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CANONICAL FORM OF A YIELD SURFACE AT REPEATED LOADING

Abstract

In the paper the canonical equation of a yield surface at any repeated loading for strengthening material is obtained. It is proved that in deviator surface holds the ideal plasticity, i.e. the strengthening doesn't happen.

It is known that stress state at the point is the main reason of changing material's mechanical state and basic problem of plasticity is to establish measure of stress state by achievement of which happens passage from elastic state to plastic, i.e. to generate plasticity criterion.

The plasticity criterion is received in [1] in the following form: "sum of maximum works of elastic deformation in two opposite directions in the space of stresses remains constant at any repeated quasistatic loading". Determine canonical equation of yield surface by means of the given criterion.

It is assumed that yield surface is a function from invariant stresses of the second order at any repeated loading, i.e.

$$J_1^2 + aJ_2 + bJ_1J_1^0 + c\sigma_{ij}^0\sigma_{ij} + d = 0 \tag{1}$$

where $a,\ b,\ c,\ d$ are functions of stress state, σ_{ij}^0 where σ_{ij}^0 are stress components at the end of previous active stress, $J_1,\ J_2$ are first and second invariants of stress tensor. Reduce (1) to canonical form. For this introduce new variables

$$\bar{\sigma}_{ij} = \sigma_{ij} - \sigma'_{ij}$$

where σ'_{ij} are coordinates of center of loading surface.

In new variables (1) will have the form

$$\bar{J}_1^2 + a\bar{J}_2 + \left(2J'_1 g_{ij} - aJ'_1 g_{ij} + a\sigma'_{ij} + bJ_1^0 g_{ij} + c\sigma_{ij}^0\right)\bar{\sigma}_{ij} +$$

$$+\left(J_1'^2 + aJ'_2 + bJ'_1 J_1^0 + c\sigma_{ij}^0\sigma'_{ij} + d\right) = 0 \tag{2}$$

where g_{ij} are metric tensor components. We require that the coefficients at $\bar{\sigma}_{ij}$ were equal to zero, i.e.

$$\left[(2-a)\,J'_1 + bJ_1^0\right]g_{ij} + a\sigma'_{ij} + c\sigma_{ij}^0 = 0. \tag{3}$$

Multiplying both parts (3) by g_{ij} and making summation by $i,\ j$ we shall get:

$$J_1' = \frac{3b+c}{L\,(a-3)} J_1^0. \tag{4}$$

Putting (4) in (3) we have

$$\sigma_{ij}' = \frac{2c - a\,(b+c)}{2a\,(3-a)} J_1^0 g_{ij} - \frac{c}{a}\sigma_{ij}^0. \tag{5}$$

In [1] was received that if the point with coerdinates g_{ij}^0 is situated on the yield surface, then the point with the coordinates $k_0\sigma_{ij}^0$ is also situated on yield surface, where

$$k_0 = \sqrt{\frac{2\sigma_T^2}{J_1^{02} + 2\,(1+\nu)\,J_2^0} - 1} \tag{6}$$

ν is Poisson coefficient, σ_T is yield point. Then coordinates of surface centre must have the form

$$\sigma_{ij}' = \frac{\sigma_{ij}^0 - k_0\sigma_{ij}^0}{2} = (1-k_0)\,\sigma_{ij}^0. \tag{7}$$

Comparing (5) with (7) we get:

$$2c - a\,(b+c) = 0, \tag{8}$$

$$-\frac{c}{a} = 1 - k_0. \tag{9}$$

Subject to (6), (7), (8) and (9) the equation (2) takes the form

$$\bar{J}_1^2 + a\bar{J}_2 = \left(\frac{k_0+1}{2}\right)^2 \left(J_1^{02} + aJ_1^0\right). \tag{10}$$

From the hypotesis accepted in [1] it follows that (1) must have the form

$$\bar{J}_1^2 + 2\,(1+\nu)\,\bar{J}_2 = \sigma_T^2. \tag{11}$$

Comparing (11) with (10) we get

$$a = 2\,(1+\nu)\,. \tag{12}$$

New yield point

$$\bar{\sigma}_T = \frac{k_0+1}{2}\sqrt{J_1^{02} + 2\,(1+\nu)\,J_1^0}. \tag{13}$$

It is natural to suppose that the material is strengthened due to the pressure, i.e. due to J_1. When $J_1 = 0$, i.e. in deviator surface the strengthening will not be, in other words, in the deviator surface $k_0 = 1$ and $\bar{\sigma}_T = \sigma_T$. Then from (8) and (9) $b = c = 0$ and from (13)$2\,(1+\nu)\,J_1^0 = \bar{\sigma}_T^2 = \sigma_T^2$ or

$$J_2^0 = \frac{\sigma_T^2}{2\,(1+\nu)}. \tag{14}$$

Allowing for in deviator surface the invariants of stress tensor coincide with the invariants of stress deviator and yield surface $\nu \approx 0,5$ from (4) we have

$$\sqrt{s_2} = \frac{\sigma_T}{\sqrt{3}} (15)$$

where s_2 is the second stress invariant. The condition (15) coincides with the Mizhes yield condition [2].

Thus the ideal plasticity holds on deviator surface. At last putting (12) to (10) for yield surface we obtain

$$\bar{J}_1^2 + 2\left(1+\nu\right)\bar{J} = \left(\frac{k_0+1}{2}\right)\left(J_1^{02} + 2\left(1+\nu\right)J_1^0\right).$$

Main results

1. The canonical equation of yield surface at any repeated loading for strengthening material considering Baushinger effect is obtained whose center is a point in the space of stresses corresponding to remainder stresses of previous active loading.

2. It is proved that in deviator surface, i.e. when $J_1 = 0$ there is no strengthening and the mechanical properties of materials change due to the pressure, i.e. due to the J_1.

References

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