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MATHEMATICAL MODELS OF COMPLEX QUEUES AND THEIR APPLICATIONS

Abstract

In this the different lift systems will be considered. Assume that there are two lifts with unbounded volumes, which give a service for customers on N floors. Denote λ_{ij} the intensity of customers (which are going from t^{th} to j^{th} floor) arriving for service. We'll assume that arrival flow is stationary and ordinary Poisson process.

In the capacity of the effeciency indexes for such systems is taken a customer average service time, which consist two components: waiting time before service and service time. Waiting time before service is measured from the instant of customer arriving into the system until the instant when customer gets service (i.e. lift comes to desired floor). Customer service time is defined as a time interval between service start instant and epoch when customer leaves lift.

Our aim is: by introducing control policy to diminish the efficiency index. There exist the different types of control policy. We'll consider control, which means delay of the beginning service. Such control for some systems gave gain in the customer average waiting time.

1. Introduction. Mathematical models of complex queues are widely used in different applications, for instance in the transportation, in biology and so on. Although such mathematical models can be formulated in the frame of standard queues, unfortunately analytical research of these models faces with some difficulties, because they have complicated structure. Important problem here is a control problem by these systems, which can diminish the values of different characteristics and get some gain.

One of the effective approaches for investigations of such systems is simulation of a behavior these systems on computer. Such approach allows calculating different characteristics of the systems and taking necessary decisions.

In this paper the different lift systems will be considered [1]. Assume that there are two lifts with unbounded volumes, which give a service for customers on N floors. Denote λ_{ij} the intensity of customers (which are going from i^{th} to j^{th} floor) arriving for service. We'll assume that arrival flow is stationary and ordinary Poisson process. There are different cases:

1. loading regime, when $\lambda_{1j} \neq 0$;
2. unloading regime, when $\lambda_{1j} = 0$;
3. mixed regime, when $\lambda_{ij} \neq 0$, $i \neq j$.

In the capacity of the efficiency indexes for such systems is taken a customer average service time [2], which consists two components: waiting time before service and service time. Waiting time before service is measured from the instant of customer arriving into the system until the instant when customer gets service (i.e. lift comes to desired floor). Customer service time is defined as a time interval between service start instant and epoch when customer leaves lift.

Our aim is: by introducing control policy to diminish the efficiency index. There exist the different types of control policy. We'll consider control, which means delay of the beginning service [2]. Such control for some systems gave gain in the customer average waiting time [3,4].

Below the different lift systems will be considered. For instance, two Independent Lifts (IL – system) in a building. Another system, (without single race) when both lifts are free and only the closer one to the call will arrive, we call it DL – system (Dependent Lifts), because motion of lifts will be dependent, "odd-even" (OE – system) one lift gives service to odd floors, the other to even floors), "upper-lower" (UL– system, one lift gives service to higher floors $(1, k, k+1, \ldots, n)$ and the other to lower floors $(1, 2, \ldots k-1)$). For the control policy UL it seems, that k should be taken closed to $n/2$. For some systems simulation shows, that k should take value between $3n/4$ and $2n/3$,[5].

Introduce the following notations:

$L_k C_{xx} F_n$- the system with K lifts, control policy xx and n floors, for instance $L_2 C_{IL} F_n$ means system with 2 lifts, control policy IL and n floors. Denote

$\mu(s)$- a customer average (expectation) service time in the system (s).

$h[j]$- a time interval, for passing distance between k floors.

h_2- stopping time at the floor (opening and closing a door);

m- a capacity, of a lift; n- number of the floors.

We consider systems with rare input flows i.e. λ_{ij} takes small values (denoted $\lambda_{ij} \approx 0$). It means that at the next customer arrival instant the previous customer was already served and the lift is free.

Denote $\lambda_1 = \sum\limits_{j=2}^{n} \lambda_{1j}$,$\lambda_2 = \sum\limits_{i=2}^{n} \lambda_{i1}$, and assume $\lambda_{ij} = 0, i, j \neq 1$.

