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MOTION OF FLUID WITH FREE SURFACE UNDER THE ACTION OF CYLINDRICAL BODY

Abstract

Based on the long waves theory, motion of cylindrical body in fluid with free surface is considered. Solutions for viscous fluid are found at the boundary conditions stipulated by the simplest solutions.

Motion of fluid in acoustical statement and elastic rigid body under the action of cylindrical rigid body is considered in a series of papers, including [1, 2]. Problems are solved by the operating methods, however, for finding original solutions one can apply the numerical methods.

Here motion of fluid with free surface in definition of theory of long waves is investigated. If viscosity is absent, then the obtained problems are analogous to ones being considered in [1, 2], but in the presence of viscosity the problems are much more exact. Below some analytical solutions are performed.

1. Motion equations

Following the theory of long waves [3, 4] in motion equations (Navier- Stockes) in the case of small bending of fluid surface, vertical speed v_z and vertical acceleration $\dfrac{dv_z}{dt}$ and also convective terms can be disregarded, which gives us

$$\frac{\partial p_l}{\partial z} = -\rho g, \tag{1}$$

$$\frac{\partial v_x}{\partial t} = -g\frac{\partial \zeta}{\partial x} + \nu \Delta v_x, \tag{2}$$

$$\frac{\partial v_y}{\partial t} = -g\frac{\partial \zeta}{\partial y} + \nu \Delta v_y, \tag{3}$$

where $p_l$is pressure, v_x and v_y are projections of velocity onto the corresponding coordinates, ζ is a height of the surface, ν is a kinematic viscosity, Δ is Laplace operator.

Equation of continuity takes the form

$$\frac{1}{h}\frac{\partial \zeta}{\partial t} = -\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}, \tag{4}$$

where h is a depth of fluid.

Equations (2), (3) and (4) imply

$$\frac{\partial^2 \zeta}{\partial t^2} = \left(a^2 + \nu\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\zeta, \tag{5}$$

where $a = \sqrt{gh}$, or in the polar system of coordinates

$$\frac{\partial^2 \zeta}{\partial t^2} = \left(a^2 + \nu\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial \theta^2}\right)\zeta, \tag{6}$$

here r is the distance of fluid particle from the pole, θ is a polar angle.

2. Boundary conditions

Let rigid cylindrical body of the radius r_0 move in the horizontal direction (along x axis) with speed V, remaining upright. Since the body moves, then equation of its surface will depend on time, but if we will introduce the moving coordinates, connected with the body, then flow equation is modified: instead of the term $a^2\dfrac{\partial^2\zeta}{\partial x^2}$ it will be $\left(a^2-V^2\right)\dfrac{\partial^2\zeta}{\partial x^2}$ (for constant V). In the case $V^2 \ll a^2$ equations (5) and (6) can be retained. On the surface of the body $r=r_0$ the condition of equality of the normal components of velocities of the fluid and cylinder holds, i.e.

$$v_r = V\cos\theta. \tag{7}$$

But in the presence of viscosity, the additional condition for the tangent component of velocity is necessary, which depends on the surface roughness of cylinder and requires a special consideration. In the case of smooth body, force P acting on cylinder is equal to

$$P = r_0\int_0^{\zeta}\int_0^{2\pi} p_l\cos\theta dz d\theta. \tag{8}$$

Integrating (1), we obtain

$$p_l = -\rho g\left(\zeta - z\right). \tag{9}$$

Substituting (9) into (8), we have

$$P = -r_0\rho g\int_0^{\zeta}\int_0^{2\pi}\left(\zeta - z\right)\cos\theta dz d\theta. \tag{10}$$

By assigning the boundary conditions, it is necessary to give the speed of cylinder $V(t)$ or the strain onto cylinder $P(t)$.

