

Semi-annual report of the department of “Function Theory” for 2024

Scientific direction: Harmonic, nonharmonic analysis and approximation theory

Topic: “Approximation of functions of many variables by ridge functions, neural networks, linear and nonlinear superpositions, embedding theorems for functional spaces”

On scientific activity

In the reporting period, 6 works connecting 7 researchers on the subject "Approximation of functions of many variables by ridge functions, neural networks, linear and nonlinear superpositions, and embedding theorems for functional spaces" were carried out. 8 papers were published. 7 of these articles were published in journals indexed in the databases Clarivate Analytics "Web of Science" and "Scopus." In addition, 1 book was published, and 1 book and 1 article were submitted for publication.

About works

Work 1: Calculating the approximation error of single hidden layer neural networks induced by a certain class of activation functions

(Executor: dr. math. sci., prof. Vugar Ismailov)

Published works:

1. A. Ismayilova, V.E Ismailov, On the Kolmogorov neural networks, Neural Networks 176 (2024), 106333;

<https://doi.org/10.1016/j.neunet.2024.106333>

2. V.E. Ismailov, Approximation error of single hidden layer neural networks with fixed weights, Information Processing Letters 185 (2024), Paper No. 106467, doi.org/10.1016/j.ipl.2023.106467

3. A.Kh. Asgarova, A.A. Huseynli, V.E. Ismailov, A Chebyshev-type alternation theorem for best approximation by a sum of two algebras, Proceedings of the Edinburgh Mathematical Society (2) 66 (2023), no. 4, 971-978, doi.org/10.1017/S0013091523000494

Work 2: Embedding theorems in Grand and Small Sobolev-Morrey type spaces
(Executor: dr. ph.-m. s., prof. Alik Najafov, PhD Aygun Omarova)

Published book:

1. A. T. Omarova, T. S. Orucov, S. N. Babusov, “Cəbr və ədədlər nəzəriyyəsinə məsələ və misallar” dərs vəsaiti, Mingəçevir 2023, səh. 319.

The book submitted for publication:

2. A.T. Omarova, R.F. Babayev, T.S. Orucov, “Kompleks dəyişənli funksiyalar nəzəriyyəsinə məsələ və misallar”, Mingəçevir 2024, səh. 184.

Work 3: Approximations of the Hilbert transform of functions integrable in the Lebesgue sense and continuous in the Hölder sense on the real line
(Executor: dr. math. sci., prof. Rashid Aliev)

4 papers were published:

1. Aliev A.R., Aliev R.A., On the Boundedness of the Fractional Maximal Operator, the Riesz Potential, and Their Commutators in Orlicz Spaces, *Mathematical Notes*, **115:4** (2024), 453-462.

<https://doi.org/10.1134/S0001434624030180>

2. Aliev R.A., Alizade L.Sh., Approximation of the Hilbert transform in the Lebesgue spaces, *Journal of numerical analysis and approximation theory*, **52:2** (2023), 139-154. <https://doi.org/10.33993/jnaat522-13122>

3. Aliev R.A., Alizade L.Sh., Approximation of the Hilbert transform in Hölder spaces, *Azerbaijan Journal of Mathematics*, **14:2** (2024).

<https://azjm.org/volumes/14-2.html>

4. Aliev R.A., Isgandarli F.M., On the representability of a smooth trivariate function by sums of generalized ridge functions, *Baku State University Journal of Math. and Comp. sciences*, **1:1** (2024), 25-36.

<http://bsuj.bsu.edu.az/en/journal/mathematics-and-computer-sciences/issue/1/1>

Work 4: “Correct” estimates in Marchaud - Type inequalities: the case of various $L_p(T)$ metrics, $1 \leq p \leq \infty$, $T = (-\pi, \pi]$

(Executor: cand. ph.-m. s., associate professor Niyazi Ilyasov)

Work 5: Investigation of the best approximation in the approximation of continuous functions by RBF neural networks

(Executor: cand. ph.-m. s., associate professor Ibrahim Maharov)

1 paper was published:

1. Aida Kh. Asgarova and Ibrahim K. Maharov, “On a formula for the approximation by RBF neural networks with two hidden nodes”, *Proceedings of the Institute of Mathematics and Mechanics* 2024, 50(1), 152-161, <https://proc.imm.az/volumes/50-1/50-01-12.pdf>

Work 6: Approximation by sums of two closed algebras containing constants in a metric compact

(Executor: PhD Aida Asgarova)

1 paper was published:

1. Aida Kh. Asgarova and Ibrahim K. Maharov, “On a formula for the approximation by RBF neural networks with two hidden nodes”, Proceedings of the Institute of Mathematics and Mechanics 2024, 50(1), 152-161,

<https://proc.imm.az/volumes/50-1/50-01-12.pdf>

On scientific-organizational activity:

In the reporting period, head of department, dr.math.sci., prof. Vugar Ismailov, made a presentation on “Some problems in the theory of approximation of ridge functions” at the institute seminar.

The chief researcher of the department, dr.ph.-m. s., prof. Alik Najafov, gave a talk with presentation entitled “Embedding theorems in generalized grand Sobolev-Morrey spaces” at the institute seminar.

In the reporting period, the chief researcher of the department, dr.math.sci. prof. Rashid Aliev, made a presentation on “The boundedness of some operators of harmonic analysis in Orlicz spaces” at the institute seminar.

The senior researcher of the department, cand. ph.-m. s., associate professor, Niyazi Ilyasov, gave a talk with presentation entitled “On equivalence of some inequalities in the approximation theory of periodic functions in $L_p(T)$ spaces” at the institute seminar.

In January of this year, mathematicians Sergio Macario and Juan Font from Spain's Jaume University referred to the joint result obtained by Vugar Ismailov and Namig Guliyev from the Institute of Mathematics and Mechanics as a striking result in their own arXiv article. They also extensively utilized this result.

(<https://arxiv.org/abs/2401.13008>)

In the first half of 2024, head of department dr.math.sci., prof. Vugar Ismailov, has become a reviewer for the journal Neural Networks three times. This journal is the official publication of neural networks societies worldwide, in Europe, and Japan.

Head of the department

Dr.Math.Sci., Prof. Vugar Ismailov