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# REGULAR SOLVABILITY CONDITIONS OF A BOUNDARY VALUE PROBLEM FOR OPERATOR-DIFFERENTIAL EQUATIONS IN HILBERT SPACE

**Abstract**

*In the paper, sufficient conditions providing the existence and uniqueness of regular solutions of a boundary value problem on a finite segment for second order operator differential equations in Hilbert space are obtained. These conditions are expressed by the properties of the coefficients of an operator-differential equation.*

Let $H$ be a separable Hilbert space, $A$ be a normal invertible operator whose spectrum is contained in the angular sector $S_\varepsilon = \{\lambda : |\arg\lambda| \le \varepsilon,\ 0 \le \varepsilon < \pi/2\}$. Suppose that $\{\lambda_k\}_{k=1}^{\infty}$ are eigen values, $\{e_k\}$ is an appropriate complete system of eigen vectors of the operator $A$:

$$Ae_k = \lambda_k e_k, \quad (e_k, e_j) = \delta_{kj} = \begin{cases} 1, & k = j, \\ 0, & k \ne j, \end{cases}$$

$$\lambda_k = \mu_k e^{i\varphi_k}, \quad |\varphi_k| \le \varepsilon,\ 0 < \mu_1 \le \mu_2 \le ... \le \mu_k \le ...$$

Then the operator may be represented in the form $A = UC$, where $C\cdot = \sum_{k=1}^{\infty} \mu_k(\cdot, e_k)e_k,\ U\cdot = \sum_{k=1}^{\infty} e^{i\varphi_k}(\cdot, e_k)e_k, \quad A\cdot = \sum_{k=1}^{\infty} \lambda_k(\cdot, e_k)e_k$. Obviously, for $\gamma \ge 0$

$$D(C^\gamma) = \left\{ x : \sum_{k=1}^{\infty} \mu_k^{2\gamma} |x, e_k)|^2 < \infty \right\}.$$

As is known, the linear set $D(C^\gamma)$ becomes a Hilbert space $H_\gamma$ with respect to the scalar product $(x, y)_\gamma = (C^\gamma x, C^\gamma y)$. Let $-\infty \le a < b \le +\infty$. Denote by $L_2((a, b); H)$ a Hilbert space of all vector functions $f(t)$ determined on the interval $(a, b)$ almost everywhere, with the values in $H$ for which

$$\|f\|_{L_2((a,b);H)} = \left( \int_a^b \|f(t)\|^2 \, dt \right)^{1/2}.$$

As in the book [1] introduce the Hilbert space

$$W_2^2((a, b); H) = \left\{ u : u'' \in L_2((a, b); H),\ C^2 u \in L_2((a, b); H) \right\}$$

with the norm

$$\|u\|_{W_2^2((a,b);H)} = \left( \|u''\|^2_{L_2((a,b);H)} + \|C^2 u\|^2_{L_2((a,b);H)} \right)^{1/2}.$$

For finite $a$ and $b$, i.e. $0<a<b<\infty$ denote by

$$\overset{\circ}{W}{}_2^2\left((a,b)\,;H\right)=\left\{u:W_2^2((a,b):H),\ u'(a)=u'(b)=0\right\}.$$

Obviously, by the traces theorem [1] $\overset{\circ}{W}{}_2^2\left((a,b)\,;H\right)$ is a complete Hilbert space.
Consider in the space $H$ the boundary value problem

$$P(d/dt)u(t)=-u''(t)+A_1u'(t)+A_2u(t)+A^2u(t)=f(t),\quad t\in(0,T),\qquad(1)$$

$$u'(0)=\varphi_0,\quad u'(T)=\varphi_1,\qquad(2)$$

where $u(t)$ and $f(t)$ take the values in $H$, $\varphi_0,\varphi_1\in H$, and the operator coefficients satisfy the conditions:

1) $A$ is a normal invertible operator in $H$ with completely continuous invertible $A^{-1}$ whose spectrum is contained in the angular sector

$$S_\varepsilon=\left\{\lambda:|\arg\lambda|\le\varepsilon,\ 0\le\varepsilon<\frac{\pi}{2}\right\};$$

2) $A_1A^{-1}$ and $A_2A^{-2}$ are bounded operators in $H$.