2. Mathematical models

2.1. Systems with one lift

$L_1 C_{IL} F_N$(one lift, no control, N floors).

Loading regime.

$\lambda_{12} = \lambda_{13} = ... = \lambda_{1n} \approx 0, \quad \lambda_1 \approx 0, \quad \lambda_{j1} = 0, \ j = 2, 3, ..., n.$

In all models it is assumed that customer chooses $i-$th floor ($i = 2, 3, ..., n-1$) according to uniform distribution i.e. with probability $1/(n-1)$. In the end of each service lift occupies i-th floor with probability $1/(n-1)$, because customer takes any floor with the same probability. Thus, at the preceding of customer arriving epoch lift occupies i-th floor with the same probability (see, Fig.1), where

$\square-$lift

t_j^{i+} - arriving instant of j-th customer, which goes from the first floor to the up (to the i-th floor);

$t_i^{(s)}$- i-th customer service end instant ;

t_i^{i+}- i-th arriving instant of lift to desired floor (to the customer);

$t_i^{(j-)}$- arriving instant of i-th customer, which comes down from j-th floor;

$t_i^{(k,a)}$- arriving instant of the k-th lift to i-th call;

$t_i^{(k,s)}$- i-th end of service instant of k-th lift;

$\left[t_k^{(a)} - t_k^{(\cdot+)}\right]$– a waiting time before service of k-th customer;

$\left[t_k^{(s)} - t_k^{(a)}\right]$ – spending time in the lift of k-th customer;

$\left[t_k^{(s)} - t_k^{(\cdot+)}\right]$– service time of k-th customer;

$(- - --)$ – lift free time interval (no customers in the system)

$\square-$ means in the figure that lift is free in this instant

Hence,

$$\mu\left(L_1 C_{IL} F_n\right) = (n-1)\, h_1 + h_2 \tag{2.1}$$

System $L_1 C_{xx} F_n$ (one lift, control policy, n –floors).

Introduce control policy, which means that in the end of service lift immediately must go to the first floor. We denote such control policy as C_1. Hence, according to this control policy at the preceding customer arriving epoch lift must occupy first floor (see, Fig.2), where t^* is the epoch when lift comes down to the first floor.

Hence,

$$\mu\left(L_1 C_1 F_n\right) = (n-1)\, h_1 \,[1]\,/2 + h_2 \tag{2.2}$$

introducing of this control policy decreases a customer service time two times.

2.2. System $L_1C_{IL}F_n$ (one lift, n floors, no control)
Unloading regime.

$$\lambda_{12} = \lambda_{13} = ... = \lambda_{1n} = 0, \quad \lambda_{21} = \lambda_{31} = ...\lambda_{n1} \approx 0, \quad \sum_{i=2}^{n} \lambda_{i1} = \lambda_2 \approx 0. \tag{2.3}$$

For this system all customers coming down from i-th ($i = 2, 3, \ldots, n$) floor to the first floor, hence in the end of customer service lift always occupies first floor (see Fig.3).

2.3. System $L_1C_{xx}F_n$ (one lift, N floors with control policy xx)
Control means that in the end of customer service lift immediately must go to the $(n/2)$-th floor and we'll denote it as $L_1C_{(n-1)/2}F_n$. Thus, at the preceding customer arriving instant lift must occupies $(n/2)$-th floor, (see, Fig.4).

Simple calculations yield

$$\mu\left(L_1C_{n/2}F_n\right) = 3\,(n-1)\,h\,[1]\,/4 + h_2 \tag{2.4}$$

i.e., customer service time for $L_1C_{n/2}F_n$ will be decreased by 25%.

