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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

APPLICATION OF OPTIMIZATION METHODS TO THE SOLUTION OF SOME INVERSE BOUNDARY VALUE PROBLEMS

Specialty: 1211.01 –Differential equations

Field of science: Mathematics

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GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the theme.

The interest in solving inverse problems and ill-posed problems started from the middle of the past century. This is in the first turn is related to the formation of these problems in many fields of science and practice such as physics, geophysics, ecology and other fields. Furthermore, in relation to mathematics own internal demand, a large number of such problems arise and are studied in the field of differential equations, mathematical physics equations, computational mathematics, etc.. Inverse problems for partial equations occupy an important place among them.

It is known that every inverse or ill-posed problem can be expressed as an problem inverse to certain direct well-posed problem. In general, under direct problems one can understand the problem of modeling of processes, phenomena and others. Therefore, in mathematical physics, the functions describing physical phenomena in direct problems are found. For solving direct problems the domain where the process is studied, the appropriate coefficients and the right-hand side of the equation, boundary conditions and initial conditions are given.

But in many cases, the right hand side of the equation (external force, source, etc.), the coefficients of the equation (properties of the medium), initial conditions (initial state of the process), boundary conditions (boundary mode) are unknown. Then there arise such inverse problems that the unknown quantities are determined according to the information about the solution of direct problems. Taking into account what has been said above, along with solving the boundary value problem in the inverse problem there arises a need to find some unknown functions involved in the direct problem.

Usually, the inverse problems are ill-posed and various methods are used for solving them, for example, Tikhonov's regularization method, Lions' quasi-inversion method, Lavrentyev's quasi-solution method and others. Beginning with early XXI century the inverse problems of finding the boundary and initial functions, right-hand sides and the coefficient of the lowest term of partial equations are reduced to optimal control problems and the obtained problems are studied by means of theory of optimal control. This time the desired right hand sides, initial and boundary functions, the coefficients of mathematical physics equations play the role of a control and by means of additional conditions the structured incompatability function is taken as an objective function or cost criterion. If the minimum value of the objective function equals zero, then additional condition or conditions in the inverse problem are satisfied. Such an approach to inverse problems is called their variational or optimizational solution method. Note that this method is widely applied to various problems.

Variational statements of inverse problems for parabolic equations were researched in the works of K.R.Ayda-zadeh, V.M.Abdullayev, O.M.Alifanov, E.A.Artyukhin, S.V.Rumyantsev, A.D.Iskenderov, R.K.Tagiyev, R.A.Gasumov, I.K.Shakenov. Variational statements of inverse problems for elliptic equations were studied in the problem of finding the lowest term in the considered equations by A.D.Iskenderov, R.A.Hamidov, in the problem of finding the coefficient in the principal part of the equation, by R.K.Tagiyev, R.S.Gasimova in the problem of finding the coefficient of the lowest term in the Helmholts equation, by A.B.Rahimov, A.Litman, G. Ferrand. Reducing the inverse problem of finding the coefficient of the lowest term for hyperbolic equations to an operator equation in Hilbert space, constructing quadratic functional by means of this operator equation, its minimization was studied by S.K.Kabanikhin, variational statements of the problem of finding the right hand side of nonlocal boundary condition wave equation, by H.F.Guliyev, Y.S.Gasimov, H.T.Tagiyev, the problem of finding the leading coefficient of the equation by H.F.Guliyev, V.N.Nasibzadeh, finding the coefficient in the acoustic problem for a string vibration equation, by H.F.Guliyev, V.N. Nasibzadeh, on a finding the coefficient of one nonlinear wave equation in the mixed problem, by Z.R.Safarova.

In the dissertation work, variational statements on some inverse problems of finding the right hand sides, initial functions, boundary functions, the coefficients of equations in boundary value problems for second order hyperbolic equations of inverse problem of finding boundary function for a second order elliptic equation of some inverse problems were given and the obtained problems were studied as optimal control problems.

K.R.Ayda-zadeh, Y.R.Ashrafova, K.R.Ayda-zadeh, S.Z.Guliyev, J.-L.Arman, K.T.Ahmadov, S.S.Akhiyev, A.G.Butkovsky, F.P.Vasiliev, K.G.Hasanov, A.I.Egorov, Y.V.Yegorov, A.D.Iskenderov, R.K.Tagiyev, S.I.Kabanikhin, K.T.Iskakov, S.I.Kabanikhin, A.L.Karchevski, V.Komkov, H.F.Guliyev, H.F.Guliyev, Y.S.Gasymov, H.T.Tagiyev, T.M.Huseynova, H.F.Guliyev, G.Q.Ismayilova, J.L.Lions, K.A.Lourie, K.B.Mansimov, M.J.Mardanov, T.K.Melikov, V.I.Plotnikov, M.A.Sadigov, J.J.Mammadova, S.Y.Serovayski, T.K.Sirazetdinov, R.K. Tagiyev, R.S. Gasymova, Y.A.Sherifov, F.T.Ibiyev, Sh.Sh.Yusubov, M.A.Yagubov, J.Sokolovski, M.B.Suryanarayana, T.Zolezzi have been engaged invarious problems of optimal control for partial differential equations.

In the introduced dissertation work, the inverse problems of finding boundary and initial conditions for second order hyperbolic and elliptic equations, on definition of the right hand side and the lowest term of the second order hyperbolic equations were reduced to optimal control problems and the obtained problems were studied by means of optimal control methods.