**Definition 1.** *If for $f(t)\in L_2((0,T);H)$ there exists the vector-function $u(t)$ satisfying equation (1), we say that $u(t)$ is a regular solution of equation (1).*

**Definition 2.** *If for any collection $f(t)\in L_2((0,T);H)$, $\varphi_0,\varphi_1\in H_{1/2}$ there exists the regular solution $u(t)$ of equation (1) that satisfies boundary conditions (2) in the sense of convergence*

$$\lim_{t\to+0}\left\|u'(t)-\varphi_0\right\|_{1/2}=0,\quad\lim_{t\to T-0}\left\|u'(t)-\varphi_1\right\|_{1/2}=0$$

*and it holds the estimation*

$$\|u\|_{W_2^2((0,T);H)}\le const\left(\|f\|_{L_2((0,T;H)}+\|\varphi_0\|_{1/2}+\|\varphi_1\|_{1/2}\right),$$

*we say that problem* (1), (2) *is called regularly solvable.*

In the present paper we'll find conditions on the coefficients of equation (1), that provide regular solvability of problem (1), (2). Note that in an infinite domain, the similar problems were investigated in many papers, for instance see [2-6], when $A$ is a positive-definite self-adjoint operator, in the papers [7,8], when $A$ is a normal operator. In a finite domain for $\varphi_0=\varphi_1=0$, when $A$ is a positive self-adjoint operator, this problem was considered in [9].

At first consider the boundary value problem

$$P_0(d/dt)u=-u''(t)+A^2u(t)=0,\quad t\in(0,T),\qquad(3)$$

$$u'(0)=\varphi_0,\ u'(T)=\varphi_1.\qquad(4)$$

**Theorem 1.** *Let condition 1) be fulfilled. Then problem (3), (4) is regularly solvable.*

**Proof.** Since $A$ is a normal invertible operator whose spectrum is contained in the sector $S_\varepsilon=\{\lambda:|\arg\lambda|\le\varepsilon,\ 0\le\varepsilon<\pi/2\}$, then $e^{-At}(t>0)$ is a strongly continuous semi-group of bounded operators. Then the general solution of equation (3) from $W_2^2((0,T);H)$ is of the form

$$u_0(t) = e^{-At}x_0 + e^{-A(T-t)}x_1, \tag{5}$$

where $x_0, x_1 \in H_{3/2}$. From condition (4) we get $-Ax_0 + Ae^{-AT}x_1 = \varphi_0$ and $-Ae^{-AT}x_0 + Ax_1 = \varphi_1$ or $-x_0 + e^{-AT}x_1 = A^{-1}\varphi_0$ and $-e^{-AT}x_0 + x_1 = A^{-1}\varphi_1$. Then with respect to $x_0$ we get the equation: $\left(E - e^{-2AT}\right)x_0 = A^{-1}e^{-AT}\varphi_1 - -A^{-1}\varphi_0 \in H_{3/2}$. Since for any $x \in H$

$$\left\|\left(E - e^{-2AT}\right)x\right\|^2 \geq \sum_{k=1}^{\infty}\left|1 - e^{-2\lambda_k T}|^2|\left(x, e_k\right)\right|^2 \geq$$

$$\geq \sum_{k=1}^{\infty}(1 - e^{-2\cos\varepsilon T})^2 \left|(x, e_k)\right|^2 = (1 - e^{-2\cos\varepsilon T})^2 \left\|x\right\|^2,$$

then the operator $E - e^{-2AT}$ is invertible in $H$ and $\left\|\left(E - e^{-2AT}\right)^{-1}\right\| \leq \leq (1 - e^{-\cos\varepsilon T})^{-1}$. Consequently, $x_0 = (E - e^{-2AT})^{-1}(e^{-A}A^{-1}\varphi_1 - A^{-1}\varphi_0)$. Obviously,

