w}{\partial \xi\partial \theta}+\frac{\partial v}{\partial \theta}\right)^2\right]\Bigg|_{\xi=\xi_j}d\theta-\frac{\sigma_x h}{2}\int_0^{\xi_1}\int_0^{2\pi}\left(\frac{\partial w}{\partial \xi}\right)^2 d\xi d\theta.
\end{aligned}
\tag{1}
$$

Here $\xi=\dfrac{x}{R}$, $\theta=\dfrac{y}{R}$; $E_h,\ G_h$ are elasticity and shift modulus of lateral ribs material; k_1 is the amount of lateral ribs; σ_x is axial contractive stress; u,v,w are displacement vector components of a shell; h and R are thickness and radius of a shell, respectively; $E,\ \nu$ is Young's modulus and Poisson ratio of shell's material; $F_h,\ I_{xh},\ I_{kp.h}$ are areas and moments of inertia of cross section of longitudinal and lateral bar with respect to the axis ox and oz, and also moment of inertia in torsion.

Kinetic energy of the shell is as follows:

$$
\begin{aligned}
K=&\frac{Eh}{2(1-\nu^2)}\int_0^{\xi_1}\int_0^{2\pi}\left[\left(\frac{\partial u}{\partial t_1}\right)^2+\left(\frac{\partial v}{\partial t_1}\right)^2+\left(\frac{\partial w}{\partial t_1}\right)^2\right]\times\\
&\times d\xi d\theta+\sum_{i=1}^{k_1}\frac{\overline{\rho}_h E_h F_h}{2R(1-\nu^2)}\sum_{i=1}^{k_1}\int_0^{2\pi}\left[\left(\frac{\partial v}{\partial t_1}\right)^2+\left(\frac{\partial w}{\partial t_1}\right)^2\right]\Bigg|_{\xi=\xi_j}d\theta.
\end{aligned}
\tag{2}
$$

Here $\rho_0,\ \rho_h$ are densities of the shell and lateral bar's materials, respectively $\overline{\rho}_h=\rho_h/\rho_0$.

Influence of a filler on a shell is determined as of external surface loads applied to the shell and is calculated as the work done by these loads when taking the system from deformed state to initial undeformed one and is represented as:

$$
A_0=-\int_0^{\xi_1}\int_0^{2\pi}\left(q_x u+q_\theta v+(1-f)q_z w\right)d\xi d\theta, \tag{3}
$$

where $q_x,\ q_\theta,\ q_z$ are pressures of the filler on the shell, f is a friction factor.

Total energy of the system is as follows:

$$\Pi = \ni + K + A_0 \tag{4}$$

Motion equation of medium in the vector form is as follows [2,3]:

$$a_e^2 qrad\,\mathrm{div}\,\overrightarrow{s} - a_t^2 rotrot\,\overrightarrow{s} + \omega^2 \overrightarrow{s} = 0 \tag{5}$$

Here $a_t = \sqrt{\frac{\lambda_s + 2\mu_s}{\rho}}$, $a_e = \sqrt{\frac{\mu_s}{\rho}}$ are propagation velocities of longitudinal and lateral waves in medium, respectively; $\overrightarrow{s}(s_x, s_\theta, s_z)$ is a displacement vector, λ, μ are Lame coefficients. We add contact conditions to the system of medium motion of equations (5). It is assumed that there is a rigid contact between the shell and medium, i.e. for $r = R$

$$u = s_x;\ v = s_\theta;\ w = s_z \tag{6}$$

$$q_x = -\sigma_{rx},\ q_y = -\sigma_{r\theta},\ q_z = -\sigma_{rr},\ w = s_r \tag{7}$$

The stress tensor components $\sigma_{rx},\ \sigma_{r\theta},\ \sigma_{rr}$ are determined in the following form [2,3]:

$$\sigma_{rx} = \mu_s\left(\frac{\partial s_x}{\partial r} + \frac{\partial s_r}{\partial x}\right);\ \sigma_{r\theta} = \mu_s\left[r\frac{\partial}{\partial r}\left(\frac{s_\theta}{r}\right) + \frac{1}{r}\frac{\partial s_r}{\partial \theta}\right]$$

$$\sigma_{rr} = \lambda_s\left(\frac{\partial s_r}{\partial x} + \frac{1}{r}\frac{\partial (rs_r)}{\partial r} + \frac{1}{r}\frac{\partial s_\theta}{\partial \theta}\right) + 2\mu_s\frac{\partial s_r}{r}\ . \tag{8}$$

Here $\lambda_s,\ \mu_s$ are Lame coefficients for medium.

Complementing the motion equations of a filler (5) by contact conditions (6), (7) we arrive at a contact problem on vibrations of a medium-filled cylindrical shell strengthened by a system of ribs. In other words, a problem on vibrations of a medium-filled cylindrical shell stiffened by cross systems of ribs under axial compression is reduced to joint integration of equations of theory of shells, medium while fulfilling the indicated conditions on their contact surface.

