Vahid Yu. BABANLY

ON MOTION OF ONE PHASE OF THREE-PHASE SYSTEM IN POROUS MEDIUM WITH TWO OBSTRUCTIONS

Abstract

It is considered the influence of obstacles of hole walls at some distance from partitions of phases of three-phase system on discharge and water-free gas-free discharge of middle (oil) phase. The numerous calculations, whose results are cited in the form of table and graphs, are made. On the basis of the obtained results some practical conclusions are made.

As practice shows during operation of natural hydrocarbon fluid fields, non-hydrocarbon phase-water is also one of the kinds of recovered production. On the other hand hydrocarbons themselves are in twophase state: gas and liquid (oil and condensate) states. In domain of flow these three phases are located by density: denser phase, and water is located below, oil in the middle and gas above. During operating use oil fields, oil is the basic kind of production. Initial stage of development is characterized by preferred oil production as more viscous phase. But in most fields water-free gas-free periods of oil well operation are short. Increase of water-free-gas-free period of oil well operation has practical importance. One of effective ways to fight against premature water cutting of well production is the creation of impermeable barriers in certain distance from oil-water and oil-gas contacts.

It should be noted that such barriers-interlayers are met in natural conditions, too.

In the present paper the problem on axisymmetric steady flow of homogeneous fluid (the other phases are accepted to be stationary) to central purtially penetrating well in the presence of obstacle at the wall of the same radius in homogeneous anisotropic medium is investigated.

The problem is put mathematically in such a way: it is required to find a solution of the equation

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial\, r}+\frac{1}{æ^2}\frac{\partial^2\phi}{\partial z^2}=0, \qquad (1)$$

under the following boundary conditions:

$$\left.\begin{array}{l}
\dfrac{\partial\phi}{\partial r}=\dfrac{q}{2\pi r_c\left(h_2-h_1\right)}=const, \quad \text{at} \quad r=r_c,\ h_1\le z\le h_2; \\
\dfrac{\partial\phi}{\partial z}=0 \ , \quad \text{at} \quad z=0 \quad \text{and} \quad h\ , \quad r_c\le r\le R_k; \\
\dfrac{\partial\phi}{\partial z}=0 \ , \quad \text{at} \quad z=h_1 \quad \text{and} \quad h_2\ , \quad r_c\le r\le R_0; \\
\phi\left(r,z\right)=\phi_k=const\ , \quad \text{at} \quad r=R_k\ , \quad 0\le z\le h,
\end{array}\right\} \qquad (2)$$

where $\phi = \frac{1}{\mu}(p + \rho_0 gh)$ is potential of filtering rate; $æ^2 = \frac{k_r}{k_z}$ is an anisotropy factor of medium; $k_r,\ k_z$ are horizontal and vertical penetrations of medium, respectively; $r_c,\ R_k$ are radius of well and external reservoir boundary (boundary of domain of flow), respectively; $p(r,z)$ is current pressure, ρ_0 is oil density, g is gravitational acceleration, q is well production, h is capacity (thickness) of oil part; $h_1,\ h_2$ are distances to obstacles; R_0 is radius of obstacle; μ is dynamic oil viscosity.

By change of variable $z_1 =æz$ the equation (1) becomes the Laplace equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{\partial^2 \phi}{\partial z_1^2} = 0. \tag{3}$$

Later on for obtaining effective solution the domain of flow is separated into two zones and under the physical considerations we assume (fig.1)

