1998 TOM IX(XVII)

МЕХАНИКА

UDC 539.3

AGALAROV J.G

TWO DIMENSIONAL AUTOMODEL PROBLEMS OF MEMBRANE DYNAMICS

Automodel movement of the membranes accompanies an impute by cone owning constant speed.

The problem of a slant impact by the normal oriented cone has been considered by Ell-Sucka A.G. [1], who obtained the movement equations of the membrane and solved ones by the iteration method. Then another authors constructed approximate and accurate solutions of the different variants of this problem.

In [2] one made an attempt to generalize the theory for non-circle line of the membrane raiding on the cone This result has been generalized in [3] for both parts of the membrane: flat and liying. Here there is shown an opportunity to reduce to integral equations of the pointed out tasks.

1.Basis equations

The motion of membrane can be written in form

$$\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1+\nu}{2}\frac{\lambda}{r}\frac{\partial^2 \vartheta}{\partial r\partial\theta} + \frac{\lambda^2}{r^2}\frac{\partial^2 u}{\partial\theta^2}\frac{1-\nu}{2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{3-\nu}{2}\frac{\lambda}{r^2}\frac{\partial\vartheta}{\partial\theta} - \frac{u}{r^2} \qquad (1)$$

$$\frac{1}{a^2}\frac{\partial^2\vartheta}{\partial t^2} = \frac{1-\nu}{2}\frac{\lambda}{r}\frac{\partial^2\vartheta}{\partial r^2}\frac{1+\nu}{2} + \frac{\lambda}{r}\frac{\partial^2 u}{\partial r\partial\theta} - \frac{\lambda^2}{r}\frac{\partial^2\vartheta}{\partial\theta} + \frac{1-\nu}{2}\frac{1}{r}\frac{\partial\vartheta}{\partial r}\frac{3-\nu}{2}\frac{\lambda}{r^2} - \frac{1-\nu}{2}\frac{\vartheta}{r^2} \qquad (2)$$

where r,θ Lagrange $\frac{1-\nu}{2}$ coordinates polar system, u,ϑ -the velocity vectors of membrane elements on a cone surface

$a = \sqrt{\frac{E}{\rho\sqrt{1-\nu^2}}}$, E-Young's modules;

ν – Poisson ratio, $\lambda = csc\ \alpha$ on cone surface, $\lambda = 1$ in plane part. Membrane's striker's velocity 5vectors on cone and could written as following:

$$u = \frac{\partial\varphi_1}{\partial r} - \frac{\lambda}{r}\frac{\partial\psi_1}{\partial\theta}; \quad \vartheta = \frac{\partial\psi_1}{\partial r} - \frac{\lambda}{r}\frac{\partial\varphi_1}{\partial\theta} \qquad (3)$$

where they are defined by equations (1)-(2) , which could be change to the next form:

$$\frac{\partial^2\varphi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_1}{\partial r}+\frac{\lambda^2}{r^2}\frac{\partial^2\varphi_1}{\partial\theta^2}=\frac{1}{a^2}\frac{\partial^2\varphi_1}{\partial t^2}$$
$$\frac{\partial^2\psi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_1}{\partial r}+\frac{\lambda^2}{r^2}\frac{\partial^2\psi_1}{\partial\theta^2}=\frac{1}{a^2}\frac{\partial^2\psi_1}{\partial t^2} \tag{4}$$

where

$$b=a\sqrt{\frac{1-\nu}{2}}.$$

2. Automodel solution.

System's (3)-(4) solution for automodel motion size «One» is

$$\varphi_1=-(a^2t^2-r^2)^{3/2}\frac{\partial}{\partial t}\frac{\varphi_0}{\sqrt{a^2t^2-r^2}}$$
$$\psi_1=-(b^2t^2-r^2)^{3/2}\frac{\partial}{\partial t}\frac{\psi_0}{\sqrt{b^2t^2-r^2}} \tag{5}$$

where φ_0,ψ_0 satisfy equations:

$$\frac{\partial^2\varphi_0}{\partial R_1^2}+\frac{1}{R_1}\frac{\partial\varphi_0}{\partial R_1}+\frac{\lambda^2}{R_1^2}\frac{\partial^2\varphi_0}{\partial\theta^2}=0,$$
$$\frac{\partial^2\psi_0}{\partial R_2^2}+\frac{1}{R_2}\frac{\partial\psi_0}{\partial R_2}+\frac{\lambda^2}{R_2^2}\frac{\partial^2\psi_0}{\partial\theta^2}=0, \tag{6}$$

where $R_1=\dfrac{1-\sqrt{1-\rho^2}}{\rho}$; $R_2=\dfrac{\bar{b}-\sqrt{\bar{b}^2-\rho^2}}{\rho}$. (7)

Unknowns functions can be written with help of analytical functions arguments $z_1=R_1^{\lambda}e^{i\theta}$ $z_2=R_2^{\lambda}e^{i\theta}$

$$\varphi_1=t\,\mathrm{Re}\left(\varphi_0+\lambda\sqrt{1-\rho^2}\,z_1\varphi_0^1\right),$$
$$\psi_1=t\,\mathrm{Re}\left(\bar{b}\,\psi_0+\lambda\sqrt{\bar{b}-\rho^2}\,z_2\psi_0^1\right). \tag{8}$$

Equation's (3) can be written in next form:

$$t(u\_i\vartheta)=\frac{\partial\varphi_1}{\partial\rho}+\frac{i\lambda}{\rho}\frac{\partial\varphi_1}{\partial\theta}-\frac{\lambda}{\rho}\frac{\partial\psi_1}{\partial\theta}+i\frac{\partial\psi_1}{\partial\rho}. \tag{9}$$

