Saftar A. HUSEYNOV, Rufat A. MAMEDZADE, Alekper Y.SHARIATI

NUMERICAL SOLUTION OF IDENTIFICATION PROBLEMS OF HEAT-EXCHANGE PROCESS IN OIL STRATA

Abstract

In the paper the inverse problem of determination of heat-conductivity and heat capacity coefficients of saturated fluid of porous medium, heat-exchange coefficient between stratum and environment is considered, the numerical algorithm of solution of the given problem is suggested.

By investigating nonisothermal filtration processes in oil stratums, inverse identification problems of parabolic equations arise. The problem of determination of coefficients of parabolic equations – heat-conductivity and heat-capacity coefficients of fluid saturation porous medium, heat-exchange coefficient between stratum and environment are of interest.

Heat-exchange process is considered in oil strata by nonisothermal radial filtration whose mathematical model has the form:

$$C(r)\frac{\partial u}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rk(r)\frac{\partial u}{\partial r}\right)+\frac{q(t)}{r}\frac{\partial u}{\partial r}-\alpha(t)u,$$

$$r_c<r<r_R,\ 0<t\le t_1\ ,\quad (1) \tag{1}$$

$$u(r,0)=f(r),\quad r_c\le r\le r_R, \tag{2}$$

$$\left(\delta_1 u-\delta_2 k\frac{\partial u}{\partial r}\right)_{r=r_c}=\upsilon(t),\quad 0\le t\le t_1 \tag{3}$$

$$\left(\delta_3 u+\delta_4 k\frac{\partial u}{\partial r}\right)_{r=r_R}=P(t),\quad 0\le t\le t_1. \tag{4}$$

In the range of the model (1)-(4), the inverse problems on determination of the functions $k(r), c(r),\ \alpha(r)$ and $u(r,t)$ under the additional conditions

$$u(r_n,t)=\varphi_n(t),\quad r_n\in(r_c,r_R),\ t\in[0,t_1],\ n=1,2,...,N \tag{5}$$

are posed.

For the approximated solution of the stated problem, we introduce the functional

$$J(k,c,\alpha)=\sum_{n=1}^{N}\int_0^{t_1}\left[u(r_n,t)-\varphi_n(t)\right]^2dt \tag{6}$$

and consider the minimization problem of the functional (6) under the conditions (1)-(4).

The gradient methods of optimization and finite-difference method are applied to numerical solution of inverse problems in the extremal statement (1)-(4), (5) which requires the solution of problems of determination of gradient of the functional (6) under the conditions (1)-(4).

For obtaining an expression for the gradient of the functional $\nabla J(k,c,\alpha)$ we use the method of increments [2].

We shall give arbitrary admissible increments $\Delta k(r), \Delta c(r),\ \Delta\alpha(r)$ by the functions $k(r), c(r),\ \alpha(r)$. Then $u(r,t)$ gets the increment satisfying the relations

$$\begin{gathered}\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial\Delta u}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\Delta k\frac{\partial u}{\partial r}\right)+\\+\frac{q}{r}\frac{\partial\Delta u}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\Delta k\frac{\partial\Delta u}{\partial r}\right)-\Delta\alpha\Delta u-\\-\Delta c\tfrac{\partial\Delta u}{\partial t}-\alpha\Delta u-\Delta\alpha u-c\tfrac{\partial\Delta u}{\partial t}-\Delta c\tfrac{\partial u}{\partial t}=0,\\r_c<r<r_R,\quad 0<t\le t_1,\end{gathered}\tag{7}$$

$$\begin{cases}\Delta u(r,0)=0,\quad r_c\le r\le r_e\ ,\\ \left(\delta_1\Delta u-\delta_2\left(\Delta k\frac{\partial u}{\partial r}+k\frac{\partial\Delta u}{\partial r}+\Delta k\frac{\partial\Delta u}{\partial r}\right)\right)_{r=r_c}=0\ ,\\ \left(\delta_3\Delta u+\delta_4\left(\Delta k\frac{\partial u}{\partial r}+k\frac{\partial\Delta u}{\partial r}+\Delta k\frac{\partial\Delta u}{\partial r}\right)\right)_{r=r_R}=0\ ,\quad 0\le t\le t_1\ .\end{cases}\tag{8}$$

At that the functional (6) gets the following increment

$$\Delta J=2\sum_{n=1}^{N}\int_0^{t_1}\left[u(r_n,t)-\varphi_n(t)\right]\Delta u(r_n,t)\ dt+\sum_{n=1}^{N}\int_0^{t_1}\left(\Delta u(r_n,t)\right)^2dt$$

or

$$\Delta J=2\sum_{n=1}^{N}\int_0^{t_1}\int_{r_c}^{r_R}\left(u(r,t)-\varphi_n(t)\right)\delta(r-r_n)\ \Delta u r dr dt+\sum_{n=1}^{N}\int_0^{t_1}\left(\Delta u(r_n,t)\right)^2dt,\tag{9}$$

where $\delta(r)$ is δ-fucntion.

