

HALF-YEAR REPORT OF 2024 ON SCIENTIFIC AND SCIENTIFIC-ORGANIZATIONAL ACTIVITY OF THE DEPARTMENT OF “EQUATIONS OF MATHEMATICAL PHYSICS” OF INSTITUTE OF MATHEMATICS AND MECHANICS OF MINISTRY OF SCIENCE AND EDUCATION OF THE REPUBLIC OF AZERBAIJAN

In the department of “Equations of Mathematical Physics” 13 workers, 10 of whom are research workers. Of them 3 **doctors of sciences, professors:**

1. Akhundov Adalat Ya. – chief researcher associate, (full time).
2. Mammadov Farman I. – chief researcher associate, (full time).
3. Kerimov Nazim B. – chief researcher associate, (a part time).

6 doctors of philosophy in mathematics:

4. Guliyev Abdurrahim F. – head of department, leading researcher associate, (full time).
5. Bagirov Shirmail H. – leading researcher associate, ass. prof., (a part time).
6. Mammadov Elchin M. – leading researcher associate, ass. prof., (full time).
7. Jafarov Nazim J. – senior researcher associate, (full time).
8. Mammadova Vafa A. – leading researcher associate, ass. prof., (full time).
9. Hasanova Aynur H. – senior researcher associate, ass. prof., (full time).
10. Mammadli Sayali M. – researcher associate, kandidat for a degru, (full time).

3 laboratory assistants:

11. Mustafayeva Lala M. – senior laboratory assistant, (full time).
12. Abdullayeva Aydan J. – laboratory assistant, (full time).
13. Jabrailova Aynur F. – laboratory assistant, doctoral student, (a part time).

I. SCIENTIFIC PART.

In 2024, according to the approved plan, the department conducts 9 research works on the topic “Solvability of initial-boundary value problems for various types of differential equations, qualitative properties of solutions and their applications”.

Work 1: ”Qualitative properties of the Wiener type of solutions to second-order parabolic equations with degenerate coefficients”.

Executor: head of department A.F. Guliyev

During the reporting period, the Dirichlet problem for the heat equation was considered, a criterion for the regularity of a boundary point was found in terms of the potential and a regularity criterion in terms of heat capacity that follows from it as a special case was obtained, and an n-dimensional analogue of the one-dimensional case of the famous Petrovsky integral criterion for symmetric regions was given.

Published papers:

1. A.F. Guliyev, *Wiener-type Criterion for the Heat Equation in terms of the Parabolic Potential and its corollaries*. Proceedings of the XI International Conference “Modern Problems of Mathematics and Mechanics” Dedicated to the Memory of a Genius Azerbaijani Scientist and Thinker Nasiraddin Tusi, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, July 03-06, Baku, pp.116-119.
<https://mpmm.imm.az/pages/abstracts>

Work 2: ”On one inverse problem for a parabolic equation of Burgers type”.

Executor: prof. A.Ya. Akhundov.

During the reporting period, a theorem on the uniqueness and stability of the solution of the posed problem was proved.

Published papers:

1. A.Ya. Akhundov, *On an inverse problem for a elliptic equation*. **Uzb. Math. Journal**, № 2, 2024, 5 pp.(in printing)
2. A.Ya. Akhundov, N.C. Pashaev, *Inverse problem for Burgers type parabolic equations*. Proceedings of the XI International Conference “Modern Problems of Mathematics and Mechanics” Dedicated to the Memory of a Genius Azerbaijani Scientist and Thinker Nasiraddin Tusi, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, July 03-06, Baku, pp. 84-86.
<https://mpmm.imm.az/pages/abstracts>

Work 3: "Qualitative theory of non-uniform elliptic and parabolic equations of the second-order. Existence and uniqueness problem for elliptic and parabolic equations with gradient in the lowest term".

Executors: prof. F.I. Mammadov, S.M. Mammadli.

During the reporting period, existence and uniqueness theorems were proven for non-uniformly degenerate elliptic and parabolic equations. The results obtained are presented in print.

Non-uniformly degenerate inequalities of Sobolev-Poincaré type containing a gradient are proved. The results will be applied to the equations.