2.4. System $L_1C_{IL}F_N$ (one lift, N floors, no control)
Mixed regime.

$$\lambda_{12} = \lambda_{13} = ... = \lambda_{1n} \approx 0, \quad \lambda_{21} = \lambda_{31} = ...\lambda_{n1} \approx 0, \quad \sum_{i=2}^{n} \lambda_{i1} = \lambda_2 \approx 0.$$

Using formula of complete probability we have

$$\mu\left(L_1C_{IL}F_n\right) = \left[\lambda_1\lambda_2\left(n-1\right)/2 + \lambda_2^2\left(n-1\right) + \right.$$

$$\left. + \lambda_1\lambda_2\left(5n^2 - 4n - 3\right)h_1/6\left(n-1\right)\right]/\left(\lambda_1 + \lambda_2\right)^2 + h_2$$

For large value n we have

$$\mu\left(L_1C_{IL}F_n\right) \approx \left[\left(\lambda_1^2 + \lambda_2^2 + 4\lambda_1\lambda_2/3\right)\right]nh_1/\left(\lambda_1 + \lambda_2\right)^2 + h_2. \tag{2.5}$$

Corollary 2.1. *If $\lambda_1 = 0$ then from (2.5) it follows (2.4) and if $\lambda_2 = 0$ then from (2.5) it follows (2.1). System $L_1C_xF_N$ (one lift, control policy x, N floors).*
Mixed regime.

$$\lambda_{12} = \lambda_{13} = ... = \lambda_{1n} \approx 0, \quad \lambda_{21} = \lambda_{31} = ...\lambda_{n1} \approx 0,$$

$$\sum_{j=2}^{n} \lambda_{1j} = \lambda_1 \approx 0, \quad \sum_{i=2}^{n} \lambda_{i1} = \lambda_2 \approx 0.$$

Let's introduce the following control. In the end of service the lift must go to k-th floor. What would be an optimal k, which minimizes value of w?

$$\mu\left(L_1C_kF_n\right) = \left[\lambda_2/\left(\lambda_1 + \lambda_2\right)\right]\left(k-1\right)^2/\left(n-1\right) +$$

$$+\left[\lambda_1/\left(\lambda_1 + \lambda_2\right) - \lambda_2/\left(\lambda_1 + \lambda_2\right)\right]\left(k-1\right) +$$

$$+\left(n-1\right)\left[\lambda_1/2\left(\lambda_1 + \lambda_2\right) + \lambda_2/\left(\lambda_1 + \lambda_2\right)\right]h_1 + h_2 \tag{2.6}$$

Corollary 2.2
If $\lambda_2 = 0$, then $k = 1$; if $\lambda_1 = 0$, then $k = \left(n-1\right)/2$ (see,Fig.4)
If $\lambda_1 \geq \lambda_2$, then $k = 1$; if $\lambda_1 < \lambda_2$, then $k = \left(1 - \lambda_1/\lambda_2\right)\left(n-1\right)/2$.
Corollary 2.3 *If $\lambda_2 = 0$ then (1.6) can be represented in the following form*

$$\mu\left(L_1C_kF_n\right) \approx \left[\left(k-1\right) + \left(n-1\right)/2\right]h_1 + h_2 \tag{2.7}$$

If $k = 1$, then from (2.7) it follows (2.3).
Corollary 2.4 *If $\lambda_1 = 0$ then from (2.6) we have*

$$\mu\left(L_1C_kF_n\right) =$$

$$= \left\{\left[\left(k-1\right)/\left(n-1\right)\right]\left[k-1\right] + \left[\left(n-k\right)/\left(n-1\right)\right]\left(N - k + k - 1\right)\right\}h_1 + h_2 =$$

$$= \left[\left(k-1\right)^2/\left(n-1\right) - \left(k-1\right) + \left(n-1\right)\right]h_1 + h_2 \tag{2.8}$$

If $k = \left(n-1\right)/2$ then $\mu\left(L_1C_{(n-1)/2}F_n\right) = 3\left(n-1\right)_1 h_1/4 + h_2$ i.e. from (2.8) it follows (2.4).