With derivating of the equations (8) and using the means, the equations (9) could be solved:

$$2(u+i\vartheta)e^{i\theta/\lambda}=\lambda z_1^{1/\lambda}\left(\lambda z_1\varphi'-\varphi\right)+\frac{\lambda}{\bar{z}_1^{1/\lambda}}\left(\lambda\cdot\overline{z_1\varphi'}+\bar{\varphi}\right)+iz_2^{1/\lambda}\left(\lambda z_2\psi'+\psi\right)+$$
$$+\frac{i\lambda}{\bar{z}_1^{1/\lambda}}\left(\lambda\overline{z_2\psi'}+\bar{\psi}\right). \tag{10}$$

The equation (10) define membranes u and ϑ vectors through analytical functions $\varphi_1(z_1)$ and $\psi(z_2)$. It has been reached that we have to find φ and ψ analytical functions in two - connection regions Ω_1 and Ω_2 of the complex domains z_1 and z_2.

In flat part of the membrane on the wave front there are also conditions

$$R_1 = 1, \quad \operatorname{Im}\varphi = 0; \quad R_2 = 1, \quad \operatorname{Im}\psi = 0,$$
$$R_1 > 1, \quad \varphi = 0; \quad R_2 > 1, \quad \psi = 0. \tag{11}$$

3. Reduction to the integral equations.

Solution on the cone is looked for like the potential of the simple layer:

$$\varphi = \frac{1}{2\pi i}\int_L \mu_1 \ln\frac{1}{\xi_1 - z_1}; \quad \psi = \frac{1}{2\pi i}\int_L \mu_2 \ln\frac{1}{\xi_2 - z_2} d\xi_2;$$

Substituted in (10) and taken z_1 and z_2 on border L_1 and L_2 correspondingly it can be obtained:

$$\begin{aligned}
2(u+i\vartheta)e^{1/\lambda} &= \lambda z_1^{1/\lambda}\left[\lambda z_1\left(\frac{1}{2}\mu + \frac{1}{2\pi i}\int_{L_1}\frac{\mu d\xi_1}{\xi_1 - z_1}\right) - \frac{1}{2\pi i}\int_{L_1}\mu_1 \ln\frac{1}{\xi_1 - z_1}d\xi_1\right] - \\
&- \frac{\lambda}{\bar{z}^{1/\lambda}}\left[\lambda\bar{z}_1\left(\frac{1}{2}\mu_1 + \frac{1}{2\pi i}\int_{\bar{L}_1}\frac{\mu d\bar{\xi}_1}{\bar{\xi}_1 - \bar{z}_1}\right) + \frac{1}{2\pi i}\int_{\bar{L}_1}\mu_1 \ln\frac{1}{\bar{\xi}_1 - \bar{z}_1}d\bar{\xi}_1\right] + \\
&+ iz_2^{1/\lambda}\lambda\left(\lambda Z_2\frac{1}{2}\mu_2 + \frac{1}{2\pi i}\int_{L_2}\frac{\mu_2 d\xi_2}{\xi_2 - z_2}\right) - \frac{1}{2\pi i}\int_{L_2}\mu_2 \ln\frac{1}{\xi_2 - z_2}d\xi_2\Bigg) - \\
&- \frac{i\lambda}{\bar{z}_2^{1/\lambda}}\left[\lambda\bar{Z}_2\left(\frac{1}{2}\mu_2 + \frac{1}{2\pi i}\int_{\bar{L}_2}\frac{\mu_2 d\bar{\xi}_2}{\bar{\xi}_2 - \bar{z}_2}\right) + \frac{1}{2\pi i}\int_{\bar{L}_2}\mu_2 \ln\frac{1}{\bar{\xi}_2 - \bar{z}_2}d\bar{\xi}_2\right]
\end{aligned} \tag{12}$$

where $\bar{L}$ border symmetric L relatively to x axis.

Thus, there is the complex singular integral equations that separating real and imaginary parts can be brought to two integral equations for unknown μ_1 and μ_2.

To provide condition (11) solution in flat part of the membrane looks like for potential the simple layer plans regular supplement:

$$\begin{aligned}
\varphi &= \frac{1}{2\pi i}\int_{L_1}\mu_1 \ln\frac{1}{\xi_1 - z_1}d\xi_1 - \frac{1}{2\pi i}\int_{\bar{L}_1}\mu_1 \ln\frac{1}{\bar{\xi}_1 - \frac{1}{z_1}}d\bar{\xi}_1 \\
\psi &= \frac{1}{2\pi i}\int_{L_2}\mu_2 \ln\frac{1}{\xi_2 - z_2}d\xi_2 - \frac{1}{2\pi i}\int_{\bar{L}_2}\mu_2 \ln\frac{1}{\bar{\xi}_2 - \frac{1}{z_2}}d\bar{\xi}_2
\end{aligned} \tag{13}$$

substituted (13) to (10) it can be obtained two singular integral equations for unknown μ_1 and μ_2.

Literature

[1]. Эль -Сакка А.Г. *О косом ударе по гибкой мембране.* Вестник МГУ , 1996, №5.

[2]. Рахматулин Х.А., Агаларов Д.Г. *К неодномерным автомодельным задачам динамики мембран.* Изв . АН Аз ССР, с.ф.т.и.м., М.: Наук, 1977 , №5.

[3]. Агаларов Д.Г ., Мамедов Ш.А., Эфендиев А. Н. *Движение гибких связей при ударе конусом.* Изв АН Аз ССР с.ф.-т.и.м., М.: Наук, 1987 , №5.