We multiply the equation (7) by the function $\psi=\psi(r,t)$ and integrate along the domain $\bar{G}=\{r_c\le r\le r_R,\ 0\le t\le t_1\}$. Rejecting second order smallness terms and using the formula of integration by parts, we obtain

$$\begin{gathered}\int_0^{t_1}\int_{r_c}^{r_R}\left(\frac{\partial}{\partial r}\left(rk\frac{\partial\psi}{\partial r}\right)-q\frac{\partial\psi}{\partial r}-\alpha(t)\psi\ r+c\frac{\partial\psi}{\partial t}r\right)\Delta u dr dt+\\+\int_0^{t_1}rk\frac{\partial\Delta u}{\partial r}\psi\Bigg|_{r_c}^{r_R}-\int_0^{t_1}rk\frac{\partial\psi}{\partial r}\Delta u\Bigg|_{r_c}^{r_R}dt+\end{gathered}$$

$$+\int_0^{t_1} r\Delta k\frac{\partial u}{\partial r}\psi\Big|_{r_c}^{r_R}dt-\int_0^{t_1}\int_{r_c}^{r_R} r\Delta k\frac{\partial\psi}{\partial r}\frac{\partial u}{\partial r}drdt+\int_0^{t_1} q\psi\Delta u\Big|_{r_c}^{r_R}dt-$$

$$-\int_0^{t_1}\int_{r_c}^{r_R}\Delta\alpha u\psi rdrdt-\int_{r_c}^{r_R} c\psi r\Delta u\Big|_0^{t_1}dr-\int_0^{t_1}\int_{r_c}^{r_R}\Delta c\frac{\partial u}{\partial t}\psi rdrdt=0. \qquad (10)$$

Let the function $\psi(r,t)$ satisfies the conditions

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(rk(r)\frac{\partial\psi}{\partial r}\right)-\frac{q}{r}\frac{\partial\psi}{\partial r}-\alpha(t)\psi+c\frac{\partial\psi}{\partial t}+ \\ +2\sum_{n=1}^{N}\left(u(r,t)-\varphi_n(t)\right)\delta(r-r_n)=0, \\ r_c<r<r_R,\quad 0\le t<t_1, \end{cases} \qquad (11)$$

$$\begin{cases} \psi(r,t_1)=0\ ,\quad r_c\le r\le r_R; \\ \left(\delta_1 r\psi-\delta_2\left(q\psi-rk\frac{\partial\psi}{\partial r}\right)\right)_{r=r_c}=0\ ; \\ \left(\delta_3 r\psi+\delta_4\left(q\psi-rk\frac{\partial\psi}{\partial r}\right)\right)_{r=r_R}=0,\ 0\le t\le t_1 \end{cases} \qquad (12)$$

Allowing for (7)-(8) and (11)-(12) in (10), the increment of the functional (9) has the following form

$$\Delta J(k,\alpha,c)=\int_0^{t_1}\int_{r_c}^{r_R}\frac{\partial\psi}{\partial r}\frac{\partial u}{\partial r}\Delta krdrdt+\int_0^{t_1}\int_{r_c}^{r_R}\psi u\Delta\alpha rdrdt+$$

$$+\int_0^{t_1}\int_{r_c}^{r_R}\psi\frac{\partial u}{\partial t}\Delta crdrdt+\sum_{n=1}^{N}\int_0^{t_1}\Delta u(r_n,t)^2dt. \qquad (13)$$

The first three addends in the right hand side (13) form principal linear part of increment of functional [2]. Consequently

$$\nabla J=grad\ J=\left(\int_0^{t_1}\frac{\partial\psi}{\partial r}\frac{\partial u}{\partial r}rdt,\ \int_{r_c}^{r_R}\psi urdr,\int_0^{t_1}\psi\frac{\partial u}{\partial t}rdt\right)\equiv(J_k,J_\alpha,J_c). \qquad (14)$$

Thus, for calculation of gradient of the functional (6) at first it is necessary to solve the direct problem (1)-(4) then (11)-(12) and to use the formula (14).

Organization of iteration process of gradient method is realized by a standard way [2] if we give admissible initial approximations $k^{(0)}(r)$, $\alpha^{(0)}(t)$, $c^{(0)}(r)$. Step of gradient method is chosen from the condition of monotonically decreasing of the functional (6) by halving method. For numerical realization of algorithm, the difference method [3] is used on nonuniform grids which are constructed based on apriori information on properties of solutions [1].