3. Solution of the problem

We find solution of the problem in the form

$$\zeta = \zeta_1\cos\theta. \tag{11}$$

Substituting into (6), we obtain

$$\frac{\partial^2\zeta_1}{\partial t^2} = \left(a^2+\nu\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-1\right)\zeta_1. \tag{12}$$

At the initial condition

$$t=0,\ \zeta=0, \tag{13}$$

applying to (12) Laplace- Carson transformation, we have

$$p^2\bar{\zeta}_1 = \left(a^2+\nu p\right)\left(\bar{\zeta}_1'' + \frac{1}{r}\bar{\zeta}_1' - \bar{\zeta}_1\right), \tag{14}$$

where p is transformation parameter, $\bar{\zeta}_1$ is image of ζ_1.

$$\zeta_1 = \bar{C}(p) K_1 \left(\frac{pr}{\sqrt{a^2 + \nu p}} \right) \tag{15}$$

will be solution of equation (14) bounded on infinity, where K_1 is McDonald function of the first order.

In order to find original of expression (15), we will use the original of function [5]

$$\sqrt{p} K_1 \left(2\sqrt{\alpha p} \right) \doteq \frac{\exp\left(-\frac{\alpha}{t} \right)}{2\sqrt{a}}. \tag{16}$$

By using the displacement theorem from p to $p-2$, we can obtain

$$\frac{p}{\sqrt{p-2}} K_1 \left[2\sqrt{\alpha (p-2)} \right] \rightarrow \frac{\exp\left(2t - \frac{\alpha}{t} \right)}{2\sqrt{a}}. \tag{17}$$

By using formula for passing from p to $p + \frac{1}{p}$ [5], from (17) we can obtain

$$\frac{\sqrt{p}}{p-1} K_1 \left[2(p-1) \sqrt{\frac{\alpha}{p}} \right] \rightarrow \frac{1}{2\sqrt{\alpha}} \int_0^t J_0 \left[2\sqrt{2(t-\tau)\tau} \right] e^{2\tau - \frac{\alpha}{\tau}} d\tau. \tag{18}$$

By the displacement theorem from p to $p+1$, (18) implies

$$\frac{1}{\sqrt{p+1}} K_1 \left[2\sqrt{\alpha} \frac{p}{\sqrt{p+1}} \right] \rightarrow \frac{e^{-t}}{2\sqrt{\alpha}} \int_0^t J_0 \left[2\sqrt{2(t-\tau)\tau} \right] e^{2\tau - \frac{\alpha}{\tau}} d\tau. \tag{19}$$

By similarity theorem, passing from p to γp, we obtain

$$\frac{1}{\sqrt{\gamma p+1}} K_1 \left[2\sqrt{\alpha} \frac{\gamma p}{\sqrt{\gamma p+1}} \right] \rightarrow \frac{e^{-\frac{t}{\gamma}}}{2\sqrt{\alpha}\gamma} \int_0^t J_0 \left[\frac{2}{\gamma} \sqrt{2(t-\tau)\tau} \right] e^{\frac{2\tau}{\gamma} - \frac{\alpha\gamma}{\tau}} d\tau. \tag{20}$$

Finally, by introducing in (20) α and γ instead of physical constants by formulae,

$$2\sqrt{\alpha} = \frac{ra}{\nu}; \ \gamma = \frac{\nu}{a^2},$$

we obtain

$$\frac{a^2}{\sqrt{a^2+\nu p}} K_1 \left[\frac{rp}{\sqrt{a^2+\nu p}} \right] \rightarrow \frac{a}{r} e^{-\frac{a^2 t}{\nu}} \int_0^t J_0 \left[\frac{2a^2}{\nu} \sqrt{(t-\tau)\tau} \right] e^{\frac{2a^2\tau}{\nu} - \frac{r^2}{4\nu\tau}} d\tau. \tag{21}$$

Expression (21) can be considered as one of the particular solutions of equation (6), because the coefficient of K_1 can be included in the boundary condition everywhere by virtue of the its independence of r.

On the boundary $r = r_0$ as $t \rightarrow 0$ it tends to zero, since $\exp\left[-r_0/(at) \right] \rightarrow 0$. Thus, we have solution (21) strongly growing beginning from zero together with all its derivatives with consequent deceleration of growth.

Having the given boundary conditions and solution (21), one can construct solution by the numerical method used in [2].

References

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Received October 10, 2002; Revised March 12, 2002.
Translated by Azizova R.A.