Object and subjects of research. The object of the study of the dissertation work are the problems on finding boundary and initial functions for hyperbolic and elliptic equations, inverse problems, optimal control problems, inverse problems on defining the right hand side and the coefficients the lowest term of second order hyperbolic equations and optimal control problems.

The subject of the dissertation work is to find boundary and initial functions of hyperbolic and elliptic equations, the approaches based on reduction of inverse problems on definition of the right hand side and the coefficient of the lowest term of second order hyperbolic equations, to optimal control problems and the methods for solving optimal control problems.

The goals and objectives of the study is to find boundary and initial functions for second order hyperbolic and elliptic equations and to reduce the inverse problems on definition of the right hand side and the lowest term of second order hyperbolic equations to optimal control problems, to apply the methods of optimal control theory to the obtained problems, to derive optimality conditions.

Research methods. In the dissertation work, the methods of mathematical theory of optimal control and optimization, the methods of mathematical physics and functional analysis are applied.

The main theses to be defended.

- reducing the problems of finding boundary and initial functions for second order hyperbolic and elliptic equations to optimal control problems;
- reducing the problems on defining the right hand side and the coefficient of the lowest term of second order hyperbolic equations to optimal control problems;
- studying the obtained optimal control problems by the methods of optimal control theory;
- calculation of differentials of functionals;
- deriving optimality condition.

Scientific novelty of the study. The following new results were obtained:

- the problems of finding the boundary and initial functions for second order hyperbolic and elliptic equations were reduced to optimal control problems;
- the problems of defining the right hand side and the coefficient of the lowest term of second order hyperbolic equations were reduced to optimal control problems;
- the obtained optimal control problems were studied;
- it was proved that the functionals are differentiable, and the expressions for their gradients were obtained;
- variational inequality type optimality conditions were derived.

Theoretical and practical importance of the study. The results obtained in the work are mainly of theoretical character. The methods used in the work can be applied to other partial equations as well. The practical importance of the work is that the obtained results can be used for the approximate solution of various inverse and ill-posed problems in vibration, wave and stationary processes.

Approbation and application. The results of the dissertation work were reported in the following scientific seminars and conferences: in the seminars of the department of "Mathematical analysis and function theory" (head: prof. Gurbanov N.T.) and "Differential equations and optimization" (head: prof. Feyziyev F.G.) of Sumgayit State University, in the seminars of the department of "Mathematical methods of control theory" (head prof. Guliyev H.F.) of Baku State University, in the V International conference "Control and Optimization with Industrial Applications" (Baku 2015), in the III Republican Scientific conference "Applied of Mathematics and Information Technologies" (Sumgait 2016), in the International Scientific conference "Applied and Theoretical Problems of Mathematics" dedicated to the 55th anniversary of Sumgayit State University (Sumgayit 2017), in the International conference "Modern Problems of Mathematics and Mechanics" devoted to the 60th anniversary of the Institute of Mathematics and Mechanics of ANAS (Baku 2019), in the Republican scientific conference "Fundamental problems of mathematics and the application of intellectual technologies in education" (Sumgayit 2020), in the Republican scientific conference "Modern problems of university science and education" dedicated to the 60th anniversary of Sumgayit State University (Sumgayit 2022), in the II Republican scientific conference "Fundamental problems of mathematics and application of intellectual technologiesin in education" in education" (Baku 2022), in the III International Istanbul Current Scientific Research Congress" (Turkey 2023).

Author’s personal contribution. All results and suggestions obtained belong to the author.

The name of the organization where the dissertation work was executed. The dissertation work was executed in the department of "Mathematical physics equations" of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

Publications. On the topic of the dissertation the applicant's 11 papers and 8 abstracts were published.

The total volume of the dissertation work indicating separately the volume of each structural units by signs.

The dissertation work consists of introduction, two chapters, result and references with 108 names. The total volume of the work consists of 226824 signs (title page - 352 signs, contents - 5852 signs, introduction - 47667 signs, the first chapter - 98000 signs, the second chapter - 74000 signs, conclusion - 953 signs).

GENERAL CHARACTERISTICS OF THE WORK

The dissertation work consists of introduction, two chapters and a list of references.

Chapter I consists of 6 sections and is devoted to the study of the problems of finding the boundary and initial conditions for second order hyperbolic and elliptic equations by the optimization methods.

In **1.1** we consider an inverse boundary value problem for a string vibration equation and its study by an optimal control method.

1.2 studies the problem on defining the initial function in a boundary value problem for a second order linear hyperbolic equation.

In the cylinder $Q = \Omega \times (0,T)$ we consider the following boundary value problem

$$\frac{\partial^2 u}{\partial t^2} + Lu = f(x,t), \ (x,t) \in Q, \tag{0.1}$$

$$u(x,0) = v(x), \ \frac{\partial u(x,0)}{\partial t} = u_1(x), \ x \in \Omega, \tag{0.2}$$

$$u\big|_s = 0, \tag{0.3}$$

here $\Omega \subset R^n$ is a bounded domain with smooth Γ boundary, $S = \Gamma \times (0,T)$ is lateral surface of the cylinder Q, $f \in L_2(Q)$,

$u_1 \in L_2(\Omega)$ are the known functions, $v(x) \in \overset{0}{W}{}_2^1(\Omega)$ is an unknown functions,

$$Lu \equiv -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j} \right) + a_0(x,t)u,$$

so that the conditions

$$a_{ij} \in C(\overline{Q}),\ \frac{\partial a_{ij}}{\partial t} \in C(\overline{Q}),\ a_{ij}(x,t) = a_{ji}(x,t),$$
$$(x,t) \in Q,\ i,j = 1,\dots,n,\ a_0 \in C(\overline{Q})$$

and

$$\sum_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j \geq \mu\sum_{i=1}^{n}\xi_i^2,\quad \mu = const > 0$$

are satisfied.