Introduce the nonuniform grid

$$\overline{W_{ij}} = \left\{ (r_i, t_j) : r_{i+1} = r_i + h_{i+1},\ i = \overline{0, N-1};\ r_0 = r_c, \right.$$

$$\left. r_N = r_R; t_{j+1} = \overline{t_j + \tau_{j+1}},\ j = \overline{0, M-1};\ t_0 = 0,\ t_M = t_1 \right\}.$$

in the domain $\bar{G} = \{r_c \le r \le r_R,\ 0 \le t \le t_1\}$.

In view of the fact that large gradients of the functions $u(r,t)$, $\psi(r,t)$ are reached near the boundaries $r = r_c$ and $r = r_R$, by constructing nonuniform grid on space variable, the indicated property of solutions was taken into account.

We may approximate the boundary value problems (1)-(4) and (11)-(12) on the grid $\bar{W}_{ij}$ by the following difference problems :

$$\begin{cases} \Big(r_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \left(u_{i+1,j+1} - u_{i,j+1} \right) / h_{i+1} - \\ - r_{i-\frac{1}{2}} k_{i-\frac{1}{2}} \left(u_{i,j+1} - u_{i-1,j+1} \right) / h_i \Big) \times \\ \times \left(r_i \hbar_i \right) + q_j \left(u_{i+1,j+1} - u_{i,j+1} \right) / \left(r_i h_{i+1} \right) - \alpha_j u_{i,j+1} = \\ = c_i \left(u_{i,j+1} - u_{ij} \right) / \tau_{j+1}, i = 1, \overline{N-1}; j = \overline{0,\ M-1}, \\ u_{i,0} = f_i\ ,\quad i = \overline{0, N}\ ; \\ \delta_1 u_{0,j} - \delta_2 k_0 \left(u_{1,j} - u_{0,j} \right) / h_1 = \upsilon_j,\quad j = \overline{0, M}\ , \\ \delta_{31} u_{Nj} + \delta_4 k_N \left(u_{N,j} - u_{N-1,j} \right) / h_N = P_j\ ,\quad j = \overline{0, M}\ , \end{cases} \tag{15}$$

$$\begin{cases} \Big(r_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \left(\psi_{i+1,j} - \psi_{i,j} \right) / h_{i+1} - r_{i-\frac{1}{2}} k_{i-\frac{1}{2}} \left(\psi_{i,j} - \psi_{i-1,j} \right) / h_i \Big) / \\ / \left(r_i \hbar_i \right) - -q_j \left(\psi_{i,j} - \psi_{i-1,j} \right) / \left(r_i h_i \right) - \alpha_{j+1} \psi_{i,j} + \\ + c_i \left(\psi_{i,j+1} - \psi_{i,j} \right) / \tau_{j+1} + 2 \sum\limits_{n=1}^{N} \left(u_{ij} - \varphi_{nj} \right) \delta \left(r_i - r_n \right) = 0 \\ , i = \overline{1, N-1}; j = \overline{M-1, 0}, \psi_{i,M} = 0\ ,\quad i = \overline{0, N}\ ; \\ \delta_1 r_c \psi_{0,j} - \delta_2 \left(q_j \psi_{0,j} - r_c k_0 \left(\psi_{1,j} - \psi_{0,j} \right) / h_1 \right) = 0\ , \\ \delta_3 r_R \psi_{N,j} + \delta_4 \left(q_j \psi_{N,j} - r_N k_R \left(\psi_{N,j} - \psi_{N-1,j} \right) / h_N \right) = 0, \end{cases} \tag{16}$$

where

$$u_{i,j} = u(r_i, t_j), \ \psi_{i,j} = \psi(r_i, t_j),$$

$$\delta(r_i - r_n) = \begin{cases} \dfrac{1}{h_i}, & \text{if} \quad r_i = r_n, \\ 0, & \text{if} \quad r_i \neq r_n . \end{cases}$$

The schemes (15) and (16) are linear with respect to the values u and ψ on the new layer and they are solved in each iteration by sweep method. For approximated calculation of integrals the quadrature method of trapezoids is used.

The numerical experiments in which at first the direct problems with the given coefficients $k^*(r)$, $\alpha^*(t)$, $c^*(r)$ were solved and it was accepted $\varphi_n(t) \equiv u^*(r_n, t)$ as the function $\varphi_n(t)$, were made. Further, the problems were solved by the suggested algorithm at chosen initial approximations $k^{(0)}(r)$, $\alpha^{(0)}(t)$, $c^{(0)}(r)$.

The results of munerical experiments showed that the suggested algorithm gives sufficiently exact (on functional) solutions of the problem (1)-(5) in a comparatively short time.

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Saftar I. Huseynov, Rufat A. Mamedzade, Alekper Y. Shariati
Azerbaijan State Oil Academy.
20, Azadlyg av.,AZ1601, Baku, Azerbaijan.
Tel.: 74-85-75(apt.).

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