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NUMERICAL METHOD OF ANALYSIS OF NONSTATIONARY PROCESSES IN ASMAIN PIPELINES

Abstract

In the paper the numerical method for analysis of nonstationary processes which occur in gas-main pipeline is assumed. The recurrence dependence based on the suggested method significantly facilities the solution of the stated problem in IBM.

One of most essential singularities which radically distinguishes gas-transport systems from the other subsystems of fuel and energy complex, is nonstationarity of processes proceeding in gas-main pipelines.

Therefore the studying of dynamics problems in gas-main pipeline is an actual problem.

In the general case, the isothermal nonstationary gas flow in gas-main pipelines with constant cross-sections is described by nonlinear system of partial differential equations [1,2]:

$$\begin{gathered}\frac{\partial}{\partial\chi}\left[(1+\beta)\,\rho\omega^2\right]+\frac{\partial(\rho\omega)}{\partial t}=-\frac{\partial P}{\partial\chi}-\rho q\frac{dz}{dx}-\lambda_0\frac{\pi\omega^2}{2D},\\ \frac{\partial(\rho\omega)}{\partial\chi}=-\frac{1}{c^2}\frac{\partial P}{\partial t}\end{gathered}\qquad(1)$$

where $P=P(\chi,t)$ is average pressure in cross-section, $\omega=\omega(\chi,t)$ is average velocity in cross-section: ρ is density of gas, λ_0 is a coefficient of pressure loss; D is inside diameter, q is acceleration of gravity, z is height on which the center of section χ; β is Koriolis correction on nonuniform velocity distribution on cross-section, c is acoustic speed in gas.

In the system of equations (1) because of small influence of the coefficient β it can be disregarded [1,2].

For horizontal gas pipeline we can represent the system of equations (1) in the following form:

$$\begin{gathered}-\frac{\partial P}{\partial\chi}=\frac{\partial(\rho\omega)}{\partial t}+\frac{\lambda_0}{2D}\omega^2,\\ -\frac{1}{c^2}\frac{\partial P}{\partial t}=\frac{\partial(\rho\omega)}{\partial\chi}\ .\end{gathered}\qquad(2)$$

As conducted analysis shows [1-6] if the length of gas pipeline is so large and pressure loss on friction exceed impact pressure by Zhukovsky formula not less than 3,5-4 times we can disregard the term $\dfrac{\partial(\rho\omega)}{\partial\chi}$.

By that we can represent the system of equations (2) in the following form:

$$-\frac{\partial P}{\partial \chi}=\frac{\lambda_0|\omega|}{2D}\rho\omega$$
$$-\frac{1}{c^2}\frac{\partial P}{\partial t}=\frac{\partial(\rho\omega)}{\partial\chi}\;. \qquad (3)$$

The exact analytical solution of the system (3) is not obtained yet therefore for the solution of the system of equations (3) we use numerical methods approximated analytical or method of relation of nonlinear equations to linear ones i.e. various methods of linearization.

I.A.Charniy [1] suggested four methods for linearization of flow equation. The widest spread occurrence got substitution of quadratic term $\frac{\lambda_0\rho\omega^2}{2D}$ by linear one $2a=\frac{\lambda_0\omega_{av}}{2D}$, $\omega_{av}=\frac{2}{3}\left(\frac{\omega_1^2+\omega_0\omega_1-2\omega_0^2}{\omega_1-\omega_0}\right)$ where ω_0,ω_1 are steady state value of average velocity corresponding before and after nonstationary process.

After such linearization the system of equations (3) is resulted to the thermal conductivity type equation with the constant coefficients:

$$=-\frac{\partial P}{\partial\chi}=2a\rho\omega,$$
$$-\frac{1}{c^2}\frac{\partial P}{\partial t}=\rho\frac{\partial\omega}{\partial\chi},\quad 0\le\chi\le l \qquad (4)$$

where l is the length of gas-main pipelines.

Thus for the solution of the stated problem it is required to solve the system of equations (4) under the accepted initial and boundary conditions.

using analytical methods [1,3-6], for this purpose, because of arbitrariness of boundary conditions for real system of gas-main principles [5,5], is connected with large mathematical difficulties.

At present the effective direction for the solution of dynamics problem in gas-main pipelines is to use numerical methods [5].