If the function $v(x) \in \overset{0}{W}{}_2^1(\Omega)$ is given, the problem (0.1)-(0.3) is a direct problem in the cylinder Q. If $v(x) \in \overset{0}{W}{}_2^1(\Omega)$ is an unknown function, then for defining the function $v(x)$ the additional information

$$u(x,T) = \chi(x), \qquad x \in \Omega, \tag{0.4}$$

is used, here $\chi(x) \in \overset{0}{W}{}_2^1(\Omega)$ is a known function. Then the problem (0.1)-(0.4) is an inverse problem corresponding to the problem (0.1)-(0.3).

The problem (0.1)-(0.4) is reduced to an optimal control problem: to minimize the inconsistency functional

$$J_0(v) = \frac{1}{2}\int_\Omega [u(x,T;v) - \chi(x)]^2 dx \tag{0.5}$$

by the solution of the problem (0.1)-(0.3), here the function $v(x)$ is called a control, the function $u(x,t;v)$ is the solution of the problem (0.1)-(0.3) from $W_{2,0}^1(Q)$ corresponding to the function $v(x) \in \overset{0}{W}{}_2^1(\Omega)$.

It should be noted that the problem (0.1)-(0.3), (0.5), in general, is an ill-posed problem.

If in problem (0.1)-(0.3), (0.5) $\min\limits_{v \in \overset{0}{W}{}_2^1(\Omega)} J_0(v) = 0$, then the additional condition (0.4) is satisfied.

At first it is shown that $\inf\limits_{v \in \overset{0}{W}{}_2^1(\Omega)} J_0(v) = 0$. Then instead of the problem (0.1)-(0.3), (0.5) we consider the following problem: to minimize the function

$$J_\alpha(v) = J_0(v) + \frac{\alpha}{2}\|v\|^2_{\overset{0}{W}{}_2^1(\Omega)} \tag{0.6}$$

within the condition (0.1)-(0.3) in the convex closed set $V_m \subset \overset{0}{W}{}_2^1(\Omega)$, here $\alpha > 0$ is a given number, V_m is a class of admissible controls.

Theorem 0.1. Assume that for the problem (0.1)-(0.3), (0.6) the conditions $f \in L_2(Q)$, $u_1 \in L_2(\Omega)$, $v(x) \in \overset{0}{W}{}_2^1(\Omega)$, $a_{ij} \in C(\overline{Q})$, $\frac{\partial a_{ij}}{\partial t} \in C(\overline{Q})$, $a_{ij}(x,t) = a_{ji}(x,t)$, $(x,t) \in Q$, $i, j = 1,\ldots,n$, $a_0 \in C(\overline{Q})$, $\sum\limits_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j \geq \mu\sum\limits_{i=1}^{n}\xi_i^2$, $\mu = const > 0$ are satisfied. Then the functional (0.6) is Frechet continuously differentiable in V_m and at the point $v \in V_m$ its differential with the increment $\delta v(x) \in \overset{0}{W}{}_2^1(\Omega)$ is determined by the expression

$$< J'_\alpha(v), \delta v >= \int\limits_\Omega \left\{\left[-\frac{\partial \psi(x,0;v)}{\partial t} + \alpha v(x)\right]\delta v(x) + \alpha\sum_{i=1}^{n}\frac{\partial v}{\partial x_i}\cdot\frac{\partial \delta v}{\partial x_i}\right\}dx .$$

Theorem 0.2. Assume that the conditions of theorem 0.1 are satisfied for the given problem (0.1)-(0.3), (0.6). Then in this problem a necessary and sufficient condition for the optimality the control $v_* = v_*(x) \in V_m$ is the satisfacton of the inequality

$$\int_\Omega \left[-\frac{\partial \psi(x,0;v_*)}{\partial t} + \alpha v_*(x) \right] (v(x) - v_*(x)) dx +$$

$$+ \alpha \int_\Omega \sum_{i=1}^{n} \frac{\partial v_*}{\partial x_i} \left(\frac{\partial v(x)}{\partial x_i} - \frac{\partial v_*(x)}{\partial x_i} \right) dx \geq 0 \qquad \forall v \in V_m$$

here for $v = v_*(x)$ the function $\psi(x,t;v_*)$ is a generalized solution of the adjoint problem

$$\frac{\partial^2 \psi}{\partial t^2} + L\psi = 0, \quad (x,t) \in Q,$$

$$\psi(x,T;v) = 0, \quad \frac{\partial \psi(x,T;v)}{\partial t} = -[u(x,T;v) - \chi(x)], \; x \in \Omega,$$

$$\delta\psi\big|_s = 0$$

from $W_{2,0}^1(Q)$.