Published papers:

1. F.I. Mamedov, N.M. Mammadzada, S.M. Mammadli, *To the Weak Solvability of Dirichlet Problem for a Fractional Order Degenerate Elliptic Equation*. *Azerbaijan Journal of Mathematics*, V. 14, NO 1, 2024, p. 69-78.
<https://azjm.org/volumes/1401/pdf/1401-6.pdf>
2. Farman Mamedov, Yusuf Zeren, Khayala Gasimova, Abdullah Salami, *Dirichlet problem for a non-uniformly elliptic equation with L^1 data*. 7th International HYBRID Conference on Mathematical Advances and Applications (ICOMAA-2024), May, 08-11, 2024, Yildiz Technical University, Istanbul, Türkiye, p. 21.
<https://2024.icomaas.com/wp-content/uploads/2024/06/ICOMAA-2024-ABSTRACT-BOOK0406.pdf>
3. F.I. Mamedov, Kh.A. Gasimova, V.A. Mamedova, *Dirichlet problem for a non-uniformly elliptic equation with small terms*. Proceedings of the XI International Conference "Modern Problems of Mathematics and Mechanics" Dedicated to the Memory of a Genius Azerbaijani Scientist and Thinker Nasiraddin Tusi, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, July 03-06, Baku, pp. 152-154.
<https://mpmm.imm.az/pages/abstracts>

Work 4: "Some spectral properties of high-order ordinary differential operators with regular boundary conditions".

Executor: prof. N.B. Kerimov.

The article considers the eigenvalue problem of the form

$$\begin{aligned}
-y + q(x)y &= \lambda y, \quad 0 < x < 1, \\
y(0) \sin \beta &= y(0) \cos \beta, \quad 0 \leq \beta < \pi, \\
y(1) &= (a\lambda^2 + b\lambda + c)y(1)
\end{aligned}$$

Here λ is a spectral parameter, $q(x)$ is a real continuous function on the interval $[0,1]$, a, b, c are real numbers. Necessary and sufficient conditions for the minimality and completeness of a system of eigenfunctions and associated functions of a suitable differential operator are studied in two cases: quadratic ($a \neq 0$) and affine ($a = 0, b < 0$).

Published papers:

1. Nazim Kerimov and Yagub Aliyev, [Minimality conditions for Sturm-Liouville problems with a boundary condition depending affinely or quadratically on an eigenparameter](https://books.google.az/books?id=atNI0AEACAAJ&printsec=frontcover&hl=ru&source=gbs_ge_summary_r&cad=0#v=onepage&q&f=false). **Contemporary Mathematics**, Advances in Functional Analysis and Operator Theory, Volume 798, 2024, pp.1-12.
https://books.google.az/books?id=atNI0AEACAAJ&printsec=frontcover&hl=ru&source=gbs_ge_summary_r&cad=0#v=onepage&q&f=false

Work 5: "Study of the existence of global solutions to quasilinear, semilinear parabolic equations and systems of high-order equations in an infinite domain".

Executor: ass. prof. Sh.H. Bagirov.

During the reporting period, in a cylindrical region whose base is the outer part of the ball, the problem of the existence of positive global solutions for has a general non-linearity second-order parabolic equation of divergent form with periodic coefficients was studied. Sufficient conditions have been found to ensure the absence of such solutions. In a half-cylinder, in the immediate vicinity of infinity, the asymptotic behavior of solutions of a semilinear elliptic equation of the second-order, satisfying the homogeneous Neumann condition on the lateral surface of the half-cylinder, is studied.

Published papers:

1. Sh.G. Bagirov, [The Absence of Positive Global Periodic Solution of a Second-Order Semi Linear Parabolic Equation With Time-Periodic Coefficients](https://azjm.org/volumes/14-2.html) **Azerbaijan Journal of Mathematics**, vol 14, no 2 (2024) (in printing).
<https://azjm.org/volumes/14-2.html>
2. Bağırov Şirmayıl Həsən, Həsənova Aytən Məlik, *Silindrik oblastda yarım xətti elliptik tənliyi həllinin asimptotikası*. Proceedings of the Republican Scientific Conference on the topic "Modern problems of mathematics, mechanics and

information technologies”, dedicated to the 101st anniversary of the birth of national leader Heydar Aliyev, May 02-03, 2024, Baku, p. 15.

3. Sh.H. Bagyrova, *Absence of global solutions to a system of high-order semilinear equations with a biharmonic operator in the main part*. Proceedings of the XI International Conference “Modern Problems of Mathematics and Mechanics” Dedicated to the Memory of a Genius Azerbaijani Scientist and Thinker Nasiraddin Tusi, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, July 03-06, Baku, pp. 98-99.