Further, we'll consider a shell with simply supported ends. We'll look for the displacement vector components in the form:

$$\begin{aligned} u &= A\cos kx \cos n\varphi \exp(i\omega_1 t_1),\\ v &= B\sin kx \sin n\varphi \exp(i\omega_1 t_1),\\ w &= C\sin kx \cos n\varphi \exp(i\omega_1 t_1), \end{aligned} \tag{9}$$

where $A,\ B,\ C$ are the unknown constants; $k = \frac{m\pi}{L}$ $(m = 1, 2, ...)$, are wave numbers in longitudinal and peripheral directions, respectively, L is shell's length,

$$\omega_1 = \frac{\omega}{\omega_0}, t_1 = \omega_0 t, \omega_0 = \sqrt{\frac{E}{(1-v^2)\rho_0 R^2}}, \omega_1 = \sqrt{\frac{(1-v^2)\rho_0 R^2\omega^2}{E}}.$$

The solution of system (5) is of the form [3]:

a) under small inertia actions from the side of a filler on the vibration process of the system:

$$s_x = \left[\left(-kr\frac{\partial I_n(kr)}{\partial r} - 4(1-v_s)kI_n(kr)\right)A_s + kI_n(kr)B_s\right]\times$$

$$\times \cos n\varphi \cos kx \exp(i\omega_1 t_1)$$

$$s_\varphi = \left[-\frac{n}{r}I_n(kr)B_s - \frac{\partial I_n(kr)}{\partial r}C_s\right]\sin n\varphi \sin kx \exp(i\omega_1 t_1) \tag{10}$$

$$s_r = \left[-k^2 r I_n(kr)A_s + \frac{\partial I_n(kr)}{\partial r}B_s + \frac{n}{r}I_n(kr)C_s\right]\times$$

$$\times \cos n\varphi \cos kx \exp(i\omega_1 t_1)$$

b) inertia actions of a filler on vibrations process of the system is essential:

$$s_x = \left[A_s k I_n(\gamma_e r) - \frac{C_s\gamma_t^2}{\mu_t}I_n(\gamma_t r)\right]\cos n\varphi \cos kx \exp(i\omega_1 t_1)$$

$$s_\varphi = \left[-\frac{A_s n}{r}I_n(\gamma_e r) - \frac{C_s nk}{r\mu_t}I_n(\gamma_t r) - \frac{B_s}{n}\frac{\partial I_n(\gamma_t r)}{\partial r}\right]\times$$

$$\times \sin n\varphi \sin kx \exp(i\omega_1 t_1)$$

$$s_r = \left[A_s\frac{\partial I_n(\gamma_e r)}{\partial r} - \frac{C_s k}{\mu_t}\frac{\partial I_n(\gamma_t r)}{\partial r} + \frac{B_s n}{r}I_n(\gamma_t r)\right]\times \tag{11}$$

$$\times \cos n\varphi \sin kx \exp(i\omega_1 t_1)$$

Here I_n is Bessel's n-th order modified function of first kind, $A_s,\ B_s,\ C_s$ are constants.

Using contact conditions (6), displacements of shells (9), the solutions of the equation of motion of medium (10) and (11), we express the constants $A_s,\ B_s,\ C_s$ by $A, B,\ C$. As a result, for $q_x,\ q_\theta,\ q_z$ we find:

$$q_x = (\widetilde{C}_{x1}A + \widetilde{C}_{x2}B + \widetilde{C}_{x3}C)\cos n\varphi \cos kx \exp(i\omega_1 t_1) \tag{12}$$

$$q_\theta = (\widetilde{C}_{\theta 1}A + \widetilde{C}_{\theta 2}B + \widetilde{C}_{\theta 3}C)\sin n\varphi \sin kx \exp(i\omega_1 t_1)$$

$$q_r = (\widetilde{C}_{r1}A + \widetilde{C}_{r2}B + \widetilde{C}_{r3}C)\cos n\varphi \sin kx \exp(i\omega_1 t_1)$$

After substitution of (12) in (3) and integration with respect to ξ and θ for the work of external pressures from the side of a filler applied to the shell, we get

$$A = -R^2\pi\left[S_2\widetilde{C}_{x1}A^2 + (S_2\widetilde{C}_{x2} + S_1\widetilde{C}_{\theta 1})AB + \right.$$
$$\left. +(S_2\widetilde{C}_{x3} + S_1\widetilde{C}_{r1})AC + S_1(\widetilde{C}_{\theta 3} + \widetilde{C}_{r2})BC + S_1\widetilde{C}_{\theta 2}B^2 + S_1\widetilde{C}_{r3}C^2\right] \tag{13}$$

Here $\widetilde{C}_{ra}$ is a constant, $S_1 = \dfrac{1}{2} - \dfrac{\sin 2k\xi_1}{4k}$, $S_2 = \dfrac{1}{2} + \dfrac{\sin 2k\xi_1}{4k}$.