In **1.3** we consider the definition of boundary conditions in a two-dimensional wave equation and its study by the optimal control method. The functions

$$(u(x_1,x_2,t), v_1(x_2,t), v_2(x_1,t)) \in W_2^1(Q) \times$$
$$\times L_2((0,l_2) \times (0,T)) \times L_2((0,l_1) \times (0,T))$$

are determined from the following relations:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2}, \quad (x_1,x_2,t) \in Q, \tag{0.7}$$

$$u(x_1,x_2,0) = u_0(x_1,x_2), \; (x_1,x_2) \in \Omega, \tag{0.8}$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = u_1(x_1,x_2), \quad (x_1,x_2) \in \Omega \tag{0.9}$$

$$\left.\frac{\partial u}{\partial x_1}\right|_{x_1=0} = 0, \quad \left.\frac{\partial u}{\partial x_1}\right|_{x_1=l_1} = v_1(x_2,t), \; (x_2,t) \in (0,l_2) \times (0,T), \tag{0.10}$$

$$\left.\frac{\partial u}{\partial x_2}\right|_{x_2=0} = 0, \quad \left.\frac{\partial u}{\partial x_2}\right|_{x_2=l_2} = v_2(x_1,t), \; (x_1,t) \in (0,l_1) \times (0,T), \tag{0.11}$$

$$u(0,x_2,t)=a_1(x_2,t),\quad (x_2,t)\in(0,l_2)\times(0,T),$$
$$u(x_1,0,t)=a_2(x_1,t),\quad (x_1,t)\in(0,l_1)\times(0,T). \tag{0.12}$$

Here $Q=\Omega\times(0,T)$ is a parallellepiped, $\Omega=\{0<x_1<l_1,\ 0<x_2<l_2\}$ is a rectangle, $l_1>0,\ l_2>0,\ T>0$ are the given numbers, $u_0\in W_2^1(\Omega)$, $u_1\in L_2(\Omega)$, $a_1(x_2,t)\in W_2^1((0,l_2)\times(0,T))$, $a_2(x_1,t)\in W_2^1((0,l_1)\times(0,T))$ are the given functions.

This problem is reduced to the following optimal control problem: it is required to find the minimum of the functional

$$J(v)=\frac{1}{2}\int_0^T\int_0^{l_2}[u(0,x_2,t;v)-a_1(x_2,t)]^2\,dx_2dt+$$
$$+\frac{1}{2}\int_0^T\int_0^{l_1}[u(x_1,0,t;v)-a_2(x_1,t)]^2\,dx_1dt, \tag{0.13}$$

within the condition (0.7)-(0.11), here the function $u(x_1,x_2,t;v)$ is the solution of the problem (0.7)-(0.12) corresponding to the vector-function $v=(v_1(x_2,t),v_2(x_1,t))$.

It should be noted that the problem (0.7)-(0.11), (0.13), in general, is an ill-posed problem.

Assume that problem (0.7)-(0.11) corresponding to each control $v=(v_1(x_2,t),v_2(x_1,t))\in L_2((0,l_2)\times(0,T))\times L_2((0,l_1)\times(0,T))$ has a unique generalized solution from $W_2^1(Q)$.

If one can find a control giving zero value to the functional (0.13), then the addiotional information (0.12) is satisfied.

Theorem 0.3. Assume that for the given problem (0.7)-(0.11), (0.13) the conditions $u_0\in W_2^1(\Omega)$, $u_1\in L_2(\Omega)$, $a_1(x_2,t)\in W_2^1((0,l_2)\times(0,T))$, $a_2(x_1,t)\in W_2^1((0,l_1)\times(0,T))$ are satisfied. Then in the optimal control problem (0.7)-(0.11), (0.13)

$$\inf J(v)=0,\ v=(v_1,v_2)\in L_2((0,l_2)\times(0,T))\times L_2((0,l_1)\times(0,T)). \tag{0.14}$$

In what follows, for the optimality condition to be obtained in future not to degenerated, along with the solution of the problem (0.7)-(0.11) the controllers, themselves and first order derivatives with respect to t in the closed convex set V_m from the space $L_2((0,l_2)\times(0,T))\times$ $L_2((0,l_1)\times(0,T))$ we consider a problem on minimization of the functional

$$J_\beta(v)=J(v)+\frac{\beta}{2}\left[\int_0^T\int_0^{l_2}v_1^2dx_2dt+\int_0^T\int_0^{l_1}v_2^2dx_1dt\right], \qquad (0.15)$$

here $\beta>0$ is a given number. This problem will be called the problem (0.7)-(0.11), (0.15). This problem has a unigue optimal control.

We introduce a problem adjoint to the problem (0.7)-(0.11), (0.15):

$$\frac{\partial^2\psi}{\partial t^2}=\frac{\partial^2\psi}{\partial x_1^2}+\frac{\partial^2\psi}{\partial x_2^2}, \quad (x_1,x_2,t)\in Q, \qquad (0.16)$$

$$\psi\big|_{t=T}=0, \frac{\partial\psi}{\partial t}\bigg|_{t=T}=0, \quad (x_1,x_2)\in\Omega, \qquad (0.17)$$

$$\frac{\partial\psi}{\partial x_1}\bigg|_{x_1=0}=-[u(0,x_2,t;v)-a_1(x_2,t)],$$

$$\frac{\partial\psi}{\partial x_1}\bigg|_{x_1=l_1}=0, \ (x_2,t)\in(0,l_2)\times(0,T), \qquad (0.18)$$

$$\frac{\partial\psi}{\partial x_2}\bigg|_{x_2=0}=-[u(x_1,0,t;v)-a_2(x_1,t)],$$

$$\frac{\partial\psi}{\partial x_2}\bigg|_{x_2=l_2}=0, \ (x_1,t)\in(0,l_1)\times(0,T). \qquad (0.19)$$

Note that the function $\psi(x_1,x_2,t;v)$ being the solution of the adjoint problem (0.16)-(0.19) is contained in $W_2^1(Q)$.