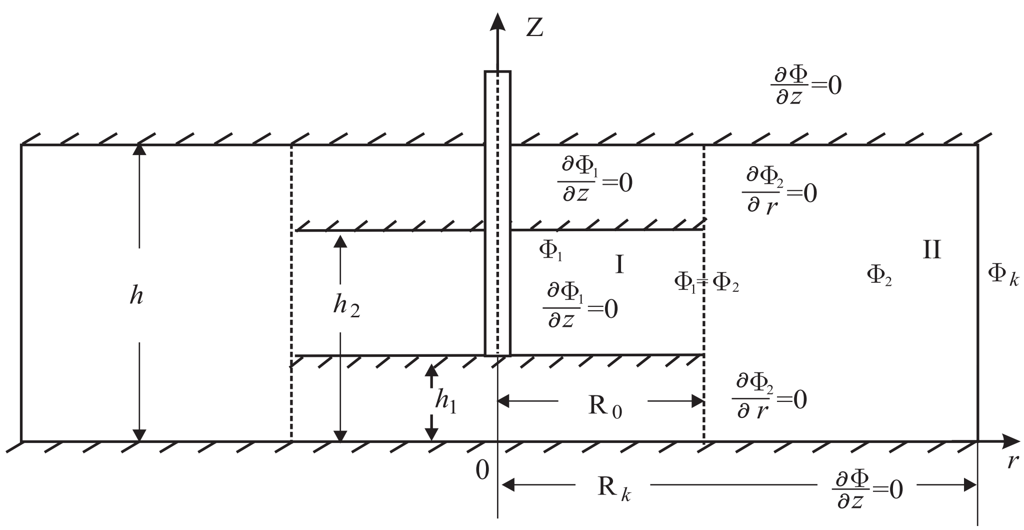

Fig.1

$$\frac{\partial \phi_2}{\partial r} = 0 \qquad \text{at} \qquad r = R_0,\ \ 0 \le z \le h_1 \ \text{ and } \ h_2 \le z \le h. \tag{4}$$

On conditional whole boundary of zone the following conditions will be fulfilled

$$\frac{\partial \phi_1}{\partial r} = \frac{\partial \phi_2}{\partial r} \quad \text{and} \quad \phi_1(r,z) = \phi_2(r,z) \quad \text{at} \quad r = R_0,\ \ h_1 \le z \le h_2, \tag{5}$$

where $\phi_1,\ \phi_2$ are potentials of rates in the first and second zones respectively.

The stated problem is solved by the method of finite integral transformations in G.A. Greenberg statement [1]. In the second zone the eigen function satisfies the following equation

$$\frac{d^2 Q_j(z)}{dz^2} + \mu_j^2 Q_j(z) = 0, \tag{6}$$

under the boundary conditions

$$\frac{dQ_j(z)}{dz} = 0\ , \quad \text{at} \quad z = 0 \quad \text{and} \quad h, \quad R_0 \le r \le R_k. \tag{7}$$

The solution of the equation (6) subject to the boundary conditions (7) has the form

$$Q_j(z) = \sqrt{\frac{2}{h}} \cos \mu_j z, \tag{8}$$

where $\mu_j = \dfrac{j\pi}{h}$ are eigen numbers.

Multiplying term-by-term the equation (3) for the function $\phi_2(r,z)$ by $Q_j(z)\,dz$ and integrating within the limits of 0 to h and denoting by

$$\phi_j(\mu_j z) = \int_0^h \phi_2(r,z)\, Q_j(z)\, dz, \tag{9}$$

we obtain the Bessel equation

$$\frac{d^2\phi_j}{dr^2} + \frac{1}{r}\frac{d\phi_j}{dr} - \mu_j^2 \phi_j = 0, \tag{10}$$

whose solution has the form

$$\phi_j(\mu_j r) = C_1 I_0(\mu_j r) + C_2 K_0(\mu_j r)\,,$$

where $I_0,\ \ K_0$ are zeroth order Bessel functions of imaginary argument of the first and second kinds.

