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ON A CHARACTERISTIC FUNCTION OF FATIGUE FAILURE UNDER ASYMMETRIC LOADING CYCLES

Abstract

The characteristic function describing the fatigue failure process of materials under stationary asymmetric loading cycles is suggested. The system of experiments for defining the unknown material constants contained in the suggested function is formulated. Some experimental data of fatigue failure of structural steel used in the facilities working in sea-water are processed.

Let's consider a cyclic loading process that occurs at some constant value of the stress S_o. Let S_a be an amplitude of the considered loading process. The fatigue curve will depend on S_o and S_a. If by N_s we denote the number of cycles to fatigue failure, then $N_s = N_s(S_0, S_a)$. This dependence is defined from the experiments for cyclic fatigue of the samples made of the material under investigation. For each material, the dependence $N_s = N_s(S_0, S_a)$ has one and the same form and therefore is assumed to be a fatigue failure characteristic under asymmetric cycles.

While constructing phenomenological theories of fatigue failure, the function $N_s = N_s(S_0, S_a)$ has an essential value, since it is contained in the relation for defining the number of cycles to fatigue failure under non-constant applitude S_a. While conducting specific strength analysis, the specific form of the characteristic function $N_s = N_s(S_0, S_a)$ is required. The following approximation formula for this function is suggested:

$$N_s(S_0, S_a) = \\ = \begin{cases} N_0 \exp\left[\alpha\left(1 - \frac{(S_0+S_a)^2}{r^2}\right) + \beta\left(1 - \frac{(S_0-S_a)^2}{r^2}\right)\right], \text{for } S \geq r; \\ \infty, \quad \text{for } S < r \end{cases} \tag{1}$$

where N_0 is a base number of cycles to fatigue failure, α and β are the constants of the material, r is an endurance limit.

In the suggested formula (1), the experimental facts that the relations of maximal and minimal stresses on endurance limit have an essential influence on N_s, are taken into account. The noted one allows to take into account the influence of asymmetry degree of the cycle on N_s.

Now consider a matter on experimental definition of the quantities N_0, α, β, r contained in relation (1). These quantities are determined while conducting experiments on fatigue failure of samples made of the material under investigation.

The endurance limit r is determined with using Weyler diagram that is a characteristic of fatigue failure of materials under symmetric loading cycles. The number of cycles to failure are plotted on the abscissa, the amplitude of alternating loads stress by the law of stress symmetric cycles are laid off along the ordinate. For many materials, the Weyler diagram has a horizontal asymptote. Therewith, the appropriate stress defines the endurance limit r. Under the amplitudes less than the endurance limit, the fatigue failure of these materials doesn't occur. Note the Weyler diagram of some materials, especially of non-ferrous metals has no asymptote. The endurance limit of these materials is defined conventionally as a stress amplitude quantity under which the sample fails after the given number of cycles that is called the test base. The test base is established subject to the required durability of the construction.[1].

For the known r the base number of the cycles N_0 may be determined from the fatigue failure experiments of the samples under the following symmetric loading cycles: $S_0 = 0,\ S_s = r$. Indeed, for $S_0 = 0,\ S_a = r$ from (1) we have: $N_0 = N_s(0, r)$. For unknown r the quantity N_0 is given as a test base.

Now note the results that allow to define the constants α and β. Under symmetric loading cycles ($S_0 = 0$) formula (1) goes into the following formula

$$N_s\,|_{S_0=0} = \begin{cases} N_0 \exp\left[\gamma\left(1 - \dfrac{S_a^2}{r^2}\right)\right], & \text{for} \quad S_a \geq r, \\ \infty \quad \text{for} \quad S_a < r. & \end{cases} \tag{2}$$

where $\gamma = \alpha + \beta$.

Relation (2) is the equation of Weyler curve. Let the Weyler diagram (curve) be known for the given material. While using this diagram and according to formula (2), we define the quantity γ, i.e. the sum of two desired quantities α and β:

$$\gamma = \frac{[ln(N_3/N_0)]_{S_0=0}}{1 - \dfrac{S_a^2}{r^2}}. \tag{3}$$

Consequently, the Weyler curve allows to define not only the endurance limit r and the base number of the cycles N_0, but also the quantity $\gamma = \alpha + \beta$. Therewith, giving different values to $S_a = S_a^{(k)}$ $(k = 1, 2, ...)$, from the Weyler curve we measure the appropriate values of $N_s^{(k)} = N_s(N_0, r, S_a^{(k)})$ and then with using one of the mathematical approximations methods we find the quantity $\gamma = \alpha + \beta$ from formula (3).

Let the value of the quantity $S_a : S_a = S_a^0 \geq r$ be fixed. Let also for the given material the dependence curve $N_s \sim S_0$ that corresponds to the fixed value $S_a = S_a^0$ be known. This curve enables under the given various values of $S_a = S_0^{(m)}$ $(m = 1, 2, ...)$ to define $N_s^{(m)} = N_s^{(m)} = N_s(S_0^{(m)}, S_a^0)$. In this case, allowing for $\beta = \gamma - \alpha$, formula (1) for $S_a = S_a^0 \geq r$ may be written in the form:

$$N_s^{(m)} = N_0 \exp\left[\alpha\left(1 - \frac{(S_0^{(m)} + S_a^0)^2}{r^2}\right) + (\gamma - \alpha)\left(1 - \frac{(S_0^{(m)} - S_a^0)^2}{r^2}\right)\right].$$

Hence we have

$$\alpha = \frac{r^2}{4S_0^{(m)}S_a^0}\left\{\gamma\left(1 - \frac{(S_0^{(m)} - S_a^0)^2}{r^2}\right) - ln\frac{N_s^{(m)}}{N_0}\right\}. \tag{4}$$

For the known r, γ, N_0 and determined from the curve $N_s = N_s(S_0^{(m)}, S_a^0)$ $(m = 1, 2, ...)$ values of $N_s^{(m)}$, formula (4) allows is find the quantity α. By using formula (4) one of the methods of mathematical approximations should be enlisted. As soon as the quantity α becomes known, the quantity β is defined from the formula $\beta = \gamma - \alpha$.

Thus, we formulated the system of experiments that allows to find the unknown constants contained in formula (1) for characteristical function of fatigue failure of materials under asymmetric loading cycles.

Some fatigue failure experimental data of structural steel used in the installations working at sea-water [2] were processed. For this steel, the endurance r limit approximately equals 7 Mra. Base number of the cycles N_0 equals 10^8. The quantity $\gamma = \alpha + \beta = 0,2$. Fatigue curve for the structural steel under consideration is recalculated after definition of the constants. Discordance of experimental and rated data didn't increase 4%.

Finally note that the suggested formula (1) for the characteristic function failure may be used in deterministic and stochastic [4,5] theories of materials fatigue failure.

References

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