Theorem 0.4. Assume that for the given problem (0.7)-(0.11), (0.15) the conditions $u_0\in W_2^1(\Omega)$, $u_1\in L_2(\Omega)$, $a_1(x_2,t)\in W_2^1((0,l_2)\times(0,T))$,

$a_2(x_1,t)\in W_2^1((0,l_1)\times(0,T))$ are satisfied. Then in V_m the functional (0.15) is Frechet continuously differentiable and at the point $v\in V_m$ its differential is determined by the formula

$$\left\langle J'_\beta(v),\delta v\right\rangle_{L_2((0,l_2)\times(0,T))\times L_2((0,l_1)\times(0,T))}=$$

$$=\int_0^T\int_0^{l_2}[\psi(l_1,x_2,t;v)+\beta v_1(x_2,t)]\delta v_1(x_2,t)dx_2dt+$$

$$+\int_0^T\int_0^{l_1}[\psi(x_1,l_2,t;v)+\beta v_2(x_1,t)]\delta v_2(x_1,t)dx_1dt\,.$$

Theorem 0.5. Assume that the conditions of theorem 0.4 are satisfied. Then the necessary and sufficient condition for the optimality of the control $v_*=(v_{1*},v_{2*})\in V_m$ in the problem (0.7)-(0.11), (0.15) is the satisfaction of the inequality

$$\int_0^T\int_0^{l_2}[\psi_{1*}(l_1,x_2,t)+\beta v_{1*}(x_2,t)](v_1(x_2,t)-v_{1*}(x_2,t))dx_2dt+$$

$$+\int_0^T\int_0^{l_1}[\psi_{2*}(x_1,l_2,t)+\beta v_{2*}(x_1,t)](v_2(x_1,t)-v_{2*}(x_1,t))dx_1dt\geq 0,$$

for arbitrary $v=(v_1(x_2,t),v_2(x_1,t))\in V_m$, here $\psi_*(x_1,x_2,t)=\psi(x_1,x_2,t;v_*)$ is the solution of the problem (0.16)-(0.19) corresponding to $v=(v_{1*}(x_2,t),v_{2*}(x_1,t))$.

In **1.4** the inverse problem an finding the initial function in the Cauchy problem for a wave equation is considered.

This problem is reduced to an optimal control problem and is studied by the methods of optimal control theory.

In **1.5** we consider reducing an inverse problem of thermoacoustics to an optimal control problem and study it.

In **1.6** we consider reducing the continuation problem for elliptic equation to an optimal control problem and study it.

In the cylinder $\Omega=\left\{(x,y)\in R^{n+1}:x\in(0,l),\ y\in D\subset R^n\right\}$ we consider the following initial boundary value problem

$$\frac{\partial^2 u}{\partial x^2} + \sum_{i,j=1}^{n} \frac{\partial}{\partial y_i}\left(a_{ij}(y)\frac{\partial u}{\partial y_j} \right) - a(y)u = f(x,y), (x,y) \in \Omega, \tag{0.20}$$

$$u(0,y) = \varphi(y), \quad \frac{\partial u(0,y)}{\partial x} = 0 \quad y \in D, \tag{0.21}$$

$$\left.\frac{\partial u}{\partial \nu_A}\right|_{\partial D} = 0, \quad x \in (0,l) \tag{0.22}$$

Here $D \subset R^n$ is a rather smooth domain with the boundary ∂D, $l > 0$ is a given number, $f \in L_2(\Omega)$, $\varphi \in L_2(D)$ are the given functions, $a_{ij}(y)$, $i,j = \overline{1,n}$ are the given functions whose coefficients and the coefficient $a(y)$ satisfy the following properties:
For $a_{ij} \in C^1(\overline{D})$, $a \in C(\overline{D})$, $a_{ij}(y) = a_{ji}(y)$, $i,j = \overline{1,n}$, $a(y) \geq 0$, $y \in \overline{D}$ and arbitrary $\xi \in R^n$ and all $y \in \overline{D}$, $\alpha = const > 0$ $\sum_{i,j=1}^{n} a_{ij}(y)\xi_i\xi_j \geq \alpha \sum_{k=1}^{n} \xi_k^2$.

Being a continuation problem, (0.20)-(0.22) is an ill-posed problem, i.e. the soluion of the problem is continuously independent of the data.

We introduce the direct problem

$$\frac{\partial^2 u}{\partial x^2} + \sum_{i,j=1}^{n} \frac{\partial}{\partial y_i}\left(a_{ij}(y)\frac{\partial u}{\partial y_j} \right) - a(y)u = f(x,y), \ (x,y) \in \Omega, \tag{0.23}$$

$$\frac{\partial u(0,y)}{\partial x} = 0, \quad \frac{\partial u(l,y)}{\partial x} = v(y), \ y \in D, \tag{0.24}$$

$$\left.\frac{\partial u}{\partial \nu_A}\right|_{\partial D} = 0, \quad x \in (0,l) \tag{0.25}$$

inverse to this problem.

The inverse problem consists of finding the function $v(y)$ determined from the additional information

$$u(0,y) = \varphi(y), \quad y \in D \tag{0.26}$$

and the relations (0.23)-(0.25).