One of effective numerical method of analysis of nonstationary processes in gas-main pipelines is the method based on the theory of impulse systems [7]. By that discrete Laplace's transformation is used as a mathematical apparatus [8].

Under such approach by substituting initial continuous system with distributed parameters by impulse system equivalent to it, the operation of continuous integration may be substituted by summation only by method of rectangles.

However as conducted analysis shows in a number of cases in analysis of dynamic processes in gas-main pipelines the using of approach connected with substitution of operation of continuous integration by summation by rectangles method finally gives significant errors.

In the given paper the further development and generalization of numerical method [7] is given for analysis of nonstationary processes in gas-main pipelines at arbitrary law of variation of pressure at the beginning and velocity of gas flow at the end of pipeline.

The essence of suggested method is based on using discrete analogy of integral equation of convolution [5].

The advantage of the suggested approach is that it allows without passage to domain of discrete transforms to realize the passage from Laplacian transforms of desired functions in domain of originals, without determining the roots of characteristic equations, to substitute the operation of continuous integration by summation by trapezoid formula which significantly simplifies mathematical manipulations an increases the excatness of analyses.

Nonstationary processes proceeding in gas-main pipelines are described by partial differential equations of the form (4).

In the considered case in the system of differential equations (4) under $P(\chi, t)$, $\omega(\chi, t)$ their abundant values over the stationary ones are subtended.

From the conditions of statement of the problem the initial conditions are accepteed as zeros

$$P(\chi, t)_{t=0} = 0 \qquad \omega(\chi, t)_{t=0} = 0 \ .$$

The boundary conditions have the following form:

$$P(\chi, t)_{\chi=0} = P_H(t) \qquad \omega(\chi, t)_{\chi=0} = \omega_k(t)$$

where $P_H(t)$ is an arbitrary law of variation of pressure at the beginning of gas pipeline, $\omega_k(t)$ is arbitrary law of variation of speed at the end of gas pipeline.

By solving the stated problem at the first step it is necessary to obtain the Laplacian transform for the functions $P(\chi, t)$ and $\omega(\chi, t)$.

In this connection, under the mentioned initial and boundary conditions we obtain the expressions for the mentioned functions in the following operator form from the solutions of system of equations (4):

$$p(\chi, s) = \frac{ch\gamma(l-\chi)}{ch\gamma l} P_H(s) - b(s) \frac{ch\gamma\chi}{ch\gamma l} \omega_k(s), \tag{5}$$

$$\omega(\chi, s) = \frac{1}{b(s)} \frac{ch\gamma(l-\chi)}{ch\gamma l} P_H(s) + \frac{ch\gamma\chi}{ch\gamma l} \omega_k(s), \tag{6}$$

where $\gamma = \sqrt{k_1 k_2}\sqrt{s}$ is operator constant wave propagation.

$b(s) = \sqrt{\frac{k_1}{k_2}\frac{1}{\sqrt{s}}}$ is operator wave resistance of gas pipeline.

s is an operator of Laplace transformation, $\omega(\chi, s), p(\chi, s), P_H(s), \ \omega_k(s)$ is a Laplacian transform of the functions $\omega(\chi, t), p(\chi, t), P_H(t), \ \omega_k(t); \ k_1 = 2a\rho, \ k_2 = \frac{1}{\rho c^2}$.

The second step of solution of the given problem is connected with realization of passage from the transform (5), (6) to the domain of originals.

As the conducted analysis shows the further using of method [7] for passage from the transforms (5), (6) to the domain of originals by substituting integration operation with summation by trapezoid method becomes impossible.

By that unlike [7] in the given paper the other approach whose essence is in the following form, was suggested.

In expressions for the functions $p(\chi, s), \omega(\chi, s)$ from (5), (6) by passing from hyperbolic functions to power functions we obtain

$$P(s,\delta) = \frac{e^{-2\gamma l\delta} + e^{-2\gamma l(1-\delta)}}{1+e^{-2\gamma l}} P_{H(s)} - b(s)\frac{e^{-\gamma l(1-\delta)} - e^{-\gamma l(1+2\delta)}}{1+e^{-2\gamma l}}\omega_{k(s)}, \qquad (7)$$

where $\delta = \frac{\chi}{2l}$ is a constant coefficient

$$\omega(s,\delta) = \frac{1}{b(s)}\frac{e^{-2\gamma l\delta} - e^{-2\gamma l(1-\delta)}}{1+e^{-2\gamma l}} P_{H(s)} + \frac{e^{-\gamma l(1-2\delta)} + e^{-\gamma l(1+2\delta)}}{1+e^{-2\gamma l}}\omega_{k(s)}. \qquad (8)$$