The unknown coefficients C_1 and C_2 are determined from the corresponding conditions (2), and for the second zone we find the solution

$$\phi_2(r,z) = Q_0\phi_0 + \sum_{j=1}^{\infty} \phi_j(\mu_j r)\, Q_j(z)\,.$$

Behaving by the same way, for the first zone we determine the corresponding solution. Gapping intermediate computations we lead final solution of the stated problem

$$\phi(r,z) = \phi_k + \frac{q}{2\pi h} \ln\frac{R_0}{R_k} + \frac{q}{2\pi(h_2 - h_1)} \ln\frac{r}{R_0} +$$
$$+ \frac{æq}{\pi R_0 h (h_2 - h_1)^2} \sum_{j=1}^{\infty} \frac{u_0(\mu_j R_0)}{\mu_j^3 u_1(\mu_j R_0)} \left(\sin\mu_j h_2 - \sin\mu_j h_1\right)^2 +$$
$$+ \frac{q}{\pi R_0 h (h_2 - h_1)} \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} \frac{u_0(\mu_j R_0)\, u_2(\lambda_i r) \left(\sin\mu_j h_2 - \sin\mu_j h_1\right)}{\mu_j \left(\mu_j^2 - \lambda_i^2\right) u_1(\mu_j R_0)\, u_2(\lambda_i R_0)} \times$$
$$\times \left[(-1)^i \sin\mu_j h_1 + \sin\lambda_i h_2\right] \cos\lambda_i (z - h_1)\ \ , \tag{11}$$

where

$$u_0(\mu_j R_0) = I_0(\mu_j R_0)\, K_0(\mu_j R_k) - I_1(\mu_j R_k)\, K_0(\mu_j R_0)\,,$$

$$u_1\left(\mu_j R_0\right) = I_1\left(\mu_j R_0\right) K_0\left(\mu_j R_k\right) + I_0\left(\mu_j R_k\right) K_1\left(\mu_j R_0\right),$$

$$u_2\left(\lambda_i r\right) = I_0\left(\lambda_i r\right) K_1\left(\lambda_i r_c\right) + I_1\left(\lambda_i r_c\right) K_0\left(\lambda_i r\right) \ ,$$

$$u_2\left(\lambda_i R_0\right) = I_1\left(\lambda_i r_c\right) K_0\left(\lambda_i R_0\right) + I_0\left(\lambda_i R_0\right) K_1\left(\lambda_i r_c\right) \ ,$$

$$\lambda_i = \frac{i\pi}{h_2 - h_1}, \quad i = 1, 2, ...; \quad \mu_j = \frac{j\pi}{h} \ , \quad j = 1, 2, ... \ ,$$

$I_1,\ K_1$ are the first order Bessel functions of the first and second kind, imaginary argument.

Averaging the obtained expression of potential of filtering rate (11) on opened part $h_2 - h_1$ at $r = r_c$ we obtain

$$\bar{\phi}_c = \frac{1}{h_2 - h_1}\int_{h_1}^{h_2} \phi\left(r_c, z\right) dz = \phi_k - \frac{q}{2\pi h}\times$$

$$\times\left[\ln\frac{R_k}{r_c} + \left(\frac{h}{h_2 - h_1} - 1\right)\ln\frac{R_0}{r_c} - C\left(R_0, h_1, h_2\right)\right],$$

whence we define discharge

$$q = \frac{2\pi h\left(\phi_k - \bar{\phi}_c\right)}{\ln\frac{R_k}{r_c} + \left(\frac{1}{\bar{h}_2 - \bar{h}_1} - 1\right)\ln\frac{K_0}{r_c} - C\left(R_0, \bar{h}_1, \bar{h}_2\right)}.$$

For establishment of effect of obstacles to discharge, the relation of discharges is considered in the presence and absence of obstacles:

$$\bar{Q} = \frac{q}{q_H} = \frac{\ln\frac{\bar{R}_k}{\bar{r}_0} - C\left(\bar{r}_c, \bar{h}_1, \bar{h}_2\right)}{\ln\frac{\bar{R}_k}{\bar{r}_c} + \left(\frac{1}{\bar{h}_2 - \bar{h}_1} - 1\right)\ln\frac{\bar{R}_0}{\bar{r}_c} - C\left(\bar{R}_0, \bar{h}_1, \bar{h}_2\right)}, \tag{12}$$