<https://mpmm.imm.az/pages/abstracts>

Work 6: “Study of global properties of a solution to a mixed problem for one class of hyperbolic equations with nonlinear boundary conditions”.

Executor: ass. prof. E.M. Mammadov.

During the reporting period, for a third-order equation with nonlinearity in the main part, results were obtained on the qualitative properties of the solution to a third-order equation specified with a nonlinear boundary condition. The destruction of the solution over a finite period of time is shown and a sufficient condition for the destruction of the solution is obtained under certain conditions imposed on the initial data, nonlinearity in the equation and the boundary condition. An article is being prepared based on the results obtained.

Published papers:

1. E.M. Mamedov, *On blow-up of solution for one nonlinear problem*. Proceedings of the XI International Conference “Modern Problems of Mathematics and Mechanics” Dedicated to the Memory of a Genius Azerbaijani Scientist and Thinker Nasiraddin Tusi, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, July 03-06, Baku, pp. 149-150.

<https://mpmm.imm.az/pages/abstracts>

Work 7: “Solvability of the first boundary value problem in the Hölder class for second-order parabolic type equations with discontinuous coefficients in paraboloid type domains”.

Executor: N.J. Jafarov.

During the reporting period, in paraboloid-type domains, an estimate for the Green's function of the first boundary value problem for second-order parabolic equations of divergent structure with discontinuous coefficients was proved. At the same time, in paraboloid-type domains, a criterion was found for the solution of the Dirichlet problem to belong to the Hölder space for second-order parabolic equations of divergent structure with discontinuous coefficients. An article is being prepared based on the results obtained.

Work 8: “Qualitative properties of non-uniform elliptic and parabolic equations”.

Executor: ass. prof. V.A. Mammadova.

During the reporting period, in order to study the qualitative properties of non-uniform elliptic and parabolic equations, weighted integral inequalities of the Sobolev-Poincaré-Friedrichs type were studied. The main activity was devoted to a new proof of Friedrichs-type inequalities. Note that these inequalities are about functions defined in convex regions and vanishing in a certain subset of this region. In general, this is not true in non-smooth areas. The subset in which the class of functions vanishes can be an arbitrary set; for the Friedrichs inequality to be true, the Lebesgue dimension of this subset must be positive.

Published papers:

1. F.I. Mamedov, Kh.A. Gasimova, V.A. Mamedova, *Dirichlet problem for a non-uniformly elliptic equation with small terms*. Proceedings of the XI International Conference “Modern Problems of Mathematics and Mechanics” Dedicated to the Memory of a Genius Azerbaijani Scientist and Thinker Nasiraddin Tusi, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, July 03-06, Baku, pp. 152-154.
<https://mpmm.imm.az/pages/abstracts>

Work 9: ”On the solution of one inverse problem for a semilinear parabolic equation”.

Executor: ass. prof. A.H. Hasanova.

During the reporting period, the existence, uniqueness and stability of a solution to one inverse problem for a semilinear equation of parabolic type of the second-order were investigated. It is expected to substantiate the proposed algorithm for an approximate solution of the inverse problem under consideration and to prove the theorem on the convergence of the approximate solution to the exact solution.

Published papers:

1. H. Hasanova, *Uniqueness and assessment stability of the solution to the inverse problem for a semilinear equation of parabolic type*. Proceedings of the XI International Conference “Modern Problems of Mathematics and Mechanics” Dedicated to the Memory of a Genius Azerbaijani Scientist and Thinker Nasiraddin Tusi, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, July 03-06, Baku, pp. 131-133.
<https://mpmm.imm.az/pages/abstracts>

II. ORGANIZATIONAL ACTIVITY.

Head of the department, Ph.D. Abdurrahim Guliyev is the scientific secretary of the Dissertation Council of the Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, is an expert in the SEC on the subject of “mathematics”.

Chief researcher associate of the department prof. Adalat Akhundov is the deputy director on Scientific affairs, a member of the Scientific Council, the deputy chairman of the Dissertation Council, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, “Scientific works” of Baku University for girls. Prof. Adalat Akhundov works as a professor at Lankaran State University.