The inverse problem on finding the function $v(y)$ is reduced to the following optimal control problem: to find such a function $v(y)$ from the class $V_m = \{v(y) : v \in L_2(D), \ a \le v(y) \le b \ \ a.e. \ in . D\}$ that together with the solution of the problem (0.23)-(0.25) gives a minimum to the inconsistency functional

$$J_0(v) = \frac{1}{2}\int_D [u(0, y; v) - \varphi(y)]^2 dy. \tag{0.27}$$

Here the function $u(x, y; v)$ is the solution of the problem (0.23)-(0.25) for $v = v(y)$, a, b are the given numbers $a < b$.

It is known that for each function $v(y)$ within the given conditions the problem (0.23)-(0.25) has a unigue generalized solution in the space $W_2^1(\Omega)$.

The function $v(y)$ will be said a control, V_m a class of admissible controls. We call this problem the problem (0.23)-(0.25), (0.27).

For the necessary and sufficient condition to be obtained to be not degenerated in future, we regularize the functional (0.27):

$$J_\beta(v) = J_0(v) + \frac{\beta}{2}\int_D |v(y)|^2 dy, \tag{0.28}$$

here $\beta > 0$ is a given number. We consider a problem on finding the minimum of the functional (0.28) within the conditions (0.23)-(0.25) in the class V_m. Since the problem (0.23)-(0.25) is linear, the functional (0.28) is quadratic and strongly convex in $L_2(D)$ the problem (0.23)-(0.24), (0.28) has a unigue optimal control in the class V_m.

We introduce the problem

$$\frac{\partial^2 \psi}{\partial x^2} + \sum_{i,j=1}^{n} \frac{\partial}{\partial y_i}\left(a_{ij}(y) \frac{\partial \psi}{\partial y_j} \right) - a(y)\psi = 0, \ \ (x, y) \in \Omega, \tag{0.29}$$

$$\frac{\partial \psi(0, y)}{\partial x} = -[u(0, y; v) - \varphi(y)], \ \ \frac{\partial \psi(l, y)}{\partial x} = 0, \ \ y \in D, \tag{0.30}$$

$$\left.\frac{\partial\psi}{\partial\nu_A}\right|_{\partial D}=0,\quad x\in(0,l), \tag{0.31}$$

adjoint to the problem (0.23)-(0.25), (0.28) and prove the following theorems.

Theorem 0.6. Assume that in the problem (0.23)-(0.25), (0.28) the above conditions imposed on the data are satisfied. Then the functional (0.28) is Frechet continuously differentiable in V_m and at the point $v\in V_m$ its differential with the increment $\delta v\in L_2(D)$ is determined by the expression

$$\left\langle J'_\beta(v),\delta v\right\rangle=\int_D[\psi(l,y)+\beta v(y)]\delta v(y)dy\,.$$

Theorem 0.7. Assume that the conditions of theorem 0.6 are satisfied. Then in the problem (0.23)-(0.25), (0.28) the necessary and sufficient condition for the optimality of the control $v_*\in V_m$ is the satisfaction of the inequality

$$\int_D[\psi_*(l,y)+\beta v_*(y)](v(y)-v_*(y))dy\geq 0$$

for arbitrary $v=v(y)\in V_m$ here the function $\psi_*(x,y)$ is the solution of the adjoint problem (0.29)-(0.31) for $v=v_*(y)$.

In the **chapter II** consisting of four sections, we consider the study of the problem of finding the right hand side and the coefficinet of the lowest term of second order hyperbolic equations by optimization methods.

In **2.1** we study the defining of the right hand side of string vibration equation with nonlocal boundary condition.

This problem is studied by reducing it to an optimal control problem.

In **2.2** in a string vibration equation we consider a problem of finding the pair $(u(x,t),v(t))\in W_2^1(Q)\times(L_\infty(0,T))^n$ from the relations

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}+\sum_{i=1}^n v_i(t)h_i(x)u=f(x,t),(x,t)\in Q=(0,\ell)\times(0,T), \tag{0.32}$$

$$u(x,0)=u_0(x),\ \frac{\partial u(x,0)}{\partial t}=u_1(x),\ 0\leq x\leq\ell, \tag{0.33}$$

$$u(0,t)=u(\ell,t)=0,\ 0\le t\le T, \tag{0.34}$$
$$u(x_i,t)=g_i(t),\ 0\le t\le T,\ i=1,...,n \tag{0.35}$$

here $\ell>0, T>0$ are the given numbers, $f\in L_2(Q), u_0\in \overset{0}{W}{}_2^1(0,\ell)$, $u_1\in L_2(0,\ell)$, $g_i\in L_2(0,T)$, $h_i\in L_\infty(0,\ell)$, $i=1,\dots,n$ are the given functions, $x_i\in(0,\ell)$, $i=1,\dots,n$ are the given various points, $v(t)=(v_1(t),...,v_n(t))$ is a vector function.

Given the vector-function $v(t)$ the problem (0.32)-(0.34) becomes a direct problem in the domain Q. The problem (0.32)-(0.35) is called an inverse problem corresponding to the problem (0.32)-(0.34).

The problem (0.32)-(0.34) is an ill-posed problem, i.e. the solution of the problem is continuously independent of the data.