$$\left\|x_0\right\|_{3/2} = \left\|C^{3/2}(E - e^{-2AT})^{-1}(e^{-A}A^{-1}\varphi_1 - A^{-1}\varphi_0)\right\| \leq$$

$$\leq \left\|(E - e^{-2AT})^{-1}\right\| \left\|C^{3/2}(e^{-A}A^{-1}\varphi_1 - A^{-1}\varphi_0)\right\| \leq$$

$$\leq const \left\|C^{1\backslash 2}\left(e^{-A}A^{-1}\varphi_1 - A^{-1}\varphi_0\right)\right\|_{3/2} \leq$$

$$\leq const \left\|e^{-A}\varphi_1 - \varphi_0\right\|_{1/2} \leq const \left(\left\|\varphi_1\right\|_{1/2} + \left\|\varphi_0\right\|_{1/2}\right),$$

i.e. $x_0 \in H_{3/2}$. We find the vector $x_1$ from the equation $x_1 = A^{-1}\varphi_1 - e^{-AT}x_0$. Obviously, $x_1 \in H_{3/2}$. Thus, $\left\|u_0(t)\right\| \leq \left(\left\|\varphi_0\right\|_{1/2} + \left\|\varphi_1\right\|_{1/2}\right)$. The theorem is proved.

Now consider the problem

$$P_0(d/dt)u(t) = -u''(t) + A^2u(t) = f(t), \quad t \in (0,T), \tag{6}$$

$$u'(0) = \varphi_0, \; u'(T) = \varphi_1. \tag{7}$$

**Theorem 2.** *Let condition 1) be fulfilled. Then problem (6), (7) is regularly solvable.*

**Proof.** After substitution of $u(t) = \omega(t) - u_0(t)$, where $u_0(t)$ is a regular solution of problem (3), (4) that is of the form (5), in order to determine $\omega(t)$ we get the problem

$$P_0(d/dt)u(t) = -\omega''(t) + A^2\omega(t) = f(t), \quad t \in (0,T), \tag{8}$$

$$\omega'(0) = 0, \; \omega'(T) = 0. \tag{9}$$

Show that problem (8), (9) is regularly solvable. We can write problem (8), (9) in the form of the equation $P_0\omega = f$, where $\omega \in \overset{\circ}{W}{}_2^2((0,T);H)$ and $f \in L_2((0,T);H)$.

From theorem 1 it follows that $KerP_0=\{0\}$. Show that the range of values of the operator $P_0$ coincides with $L_2((0,T);H)$. It is easy to see that

$$\omega_1(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\xi^2E+A^2)^{-1}\left(\int_0^1 f(s)e^{-i\xi s}ds\right)e^{i\xi t}dt,\quad t\in R$$

belongs to the space $W_2^2(R;H)$ $(R=(-\infty,+\infty))$ and satisfies the equation $P_0(d/dt)\omega(t)=f(t)$ in $R$. Denote the contraction of $\omega_1(t)$ on $[0,T]$ by $\xi_1(t)$. Then we'll look for $\omega(t)$ in the form

$$\omega(t)=\xi_1(t)+e^{-tA}x_0+e^{-(T-t)A}x_1,$$

where the vectors $x_0,x_1\in H_{3/2}$ are determined from the condition $\omega'(0)=\omega;(T)=0$. Since $\xi_1(t)\in W_2^2((0,T);H)$, then by the traces theorem $\xi_1'(0),\ \xi_1'(T)\in H_{1/2}$ [1]. Then in order to determine $x_0$ and $x_1$, we get the equations $-Ax_0+Ae^{-AT}x_1=-\xi_1'(0)$ and $-Ae^{-AT}x_0+Ax_1=-\xi_1'(T)$ . Hence we find

$$x_0=(E-e^{-2AT})(e^{-AT}A^{-1}\xi_1'(T)+A^{-1}\xi_1'(0)\in H_{3/2}.$$

Then $x_1=e^{-AT}x_0-A^{-1}\xi_1'(T)\in H_{3/2}$.. Consequently, $\omega(t)\in \overset{\circ}{W}{}_2^2((0,T);H)$. From the inequality $\|P_0\omega\|^2_{L_2((0,T;H)}\le 2\|\omega\|^2_{W_2^2((0,T);H)}$ it follows that the operator $P_0:\overset{\circ}{W}{}_2^2(R_+;H)\to L_2((0,T);H)$ is bounded. Then from the Banach theorem it follows that the operator $P_0^{-1}:L_2((0,T);H)\to \overset{\circ}{W}{}_2^2((0,T);H)$ is also bounded.