We can represent the expressions (7), (8) in the following form

$$P(\delta,s)\left[\frac{1}{s} + k_1'(s)\right] = \left[k_2'(s) + k_3'(s)\right] P_H(s) - k_1'\left[k_4'(s) - k_5'(s)\right]\omega_k(s), \qquad (9)$$

$$\omega(\delta,s)\left[\frac{1}{s} + k_1'(s)\right] = \frac{1}{k_1'}\left[k_6'(s) - k_7'(s)\right] P_H(s) + \left[k_8'(s) - k_9'(s)\right]\omega_k(s). \qquad (10)$$

$$k_1'(s) = \tfrac{1}{s}e^{-R\sqrt{s}}, \qquad k_2'(s) = \tfrac{1}{s}e^{-R\delta\sqrt{s}}, \qquad k_3'(s) = \tfrac{1}{s}e^{-R(1-\delta)\sqrt{s}},$$

$$k_4'(s) = \tfrac{1}{s\sqrt{s}}e^{-\frac{R}{2}(1-2\delta)\sqrt{s}}, \qquad k_5'(s) = \tfrac{1}{s\sqrt{s}}e^{-\frac{R}{2}(1+2\delta)\sqrt{s}}, \qquad k_6'(s) = \tfrac{\sqrt{s}}{s}e^{-R\delta\sqrt{s}},$$

$$k_7'(s) = \tfrac{\sqrt{s}}{s}e^{-R(1-\delta)\sqrt{s}}, \qquad k_8'(s) = \tfrac{1}{s}e^{-\frac{R}{2}(1-2\delta)\sqrt{s}}, \qquad k_9'(s) = \tfrac{1}{s}e^{-\frac{R}{2}(1+2\delta)\sqrt{s}}$$

$$R = 2\sqrt{k_1k_2l}, \qquad k_1' = \sqrt{\frac{k_1}{k_2}}.$$

Based on convolution theorem by passing from the equations (9), (10) with respect to transforms to the equations with respect to originals we obtain

$$\int_0^t P(t-\theta,\delta)\,l(\theta)\,d\theta + \int_0^t P(t-\theta,\delta)\,k_1'(\theta)\,d\theta =$$

$$= \int_0^t P_H(t-\theta,\delta)\,k_2'(\theta)\,d\theta + \int_0^t P_H(t-\theta,\delta)\,k_3'(\theta)\,d\theta - \qquad (11)$$

$$-\int_0^t \omega_k(t-\theta,\delta)\,k_4'(\theta)\,d\theta + k_1'\int_0^t \omega_k(t-\theta,\delta)\,k_5'(\theta)\,d\theta$$

$$\int_0^t \omega(t-\theta,\delta)\,l(\theta)\,d\theta + \int_0^t \omega(t-\theta,\delta)\,k_1'(\theta)\,d\theta =$$

$$= \tfrac{1}{k_1'}\int_0^t \omega_{1H}(t-\theta,\delta)\,k_6'(\theta)\,d\theta - \tfrac{1}{k_1'}\int_0^t \omega_{1H}(t-\theta,\delta)\,k_7'(\theta)\,d\theta + \qquad (12)$$

$$+\int_0^t \omega_k(t-\theta,\delta)\,k_8'(\theta)\,d\theta + \int_0^t \omega_k(t-\theta,\delta)\,k_9'(\theta)\,d\theta$$

The integral equations (11), (12) may be solved numerically if we substitute the integrals by sums.

In this connection by using the relation between continuous time t and discrete n $(n = 0, 1, 2, ...)$ in the form $t = nT/\lambda$ (where $T = 2\tau$, $\tau = 1/c$ time of wave propagation, λ is any integer) we make discretization of the equations (11), (12) at chosen interval T/λ by substituting the operation of continuos integration by summation by trapezoid formulae.