Corollary 2.5 *If* $\lambda_1 = \lambda_2$ *then*

$$\mu\left(L_1C_{(n-1)/2}F_n\right) = \left[\lambda_2/\left(\lambda_1+\lambda_2\right)\right](k-1)^2/(n-1)+$$
$$+(n-1)\left[\lambda_1/2\left(\lambda_1+\lambda_2\right)+\lambda_2/\left(\lambda_1+\lambda_2\right)\right]h_1+h_2=$$
$$=\left[(k-1)^2/2(n-1)+3(n-1)/4\right]h_1+h_2 \qquad (2.9)$$

and it follows from (2.9)

$$k=1, \mu\left(L_1C_1F_n\right)=3(n-1)h_1/4+h_2$$

Corollary 2.6 *If* $\lambda_1 > \lambda_2$ *then*

$$\mu\left(L_2C_kF_n\right)=\left\{\left[\lambda_2/\left(\lambda_1+\lambda_2\right)\right](k-1)^2/(n-1)+\right.$$
$$\left[\lambda_1/\left(\lambda_1+\lambda_2\right)-\lambda_2/\left(\lambda_1+\lambda_2\right)\right](k-1)+(n-1)\times$$
$$\left.\times\left[\lambda_1/2\left(\lambda_1+\lambda_2\right)+\lambda_2/\left(\lambda_1+\lambda_2\right)\right]\right\}h_1+h_2 \qquad (2.10)$$

Hence, it follows from (2.10)

$$k=1,\ \mu\left(L_1C_1F_n\right)=(n-1)h_1/\left[\lambda_1/2\left(\lambda_1+\lambda_2\right)+\lambda_2/\left(\lambda_1+\lambda_2\right)\right]+h_2$$

3. System $L_2C_{IL}F_N$ (Two lifts, N floors, no control)
Loading regime.

$$\lambda_{12}=\lambda_{13}=...=\lambda_{1n}\approx 0,\quad \sum_{i=2}^{n}\lambda_{1j}\approx 0, \lambda_{21}=\lambda_{31}=...\lambda_{n1}=0.$$

We assume that if both lifts are free then to the next customer call both lifts are going. Such situations can be observed in buildings where each lift has and individual call button and when those buttons are pushed simultaneously.

Thus, both lifts will independently going to a call. Then, for that system at the preceding customer arriving epoch one lift occupies the first floor, the other i-th $(i = 2, 3, .., n)$ floor, (see, fig.5).

Hence,

$$\mu\left(L_2 C_{IL} F_n\right) = (n-1)\, h_1/2 + h_2 \tag{3.1}$$

Consider the system $L_2 C_{DL} F_n$. For customer call only one lift is going, i.e. nearest lift.

Then starting from third customer at the preceding of customer arriving epoch with the same probability one lift occupies i-th $(i = 2, 3, \ldots;)$ floor another j-th $(j = 2, 3, ..;)$ floor (see Fig.6).

Hence,

$$\mu\left(L_2 C_{DL} F_n\right) = (n-1)\, h_1 + h_2 \tag{3.2}$$

Comparison of (3.1) and (3.2) shows that for small values of intensity the system $L_2 C_{IL} F_n$ preferable that the system $L_2 C_{DL} F_n$ as $\mu\left(L_2 C_{IL} F_n\right) < \mu\left(L_2 C_{DL} F_n\right)$

3.1. System $L_2 C_{IL} F_N$ (two lifts, N floors, no control)

Mixed regime.

$$\lambda_{12} = \lambda_{13} = ... = \lambda_{1N} \approx 0,\ \sum_{i=2}^{n} \lambda_{1j} \approx 0, \lambda_{21} = \lambda_{31} = ...\lambda_{n1} \approx 0,\ \sum_{i=2}^{n} \lambda_{i1} = \lambda_2 \approx 0.$$