Let us reduce the problem (0.32)-(0.35) to the following optimal control problem:

Find in $V_m=\{v(t)\in(L_2(0,T))^n, v(t)=(v_1(t),\dots,v_n(t)):\alpha_i\le v_i(t)\le\beta_i$ $i=1,\dots,n,(0,T)$ almost everywhere} such a vector-function $v(t)$ that gives minimum to the inconsistency functional

$$J_0(v)=\frac{1}{2}\int_0^T\sum_{i=1}^n[u(x_i,t;v)-g_i(t)]^2\,dt \tag{0.36}$$

within the restrictions (0.32)-(0.34), here the function $u(x,t;v)$ is the solution to problem (0.32)-(0.34) for $v=v(t)$, $\alpha_i,\beta_i,\alpha_i<\beta_i$, $i=1,\dots,n$ are the given numbers. We call the vector-function $v=v(t)$ a control, the class V_m a class of admissible controls. Note that if $\min\limits_{v\in V_m} J_0(v)=0$ the additional condition (0.35) is satisfied.

Let us regularize the problem (0.32)-(0.34), (0.36): Find such a control $v(t)\in V_m$ that gives minimum to the functional

$$J_\beta(v)=\frac{1}{2}\int_0^T\sum_{i=1}^n[u(x_i,t;v)-g_i(t)]^2\,dt+\frac{\beta}{2}\int_0^T\sum_{i=1}^n|v_i(t)-\omega_i(t)|^2\,dt \tag{0.37}$$

here $\beta > 0$ is a given number, $\omega(t) = (\omega_1(t), \ldots, \omega_n(t)) \in (L_2(0,T))^n$ is the given vector-function. This problem will be called the problem (0.32)-(0.34), (0.37).

Theorem 0.8. Assume that the data of the problem (0.32)-(0.34), (0.37) satisfy the conditions $f \in L_2(Q)$, $u_0 \in \overset{0}{W}{}_2^1(0,l)$, $u_1 \in L_2(0,l)$, $g_i \in L_2(0,T)$, $h_i \in L_\infty(0,l)$, $i = 1,\ldots,n$, $x_i \in (0,l)$. Then the space $(L_2(0,T))^n$ has such a dense subset G that for arbitrary $\omega \in G$ at $\beta > 0$ the optimal control problem (0.32)-(0.34), (0.37) has a unigue solution.

Assume that the function $\psi = \psi(x,t;v)$ is the solution of the adjoint boundary value problem

$$\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} + \sum_{i=1}^n v_i(t) \cdot h_i(x)\psi = -$$

$$-\sum_{i=1}^n [u(x_i,t;v) - g_i(t)]\delta(x - x_i), \ (x,t) \in Q, \qquad (0.38)$$

$$\psi\big|_{t=T} = 0, \quad \frac{\partial \psi}{\partial t}\bigg|_{t=T} = 0, \quad 0 \le x \le l,$$

$$\psi(0,t) = \psi(l,t) = 0, \ 0 \le t \le T. \qquad (0.39)$$

Theorem 0.9. Let all the conditions of theorem 0.8 be satisfied. Then the functional (0.37) is Ferchet continuously differentiable in V_m and at its point $v \in V_m$ the differential with the increment δv is determined by the expression

$$\left\langle J'_\beta(v), \delta v \right\rangle = \int_0^T \sum_{i=1}^n \left(\int_0^l u(x,t)\psi(x,t)h_i(x)dx \right) \delta v_i(t)dt +$$

$$+\beta \int_0^T \sum_{i=1}^n (v_i(t) - w_i(t))\delta v_i(t)dt.$$

Theorem 0.10. Let the conditions of theorem 0.9 be satisfied. Then the necessary condition for the optimality of the control

$v_*(t)=\left(v_1^*(t),\dots,v_n^*(t)\right)\in V_m$ in the problem (0.32)-(0.34), (0.37) is the satisfaction of the inequality

$$\int_0^T \sum_{i=1}^n \left[\int_0^l u_*(x,t)\psi_*(x,t)h_i(x)dx + \right.$$
$$\left. + \beta\left(v_i^*(t)-\omega_i(t)\right)\right]\cdot\left(v_i(t)-v_i^*(t)\right)dt \ge 0 \qquad \forall v\in V_m$$

here the functions $u_*(x,t)=u(x,t;v_*)$, $\psi_*(x,t)=\psi(x,t;v_*)$ are the solutions of the problems (0.32)-(0.34) and (0.38)-(0.39) for $v(t)=v_*(t)$.

In **2.3** we study a problem on defining the coefficient of the lowest term of a second order many-dimensional hyperbolic equation.

In the cylinder $Q=\Omega\times(0,T)$ we consider a problem on defining the pair of functions $\left(u(x,t),v(x)\right)\in W_2^1(Q)\times L_\infty(\Omega)$ from the system

$$\frac{\partial^2 u}{\partial t^2}-\sum_{i,j=1}^n \frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)+v(x)u=f(x,t), \quad (x,t)\in Q, \qquad (0.\ 40)$$

$$u(x,0)=u_0(x), \qquad \frac{\partial u(x,0)}{\partial t}=u_1(x), \quad x\in\Omega, \qquad (0.41)$$

$$u|_S=0, \qquad (0.42)$$

$$\int_0^T K(x,t)u(x,t)dt=\chi(x), \quad x\in\Omega \qquad (0.43)$$

Here Ω is a bounded domain with a smooth boundary Γ in R^n, $S=\Gamma\times(0,T)$ is the lateral surface of the cylinder Q. $f\in L_2(Q)$, $u_0\in \overset{0}{W}{}_2^1(\Omega)$, $u_1\in L_2(\Omega)$, $K(x,t)\in L_\infty(Q)$, $\chi(x)\in L_2(\Omega)$, $a_{ij}(x,t)\in L_\infty(Q)$, $i,j=\overline{1,n}$ are the given functions and almost everywhere in for $\forall\xi\in R^n$ the conditions $\sum_{i,j=1}^n a_{ij}(x,t)\xi_i\xi_j \ge \alpha\sum_{i=1}^n \xi_i^2$,