where

$$C\left(\bar{r}_c, \bar{h}_1, \bar{h}_2\right) = \frac{2}{\pi^3 \bar{r}_c\left(\bar{h}_2 - \bar{h}_1\right)^2}\sum_{j=1}^{\infty}\frac{u_0\left(j\pi\bar{r}_c\right)}{j^3 u_1\left(j\pi\bar{r}_c\right)}\left(\sin j\pi\bar{h}_2 - \sin j\pi\bar{h}_1\right)^2,$$

$$C\left(\bar{R}_0, \bar{h}_1, \bar{h}_2\right) = \frac{2}{\pi^3 \bar{R}_0\left(\bar{h}_2 - \bar{h}_1\right)^2}\sum_{j=1}^{\infty}\frac{u_0\left(j\pi\bar{R}_0\right)}{j^3 u_1\left(j\pi\bar{R}_0\right)}\left(\sin j\pi\bar{h}_2 - \sin j\pi\bar{h}_1\right)^2,$$

$$\bar{R}_k = \frac{R_k}{æh}, \quad \bar{r}_c = \frac{r_c}{æh}, \quad \bar{R}_0 = \frac{R_0}{æh}, \quad \bar{h}_1 = \frac{h_1}{h}, \quad \bar{h}_2 = \frac{h_2}{h}$$

q_H is discharge without obstacles.

The numerical calculations for various values of R_0, $\bar{h}_1$ and $\bar{h}_2$ at $\bar{R}_k = 5$, $\bar{r}_c = 0,005$ were led by the formula (12). The results of calculations are cited in Table 1.

Table 1
The values of $\bar{Q}$

h_1	h_2	$\bar{R}_0$						
		0,1	0,2	0,25	0,4	0,5	0,75	1,0
0,1	0,8	0,9697	0,9517	0,9450	0,9297	0,9220	0,9074	0,8969
	0,9	0,9768	0,9639	0,9592	0,9485	0,9432	0,9331	0,9259
0,2	0,8	0,9691	0,9358	0,9267	0,9061	0,8956	0,8765	0,8627
	0,9	0,9697	0,9517	0,9450	0,9297	0,9220	0,9074	0,8969
0,3	0,7	0,9328	0,8938	0,8795	0,8478	0,8323	0,8038	0,7838
	0,8	0,990	0,9182	0,9063	0,8810	0,8681	0,8443	0,8274
	0,9	0,9619	0,9889	0,9302	0,9104	0,9004	0,8817	0,8681
0,4	0,6	0,8657	0,8033	0,7823	0,7378	0,7171	0,6806	0,6561
	0,8	0,9338	0,8954	0,8814	0,8501	0,8347	0,8065	0,7866
	0,9	0,9520	0,9232	0,9124	0,8879	0,8756	0,8526	0,8360

As I.A. Chanriy [2] showed we can take as a quantity of maximum possible limiting water free charge, the charge of partially penetrating well by stable motion of homogeneous fluid in domain whose thickness is equal to the thickness of oil part of the domain.

Assuming vertical pressure distribution to be static we have the following relation

$$\left(\rho_0 - \rho_g\right)(h - h_2) = (\rho_w - \rho_0)\, h_1,$$

or

$$\bar{h}_2 = 1 - \frac{\bar{h}_1}{\gamma}, \tag{13}$$

where $\gamma = \dfrac{\rho_0 - \rho_g}{\rho_w - \rho_0}$, ρ_0, ρ_w, ρ_g are oil, water, and gas densities respectively.