In this case at the preceding customer arriving epoch one lift occupies first floor and another i-th floor $(i = 2, 3, \ldots, n)$. The probability to have customer at the first floor is $\lambda_1/\left(\lambda_1 + \lambda_2\right)$ and at the other floor $\lambda_2/\left(\lambda_1 + \lambda_2\right)$. Thus, an expectation of customer service time comes to

$$\mu\left(L_2 C_{IL} F_n\right) = \sum \left\{ \left(\lambda_1/\left(\lambda_1 + \lambda_2\right)\right)(n-1)^2 + \left(\lambda_2/\left(\lambda_1 + \lambda_2\right)\right)\left[1/\left(n-1\right)\right]^2 \times \right.$$

$$\times \left[3n\left(n-1\right)\left(2n-1\right)/24+\left(n-1\right)^{2}\left(n-2\right)/2\right]\Big\}\, h_1+h_2$$

For large n we have

$$\mu\left(L_2C_{IL}F_n\right)\approx\left(n/4\right)\left(2\lambda_1+3\lambda_2\right)h_1\left(\lambda_1+\lambda_2\right)+h_2.$$

Corollary 3.1 *If* $\lambda_1=\lambda_2$, *then*

$$\mu\left(L_2C_{IL}F_n\right)\approx\left(5/8\right)nh_1+h_2 \tag{3.3}$$

System $L_2C_{xx}F_N$ (two lifts, control policy xx, N floors).

Introduce the control policy, which means that at the preceding customer arriving epoch one lift occupies k_1-th floor, another k_2- th floor. Our aim is to find k_1 and k_2, which minimizes the value of $\mu\left(L_2C_{k_1k_2}F_n\right)$. Similarly (3.4) we have

$$\mu\left(L_2C_{k_1k_2}F_n\right)=\left\{\left(\lambda_1/\left(\lambda_1+\lambda_2\right)\right)\left(k_1-1+\left(n-1\right)/2\right)\left(\lambda_2/\left(\lambda_1+\lambda_2\right)\right)\times\right.$$

$$\times\left\{\left[\left(k_1-1\right)/\left(n-1\right)\right]\left(k_1-1\right)+\left[\left(k_2-k_1\right)/2\left(n-1\right)\right]\left[\left(k_2-k_1\right)/2+\left(k_1-1\right)\right]+\right.$$

$$+\left[\left(k_2-k_1\right)/2\left(n-1\right)\right]\left(k_2-1\right)+$$

$$\times\left[\left(n-k_2\right)/\left(n-1\right)\right]\left[\left(n-k_2\right)+\left(k_2-1\right)\right]\}\, h_1+h_2 \tag{3.4}$$

For large N we have

$$k_1\approx\max\left[1,\left(n/4\right)\left(1-3\lambda_1/\lambda_2\right)\right],\ k_2\approx\left(k_1+2n\right)/3 \tag{3.5}$$

Using (3.4) and (3.5) we have

$$\mu\left(L_2C_{k_1k_2}F_n\right)=\left\{\lambda_1\left[\left(k_1-1\right)+\left(n-1\right)/2\right]+\right.$$

$$\left.+\,\lambda_2\left[3\left(k_1-1\right)^{2}+\left(n-k_1\right)\left(k_1+2n-4\right)\right]/3\left(n-1\right)\right\}/\left(\lambda_1+\lambda_2\right). \tag{3.6}$$

Remark 3.1 If $\lambda_2=0$ then it follows from (3.5) $k_1=1$. In this case, in fact only one lift operates, because in the end of service it comes to the first flow and there are no customers in another floor. Therefore, it does not matter location of the second lift.

For large n it follows from (3.6) $\mu\left(L_2C_{11}F_n\right)\approx\left(n-1\right)/2$, $e=0$.

Remark 3.2 If $\lambda_1=0$ then for large N using (3.5) and (3.6) we have

$$k_1=n/4, k_2=3n/4,\ \mu\left(L_2C_{k_1k_2}F_n\right)=5n/8 \tag{3.7}$$

i.e. at the preceding customer arriving epoch one lift occupies $[n/4]$ –th floor another $[3n/4]$ –th floor (see Fig.6). Comparison (2.3) and (2.7) shows

that control gives the gain in the expectation of service time 16% and in the single race time 10%.