$\alpha = const > 0, \qquad a_{ij}(x,t) = a_{ji}(x,t), \qquad i,j = \overline{1,n}, \qquad \left|\dfrac{\partial a_{ij}(x,t)}{\partial t}\right| \le M,$

$i,j = \overline{1,n}$ are satisfied.

For the given function $v(x) \in L_\infty(\Omega)$ the problem (0.40)-(0.42) is a direct problem.

If the function $v(x)$ is known, then the problem (0.40)-(0.43) is an inverse problem corresponding to the problem (0.40)-(0.42).

The problem (0.40)-(0.42) is an ill-posed problem, i.e. the solution of the problem is continuously independent of the data.

We reduce the inverse problem (0.40)-(0.43) to the following optimal control problem: to find such a function $v(x)$ from the class $V_m = \{v(x) : v(x) \in L_2(\Omega), \ \alpha \le v(x) \le \beta \ a.e. \ in \ \Omega\}$ that gives a minimum value to the inconsistency functional

$$J(v) = \frac{1}{2}\int_\Omega \left(\int_0^T K(x,t) u(x,t;v) dt - \chi(x) \right)^2 dx. \qquad (0.44)$$

Here the function $u(x,t;v)$ is the solution of the problem (0.40)-(0.42) corresponding to the function $v(x)$.

We call the function $v(x)$ a control, the set V_m is a class of admissible controls. If we can find a control giving the zero value to the functional (0.44), then the additional condition (0.43) is satisfied.

Theorem 0.11. Assume that for the problem (0.40)-(0.43) the conditions $f \in L_2(Q)$, $u_0 \in \overset{0}{W}{}_2^1(\Omega)$, $u_1 \in L_2(\Omega)$, $K(x,t) \in L_\infty(Q)$, $\chi(x) \in L_2(\Omega)$ are satisfied. Then in the optimal control problem (0.40)-(0.42), (0.44) the set $V_* = \left\{ v_* \in V_m : J(v_*) = \inf_{v \in V_m} J(v) \right\}$ is not empty, is a weak compact in $L_2(\Omega)$ and arbitrary minimizing sequence from V_m converges weakly to the set V_* in $L_2(\Omega)$.

Theorem 0.12. Assume that the conditions of theorem 0.11 are satisfied. Then the functional (0.44) is Frechlet continuously differentiable in V_m and at its point $v \in V_m$ the differential with the increment $\delta v \in L_\infty(Q)$ is determined by the expression

$$\langle J'(v), \delta v \rangle = \int_Q u\,\psi\,\delta v\,dxdt,$$

here $\psi = \psi(x,t;v)$ is the generalized solution of the adjoint problem

$$\frac{\partial^2 \psi}{\partial t^2} - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t) \frac{\partial \psi}{\partial x_j} \right) + v\psi = -K(x,t) \times$$

$$\times \left[\int_0^T K(x,t) u(x,t;v) dt - \chi(x) \right], \ (x,t) \in Q, \qquad (0.45)$$

$$\psi\big|_s = 0, \quad \psi\big|_{t=T} = 0, \quad \left.\frac{\partial \psi}{\partial t}\right|_{t=T} = 0, \quad x \in \Omega \qquad (0.46)$$

from $W_2^1(Q)$.

Theorem 0.13. Assume that the conditions of theorem 0.12 are satisfied. Then in the problem (0.40)-(0.42), (0.44) the necessary condition for the optimality of the control $v_*(x) \in V_m$ is the satisfaction of the inequality

$$\int_\Omega \left(\int_0^T u_*(x,t) \psi_*(x,t) dt \right) (v(x) - v_*(x)) dx \geq 0 \quad \forall v \in V_m$$

here the functions $u_*(x,t) = u(x,t;v_*)$, $\psi_*(x,t) = \psi(x,t;v_*)$ are the solutions of the problems (0.40)-(0.42) and (0.45)-(0.46) corresponding to the control $v = v_*(x)$.

In 2.4 in a mixed problem for a weak nonlinear wave equation an optimal control problem is studied by means of the coefficient of the lowest term.

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CONCLUSION

In the introduced dissertation work, the inverse problems of finding boundary and initial conditions for second order hyperbolic and elliptic equations, on definition of the right hand side and the lowest term of the second order hyperbolic equations were reduced to optimal control problems and the obtained problems were studied by means of optimal control methods.

The following new results were obtained:

- the problems of finding the boundary and initial functions for second order hyperbolic and elliptic equations were reduced to optimal control problems;
- the problems of defining the coefficient of the right hand side and the lowest term of second order hyperbolic equations were reduced to optimal control problems;
- the obtained optimal control problems were studied;
- it was proved that the functionals are differentiable, and the expressions for their gradients were obtained;
- variational inequality type optimality conditions were derived.

The main results of the dissertation were published in the following works:

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