Using (1), (2), (3), for the total energy of the system we get a second order polynomial for the parameters of constants $A,\ B,\ C$:

$$\Pi = (\widetilde{\varphi}_{11} - S_2\widetilde{C}_{x1} - \psi_{11}\omega_1^2)A^2 + (\widetilde{\varphi}_{22} - S_1\widetilde{C}_{\theta 2} - \psi_{22}\omega_1^2)B^2 +$$

$$+(\widetilde{\varphi}_{33} - S_1\widetilde{C}_{r3} - \psi_{33}\omega_1^2 + l_1\sigma_x)C^2 + (\widetilde{\varphi}_{44} + S_2\widetilde{C}_{x2} + S_1\widetilde{C}_{\theta 1})AB +$$

$$+(\widetilde{\varphi}_{55} + S_2\widetilde{C}_{x3} + S_1\widetilde{C}_{r1})AC + S_1(\widetilde{\varphi}_{66} + \widetilde{C}_{\theta 3} + \widetilde{C}_{r2})BC$$

Note that the quantities $\varphi_{ii}(i = 1, 2,...,6)$, $\psi_{ii}(i = 1, 2, 3)$, $l_i(i = 1, 2)$ are of bulky form and we don't cite them here.

Conditions of extremum Π in parameters A, B, C reduce the solution of a problem on vibrations of a medium-filled shell stiffened by a lateral system of ribs and subjected to lateral compression, with regard to friction in contact, to homogeneous system of linear algebraic equations of third order. Non-trivial solutions of these systems are possible only if the determinant of this system equals zero. In sequel, equating the determinants of the indicated systems to zero, we get the following frequency equation:

$$\begin{cases} 2(\widetilde{\varphi}_{11} - S_2\widetilde{C}_{x1} - \psi_{11}\omega_1^2)A + (\widetilde{\varphi}_{44} + S_2\widetilde{C}_{x2} + S_1\widetilde{C}_{\theta 1})B + \\ \qquad +(\widetilde{\varphi}_{55} + S_2\widetilde{C}_{x3} + S_1\widetilde{C}_{r1})C = 0 \\ (\widetilde{\varphi}_{44} + S_2\widetilde{C}_{x2} + S_1\widetilde{C}_{\theta 1})A + 2(\widetilde{\varphi}_{22} - S_1\widetilde{C}_{\theta 2} - \psi_{22}\omega_1^2)A + \\ \qquad +(\widetilde{\varphi}_{66} + \widetilde{C}_{\theta 3} + \widetilde{C}_{r2})C = 0 \\ (\widetilde{\varphi}_{55} + S_2\widetilde{C}_{x3} + S_1\widetilde{C}_{r1})A + (\widetilde{\varphi}_{66} + \widetilde{C}_{\theta 3} + \widetilde{C}_{r2})B + \\ \qquad +2(\widetilde{\varphi}_{33} - S_1\widetilde{C}_{r3} - \psi_{33}\omega_1^2 + l_1\sigma_x)C = 0 \end{cases} \tag{14}$$

It is easy no notice that in the case a) the system of equations (14) is reduced to cubic equation with respect to ω_1^2, otherwise it is transcendental. Since in future we'll be interested only in lower frequencies of flexural vibrations, in the case a) we can simplify this equation, rejecting the addends ω_1^4 and ω_1^6. As a result we get $(\omega_1^2 = \lambda_a)$:

$$\lambda_a = \frac{f_3^2 f_4 + f_1 f_5^2 + f_2^2 f_6}{2f_5^2\Psi_{11} + f_2^2\Psi_{33} - 4f_1 f_4\Psi_{33} - 0,5f_6(f_1\Psi_{22} + f_4\Psi_{11})} \tag{15}$$

$f_1 = \widetilde{\varphi}_{11} - S_2\widetilde{C}_{x1}$; $f_2 = \widetilde{\varphi}_{44} + S_2\widetilde{C}_{x2} + S_1\widetilde{C}_{\theta 1}$; $f_3 = \widetilde{\varphi}_{55} + S_2\widetilde{C}_{x3} + S_1\widetilde{C}_{r1}$;
$f_4 = \widetilde{\varphi}_{22} - S_1\widetilde{C}_{\theta 2}$; $f_5 = \widetilde{\varphi}_{66} + \widetilde{C}_{\theta 3} + \widetilde{C}_{r2}$; $f_6 = \widetilde{\varphi}_{33} - S_1\widetilde{C}_{r3} + l_1\sigma_x$
λ_b is similarly determined for the case b).

Fig.1.

Cite the results of investigation of influence of the number of ribs and rigidity of media on axial compression critical stress. Calculations are carried out for a shell, medium and ribs with the parameters:

$$E = E_h = 6,67 \cdot 10^9 N/m^2;\ \nu = 0,3;\ \chi = 1;\ n = 8;\ h_h = 1,39mm;\ R = 160mm;$$

$$L_1 = 800mm;\ h = 0,45mm;\ F_h = 5,75mm^2;$$

$$I_{xh} = 19,9mm^4;\ I_{kp.h} = 0,48mm^2;\ f = 0,25.$$

λ_b similarly determined for the case b).

The results are represented in figure 1. Here, the dependence of ω_1 on contractive force is shown. It is seen from figure 1 that frequencies of the system decreases due to increase of contractive force. Furthermore, account of friction leads to decrease of eigen frequency value of the construction under investigation.

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