Chief researcher associate of the department prof. Farman Mammadov is a member of the Dissertation Council, a member of the Expert Council of the HAC (Higher Attestation Commission), a member of the editorial board of the journals Azerbaijan Journal of Mathematics, Proceedings of Mathematics and Mechanics Institute, Journal of Contemporary Applied Mathematics, Universal Journal of Applied Mathematics, a reviewer of the journal of Mathematical Reviews of American Mathematical Society. Prof. Farman Mammadov works as a professor at Azerbaijan State Oil and Industry University (ASOIU).

Chief researcher associate of the department prof. Nazim Kerimov is a professor at the Khazar University, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, Transactions issue mathematics of Mathematics and Mechanics Institute, Azerbaijan Journal of Mathematics.

Leading researcher of the department associate professor Shirmail Bagirov is the deputy dean for scientific affairs of the Faculty of Mechanics and Mathematics of the Baku State University, works as an associate professor at Baku State University and at the National Aviation Academy.

Leading researcher of the department associate professor Elchin Mammadov is a member of the commission for control of the IMM trade union organization, works as an associate professor at Baku State University.

Senior researcher of the department associate professor Aynur Hasanova is a member of the working group created to use the platform Web of Science of the Clarivate Analytics and collect information.

Doctoral students and dissertators of the department under the guidance of scientific leaders (Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Shirmail Bagirov) continue their research on the approved topics.

On January 31, 2024, Chief Researcher of the Department Prof. Nazim Kerimov in the all-institute seminar of IMM gave a report on the topic “Spectral properties of higher-order differential operators with periodic boundary conditions”.

On May 29, 2024, Senior Researcher of the Department Nazim Jafarov in the all-institute seminar of IMM gave a report on the topic “Criterion for belonging to Hölder space for solutions of the first boundary value problem for parabolic equations of the second-order of divergent structure with discontinuous coefficients in paraboloid-type domains”.

This year, the Chief Researcher of the Department, prof. Farman Mamedov acted as an official opponent at the plaintiff’s defense, which took place at BSU.

This year, the leading researcher of the department, Assoc. Elchin Mamedov wrote a review of dissertations submitted by two applicants for the degree of Doctor of Philosophy in mathematics, in addition, he was the official opponent of one of the plaintiffs, another dissertation was given to him for review.

On May 31 and June 28, 2024, Chief Researcher Prof. Nazim Kerimov served as the official opponent for the plaintiff during the defense.

<https://www.imm.az/exp/wp-content/uploads/2024/04/Avt.-az-1-mohurlu.pdf>

<https://www.imm.az/exp/wp-content/uploads/2024/05/Leyla-avt.-%E2%80%94-az-SCAN.pdf>

On May 17, 2024, Senior Researcher Nazim Jafarov served as the official opponent for the plaintiff during the defense.

<https://www.imm.az/exp/wp-content/uploads/2024/04/Avtor.-az.-pecat.pdf>

Head of the department, Abdurrahim Guliyev and prof. Farman Mammadov continuing their cooperation with Turkish scientists, expanded their scientific ties. Also,

prof. Farman Mammadov continues to cooperate with Italian scientists. In addition, employees of our department cooperate with scientists from Russia, Ukraine, Turkey, Uzbekistan, America, Sweden and other countries.

Every week, on Wednesdays, under the leadership of the head of the department Abdurrahim Guliyev, and on Mondays, under the leadership of prof. Farman Mammadov, the department's seminar on the topic “Modern problems of mathematical physics” is traditionally held.

KONFERENCES

1. Chief Researcher of the Department prof. Farman Mammadov spoke as an invited speaker on the topic “*On Solvability of the L^1 Data nonuniformly Parabolic Equations*” at the International Conference “7th International HYBRID Conference on Mathematical Advances and Applications”, organized by the Yildiz Technical University in Istanbul, Turkey, May 08-11, 2024.
<https://2024.icomaas.com/invited-speakers/>
2. Leading Researcher of the Department Assoc. Shirmail Bagirov took part in the Republican Scientific Conference on the topic “Modern problems of mathematics, mechanics and information technology”, dedicated to the 101st anniversary of the birth of national leader Heydar Aliyev, Baku, May 02-03, 2024.

Thus, in the first half of 2024, employees of the department published 10 articles, of which 2 were scientific articles (1 abroad), published in journals included in the list of the Web of Science database and Scopus, 8 abstracts (1 abroad).

Head of department:

Ph.D. Abdurrahim Guliyev