Using I.A.Charney's idea and for our case subject to (13), at that we can obtair the formula for the relation of upper limit of maximum possible water-free gas-free discharges in the presence and absence of obstacles (fig.2):

$$Q_\delta = \frac{q_\delta}{q_{H\delta}} = \frac{\ln\dfrac{\bar{R}_k}{\bar{r}_c} - C\left(\bar{r}_c, \bar{h}_1, \bar{h}_2\right)}{\ln\dfrac{\bar{R}_k}{\bar{R}_0} - C\left(\bar{R}_0, \bar{h}_1, \bar{h}_2\right)}. \tag{14}$$

Numerical calculations were led by the formula (14) subject to (13) for the following conditions: $\gamma = 1 \div 10$; $\bar{h}_1 = 0,1;\ 0,2;\ 0,3;\ 0,4;\ 0,5;$ $\bar{R}_0 = 0,05;$ $0,1;\ 0,2;\ 0,25;\ 0,4;\ 0,5;\ 0,75;\ 1,0.$

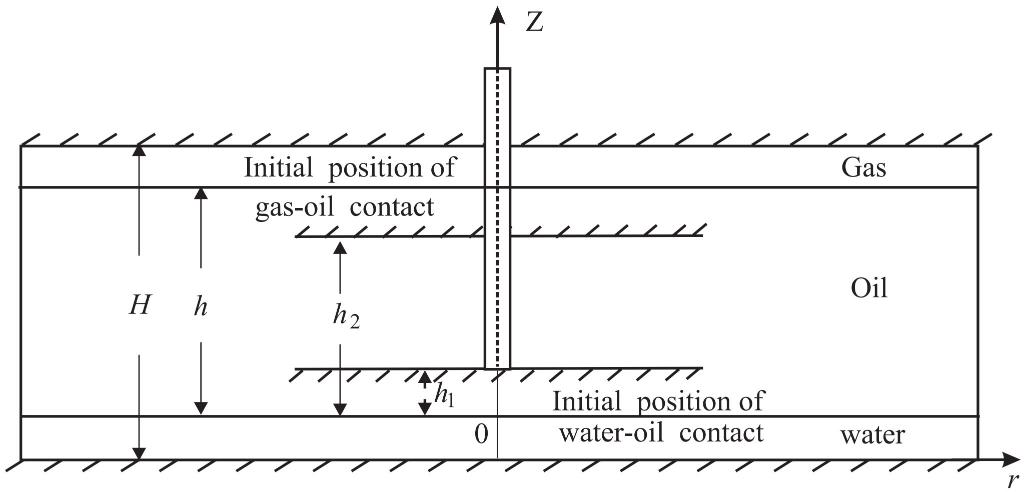

Fig.2

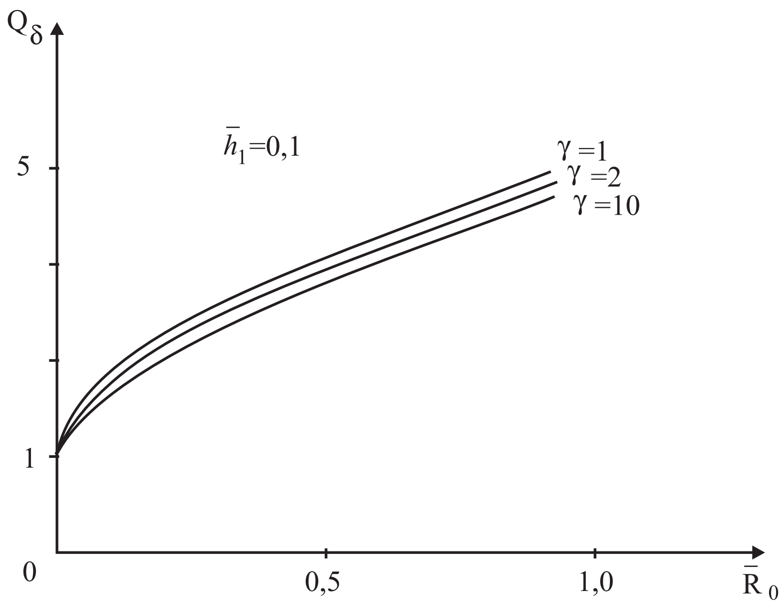

Fig.3

The results of calculations are cited in figures 3-7. On the basis of the led calculations we can conclude:

1) Q_δ increases with increasing of $\bar{R}_0$, where the rate of its increase with the growth $\bar{R}_0$ (at constant æh) slows down;

2) Q_δ decreases with increasing of γ where this fact is counted for small values of γ. Further increasing of γ does not affect the process;

3) by increasing of $\bar{h}_1$ the effect of γ to Q_δ increases. Maximum effect is observed at $\bar{h}_1 = 0,4$;

4) the effect of $\bar{R}_0$increases for large values of $\bar{h}$ and small values of γ;

5) anisotropic effect æ is equivalent to thickness effect under the other equivalent conditions.

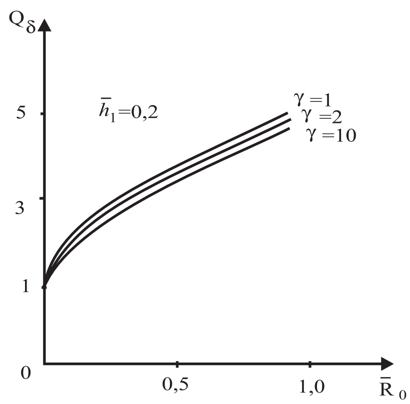

Fig.3

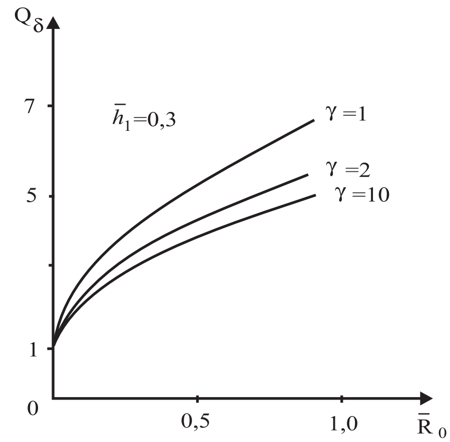

Fig.5

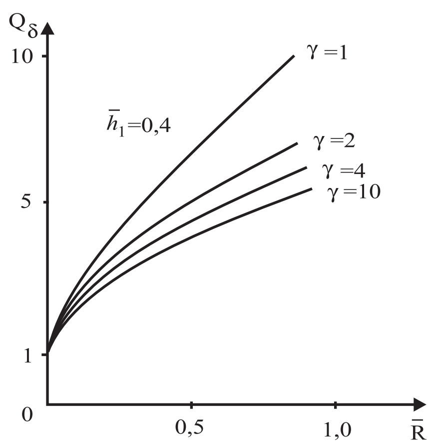

Fig.6

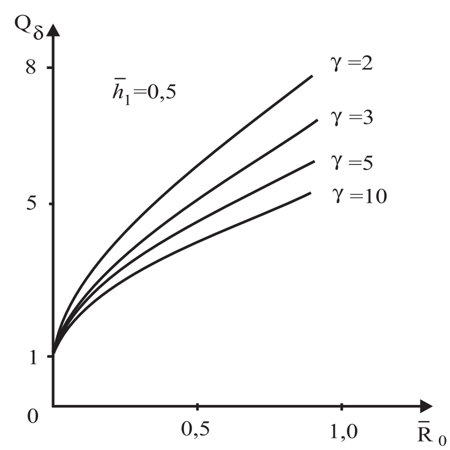

Fig.7

References

[1]. Greenberg G.A. *Selected questions of mathematical theory of electric and magnetic phenomenon.* Pub. House AN SSSR, M., 1948. (Russian)

[2]. Charniy I.A. *Subsurface hydro gas dynamics.* Gostoptekhizdat, 1963, M., 396p. (Russian)

Vahid Yu. Babanly
Institute of Applied Mathematics of Baku State University.
23, ZI. Khalilov str., AZ1148, Baku, Azerbaijan.
Tel.: 39-15-95 (off.).

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