Thus, $\|\omega(t)\|_{W_2^2((0,T);H)}\le const\,\|f\|_{L_2((0,T);H)}$ . Consequently,

$$\|u(t)\|_{W_2^2((0,T);H)}\le\|\omega(t)\|_{W_2^2((0,T);H)}+\|u_0(t)\|_{W_2^2((0,T);H)}\le$$

$$\le const\left(\|f\|_{L_2((0,T);H)}+\|\varphi_0\|_{1/2}+\|\varphi_1\|_{1/2}\right).$$

The theorem is proved.

Now prove the important lemma.

**Lemma.** *For any $u(t)\in \overset{\circ}{W}{}_2^2((0,T);H)$ there hold the following inequalities:*

$$\left\|A^2u\right\|_{L_2((0,T);H)}\le c_0(\varepsilon)\left\|P_0u\right\|_{L_2((0,T);H)},\tag{10}$$

$$\left\|Au'\right\|_{L_2((0,T);H)}\le c_1(\varepsilon)\left\|P_0u\right\|_{L_2((0,T);H)},\tag{11}$$

*where*

$$c_0(\varepsilon)=\begin{cases}1, & 0\le\varepsilon\le\pi/4\\ \dfrac{1}{\sqrt{2}\cos\varepsilon}, & \pi/4\le\varepsilon<\pi/2\end{cases},\quad c_1(\varepsilon)=\frac{1}{2\cos\varepsilon},0\le\varepsilon<\frac{\pi}{2}\tag{12}$$

**Proof.** Let $u(t)\in \overset{\circ}{W}{}_2^2((0,T);H)$. Then

$$\|P_0u\|^2_{L_2((0,T);H)}=\left\|u''\right\|^2_{L_2((0,T);H)}+\left\|A^2u'\right\|^2_{L_2((0,T);H)}-$$

$$-2\operatorname{Re}(u'', A^2u)_{L_2((0,T);H)}, \tag{13}$$

Since

$$\left(u'', A^2u\right)_{L_2((0,T);H)} = \int_0^T (u'', A^2u)dt =$$

$$= (C^{1/2}u'(t), U^2C^{3/2}u(t))\Big|_0^T - \int_0^T (Cu'(t), U^2Cu'(t))dt =$$

$$= -\int_0^T (A^*u'(t), Au'(t))dt = -(A^*u'(t), Au'(t))_{L_2((0,T);H)},$$

then it follows from (13) that

$$\|P_0u\|^2_{L_{2((0,T);H)}} = \|u\|^2_{W_2^2((0,T);H)} + 2\operatorname{Re}(A^*u'(t), Au'(t))_{L_2((0,T);H)}.$$

Since for any $x \in D(A)$

$$\operatorname{Re}(A^*x, Ax) = \operatorname{Re}\sum_{k=1}^{\infty} \overline{\lambda}_k^2 \left|(x, e_k)\right|^2 =$$

$$= \sum_{k=1}^{\infty} \mu_k^2 \cos 2\varphi_k \left|(x, e_k)\right|^2 \geq \sum_{k=1}^{\infty} \mu_k^2 \cos 2\varepsilon \left|(x, e_k)\right|^2 \geq \cos 2\varepsilon (Ax, Ax),$$

then

$$\|P_0u\|^2_{L_2((0,T);H)} \geq \|u\|^2_{W_2^2((0,T);H)} + 2\cos 2\varepsilon (Au', Au')_{L_2((0,T);H)}. \tag{14}$$