Remark 3.3 If $\lambda_1=\lambda_2$ then for large N it follows from (3.6) and (3.7)

$$k_1=1,\ k_2=2n/3, \mu\left(L_2C_{k_1k_2}F_n\right)=7n/12 \tag{3.8}$$

i.e. at the preceding customer arriving epoch one lift must occupy first floor another $2n/3$ floor, (see, Fig.7). We assume that $2n/3$ is an integer number.

Comparison (3.8) with (3.4) shows that control gives the gain in the expectation of customer service time 4% and in the single race time 2%.

$$\lambda_{12} = \lambda_{13} = ... = \lambda_{1n} \approx 0,\ \lambda_1 = \sum_{j=2}^{n} \lambda_{1j} \approx 0, \lambda_{j1} = 0,\ j = 2, 3, ..., n.$$

Denote i_1, i_2 coordinates first and second lifts at the customer preceding epoch into the system. As a customer from the first floor uniformly takes floors $2, 3, \ldots, n$, then $i_1 = 1,\ i_2 = j,\ j \neq 1$ or $i_1 = j,\ i_2 = 1,\ j \neq 1$ with the same probabilities. Hence, $y = 0,\ w^{(L_2C_{IL}F_n)} = 0$.

For $L_2C_{DL}F_n-$ system (without single race we have $i_1 = j_1,\ i_2 = j_2,\ (j_1, j_2 \neq 1)$ or $i_1 = j_2,\ i_2 = j_1,\ (j_1, j_2 \neq 1)$ with the same probabilities. Hence, $\mu\left(L_2C_{IL}F_n\right) = \left[n\left(n+1\right)\right] / \left[3\left(n-1\right)\right] \approx n/3$.

For $L_2C_{DL}F_n-$ system we have $i_1 = j_1, i_2 = j_2, j_1, j_2 = 2, 3, .., n$. Hence,

$$\mu\left(L_2C_{DL}F_n\right) = n/2.$$

So for this case (small intensity at the first floor and no customers at the other floors) $L_2C_{IL}F_n-$ system is preferable than $L_2C_{DL}F_n-$ system and it would be a right idea to keep always one lift at the first floor.

Unloading regime. For the $L_2C_{IL}F_n$ system we have $i_1 = 1, i_2 = j, j = 2, 3, .., n$ or $i_1 = j,\ i_2 = 1,\ j = 2, 3, ..., n$ with the same probabilities. Hence, simple calculations yield $n/5 < \mu\left(L_2C_{IL}F_n\right) < n/4$ i.e. it means that in fact only one lift operates in this system.

For $L_2C_{DL}F_n$ – system we have $i_1 = j_1,\ i_2 = j_2;\ j_1,\ j_2 = 2, 3, ..., n$. Hence,

$$\mu\left(L_2C_{DL}F_n\right) = n/2.$$

i.e. for this case $L_2C_{IL}F_n-$ system is preferable.

Mixed regime. $\lambda_1 \approx 0$, $\lambda_2 \approx 0$, $\lambda_1 + \lambda_2 \approx 0$. For system we have $L_2C_{IL}F_n$, $i_1 = 1$, $i_2 = j$, $j = 2, 3, ..., n$ or $i_1 = j$, $i_2 = 1$, $j = 2, 3, ..., n$.

Simple calculations yield $\mu\left(L_2C_{DL}F_n\right) = \left(\lambda_1/\left(\lambda_1+\lambda_2\right)\right)A$, where $n/4 < A < n/3$ and for - system we have

$$\mu\left(L_2C_{DL}F_n\right) = \left[\lambda_1/\left(\lambda_1+\lambda_2\right)\right](n/3) + \left[\lambda_2/\left(\lambda_1+\lambda_2\right)\right](n/2)$$

It follows from formulas that for some cases $(\lambda_1 \approx 0, \quad \lambda_2 \approx 0, \ \lambda_1 + \lambda_2 \approx 0)$ $L_2C_{IL}F_n-$ system is preferable (customer service time is less) than $L_2C_{DL}F_n-$ system, but it is obviously that generally $L_2C_{DL}F_n-$ system more effectively operates and moreover it spends less energy.

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