On the other hand, $(u'(0) = u'(T) = 0)$,

$$\left\|Au'\right\|^2_{L_2((0,T);H)} = \left\|Cu'\right\|^2_{L_2((0,T);H)} = \int_0^T (Cu'(t), Cu'(t))dt =$$

$$= (C^{1/2}u'(t), C^{3/2}u(t))\,\Big|_0^T - \int_0^T (u''(t), C^2u(t))dt =$$

$$= -(u''(t), C^2u(t))_{L_2((0,T);H)} \leq \left\|u''\right\|_{L_2((0,T);H)} \left\|C^2u\right\|_{L_2((0,T);H)} \leq$$

$$\leq \frac{1}{2}\left(\left\|C^2u\right\|^2_{L_2((0,T);H)} + \left\|u''\right\|^2_{L_2((0,T);H)}\right) = \frac{1}{2}\|u\|^2_{W_2^2((0,T);H)}.$$

Then from (14) we get

$$\left\|Au'\right\|^2_{L_2((0,T);H)} \leq \frac{1}{2}\left(\|P_0u\|^2_{L_2((0,T);H)} - 2\cos 2\varepsilon \left\|Au'\right\|^2_{L_2((0,T);H)}\right)$$

or

$$(1 + \cos 2\varepsilon)\left\|Au'\right\|^2_{L_2((0,T);H)} \leq \frac{1}{2}\|P_0u\|^2_{L_2((0,T);H)}.$$

Hence we get that

$$\left\|Au'\right\|_{L_2((0,T);H)} \leq \frac{1}{2\cos\varepsilon} \left\|P_0 u\right\|_{L_2((0,T);H)},$$

i.e. the validity of inequality (11) is proved.

For $0 \leq \varepsilon \leq \pi/4$ , from inequality (14) it follows that

$$\left\|A^2 u\right\|_{L_2((0,T);H)} \leq \left\|P_0 u\right\|_{L_2((0,T);H)}. \tag{15}$$

And for $\pi/4 \leq \varepsilon < \pi/2$ the number $\cos 2\varepsilon \leq 0$. Therefore, taking into account inequality (11) in inequality (14), we get

$$\left\|P_0 u\right\|^2_{L_2((0,T);H)} \geq \left\|u\right\|^2_{L_2((0,T);H)} + 2\cos 2\varepsilon \frac{1}{4\cos^2\varepsilon} \left\|P_0 u\right\|^2_{L_2(0,T);H)} =$$

$$= \left\|u\right\|^2_{W_2^2((0,T);H)} + \frac{\cos 2\varepsilon}{2\cos^2\varepsilon} + \left\|P_0 u\right\|^2_{L_2((0,T);H)}.$$

Thus,

$$\left(1 - \frac{\cos 2\varepsilon}{2\cos^2\varepsilon}\right) \left\|P_0 u\right\|^2_{L_2((0,T);H)} \geq \left\|u\right\|^2_{W_2^2((0,T);H)}$$

or

$$\left\|u\right\|_{W_2^2((0,T);H)} \leq \frac{1}{\sqrt{2}\cos\varepsilon} \left\|P_0 u\right\|_{L_2((0,T);H)}.$$

Hence it follows that for $\pi/4 \leq \varepsilon < \pi/2$

$$\left\|A^2 u\right\|_{L_2((0,T);H)} \leq \frac{1}{\sqrt{2}\cos\varepsilon} \left\|P_0 u\right\|_{L_2((0,T);H)}. \tag{16}$$

The validity of inequality (10) follows from (15) and (16).

The lemma is proved.

**Theorem 3.** *Let conditions 1), 2) be fulfilled and it hold the inequality*

$$\alpha(\varepsilon) = c_1(\varepsilon) \left\|A_1 A^{-1}\right\| + c_0(\varepsilon) \left\|A_2 A^{-2}\right\| < 1,$$

*where the numbers* $c_0(\varepsilon)$ *and* $c_1(\varepsilon)$ *were determined in (12).*

Then problem (1), (2) is regularly solvable.

**Proof.** After substitution of $u(t) = \omega(t) - u_0(t)$, where $u_0(t)$ is the solution of problem (3), (4), we get the following boundary value problem:

$$P(d/dt)\omega(t) = -A_1 u_0'(t) - A_2 u_0(t) + f(t), \quad t \in (0,T), \tag{17}$$

$$\omega'(0) = 0, \ \omega'(T) = 0. \tag{18}$$

Since

$$\left\|g(t)\right\|_{L_2((0,T);H)} = \left\|-A_1 u_0'(t) - A_2 u_0(t) + f(t)\right\|_{L_2 9(0,T);H)} \leq$$

$$\leq \left\|A_1 A^{-1}\right\| \left\|A u_0'(t)\right\|_{L_2((0,T);H)} +$$

$$+ \left\|A_2 A^{-2}\right\| \left\|A^2 u_0(t)\right\|_{L_2((0,T);H)} + \left\|f(t)\right\|_{L_2((0,T);H)} \leq$$

$$\leq const\,\|u_0(t)\|_{W_2^2((0,T);H)} + \|f(t)\|_{L_2((0,T);H)} \leq$$

$$\leq const\left(\|\varphi_0\|_{1/2} + \|\varphi_1\|_{1/2} + \|f(t)\|_{L_2((0,T);H)}\right),$$

then the vector function $g(t) = -A_1u_0'(t) - A_2u_0(t) + f(t) \in L_2((0,T);H)$. Thus, we can write problem (17),(18) in the form of the equation $P\omega = P_0\omega + P_1\omega = g$, where $\omega \in \overset{\circ}{W}{}_2^2((0,T);H)$, $g \in L_2((0,T);H)$. Since the operator $P_0 : \overset{\circ}{W}{}_2^2((0,T);H) \to L_2((0,T);H)$ is an isomorphism, then after substitution of $\omega = P_0^{-1}\upsilon$ we get the equation $\upsilon + P_1P_0^{-1}\upsilon = g$, in the space $L_2((0,T);H)$. On the other hand,

$$\left\|P_1P_0^{-1}\upsilon\right\|_{L_2((0,T);H)} = \|P_1\omega\|_{L_2((0,T);H)} = \left\|A_1\omega' + A_2\omega\right\|_{L_2((0,T);H)} \leq$$

$$\leq \left\|A_1A^{-1}\right\|\left\|A\omega'\right\|_{L_2((0,T);H)} + \left\|A_2A^{-2}\right\|\left\|A^2\omega\right\|_{L_2((0,T);H)} \leq$$

$$\leq \left(c_1(\varepsilon)\left\|A_1A^{-1}\right\| + c_0(\varepsilon)\left\|A_2A^{-2}\right\|\right)\|P_0\omega\|_{L_2((0,T);H)} = \alpha(\varepsilon)\,\|\upsilon\|_{L_2((0,T);H)}\,.$$

Here we used inequalities (10) and (11) from the lemma. Thus from the condition $\alpha(\varepsilon) < 1$ it follows that the operator $(E + P_1P_0^{-1})$ exists in $L_2((0,T);H)$ and is bounded. Then $\omega = P_0^{-1}(E + P_1P_0^{-1})^{-1}g$ and

$$\|\omega\|_{W_2^2((0,T);H)} \leq const\,\|g\|_{L_2((0,T);H)} \leq const\left(\|\varphi_0\|_{1/2} + \|\varphi_1\|_{1/2} + \|f\|_{L_2((0,T);H)}\right).$$

Thus, the regular solution of problem (1), (2) is $u = \omega - u_0$. Therefore,

$$\|u\|_{W_2^2((0,T);H)} \leq \|\omega\|_{W_2^2((0,T);H)} + \|u_0\|_{W_2^2((0,T);H)} \leq$$

$$\leq const\left(\|\varphi_0\|_{1/2} + \|\varphi_1\|_{1/2} + \|f\|_{L_2((0,T);H)}\right).$$

The theorem is proved.

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