By that instead of (11), (12) we obtain

$$\begin{aligned}
&+\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(P\left[n-m,\delta\right]k_1'\left[m\right]+k_1'\left[n-m+1\right]P\left[m-1,\delta\right]\right)=\\
&=\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(P_H\left[n-m\right]k_2'\left[m\right]+k_2'\left[n-m+1\right]P_H\left[m-1\right]\right)+\\
&+\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(P_H\left[n-m\right]k_3'\left[m\right]+k_3'\left[n-m+1\right]P_H\left[m-1\right]\right)-\\
&-k_1'\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}k_4'\left[m\right]\omega_k\left(\left[n-m\right]+k_4'\left[n-m+1\right]\omega_k\left[m-1\right]\right)+\\
&+k_1'\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(\omega_k\left[n-m\right]+k_5'\left[m\right]+k_5'\left[n-m+1\right]\omega_k\left[m-1\right]\right)
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
&\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(\omega\left[n-m,\delta\right]l\left[m\right]+1\left[n-m+1\right]\omega\left[m-1,\delta\right]\right)+\\
&+\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(\omega\left[n-m\right]k_1'\left[m\right]+k_1'\left[n-m+1\right]P_H\left[m-1,\delta\right]\right)=\\
&=\tfrac{1}{k_1'}\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(P_H\left[n-m\right]k_6'\left[m\right]+k_6'\left[n-m+1\right]P_H\left[m-1\right]\right)-\\
&-\tfrac{1}{k_1'}\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(P_H\left[n-m\right]k_7'\left[m\right]+k_7'\left[n-m+1\right]P_H\left[m-1\right]\right)+\\
&+\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(k_8'\left[m\right]\omega_k\left[n-m\right]+k_8'\left[n-m+1\right]\omega_k\left[m-1\right]\right)+\\
&++\tfrac{1}{2}\tfrac{T}{\lambda}\sum_{m=0}^{n}\left(\omega_k\left[n-m\right]k_9'\left[m\right]+k_9'\left[n-m+1\right]\omega_k\left[m-1\right]\right)\ ,
\end{aligned} \tag{14}$$

$$k_1'\left[n\right]=e\ r\ f\ c\frac{R}{2\sqrt{\frac{nT}{\lambda}}}\qquad k_2'\left[n\right]=e\ r\ f\ c\frac{R\delta}{2\sqrt{\frac{nT}{\lambda}}}\qquad k_3'\left[n\right]=e\ r\ f\ c\frac{R\left(1-\delta\right)}{2\sqrt{\frac{nT}{\lambda}}}$$

$$k_4'\left[n\right]=2\sqrt{\frac{nT}{\lambda}}ie\ r\ f\ c\frac{R\left(1-2\delta\right)}{2\sqrt{\frac{nT}{\lambda}}},\qquad k_5'\left[n\right]=2\sqrt{\frac{nT}{\lambda}}ie\ r\ f\ c\frac{R\left(1+2\delta\right)}{2\sqrt{\frac{nT}{\lambda}}},$$

$$k_6'[n] = \frac{e^{\frac{-(R\delta)^2}{4n\frac{T}{\lambda}}}}{2\sqrt{\pi n\frac{T}{\lambda}}}, \qquad k_7'[n] = \frac{e^{-\frac{R(1-2\delta)}{4n\frac{T}{\lambda}}}}{\sqrt{\pi n\frac{T}{\lambda}}},$$

$$k_8'[n] = e\ r\ f\ c\frac{R(1-2\delta)}{4\sqrt{\frac{nT}{\lambda}}}\ , \qquad k_9'[n] = e\ r\ f\ c\frac{R(1-2\delta)}{4\sqrt{\frac{nT}{\lambda}}}\ ,$$

$$e\ r\ f\ t = \frac{2}{\sqrt{\pi}}\int_0^t e^{-t_1^2}dt_1 \text{ is Gauss error function, integral}$$

$$\int_0^t e\ r\ f\ \ c\ t\ dt = i\ e\ r\ f\ c\ t = \frac{1}{\sqrt{\pi}} - t\ e\ r\ f\ c\ t$$

Here

$$\sum_{m=0}^{n}\left(P[n-m,\delta]\,l[m] + l[n-m+1]\,P[m-1,\delta]\right) = P(n,\delta) + \\ + \sum_{m=1}^{n}\left(P[n-m,\delta]\,l[m] + l[n-m+1]\,P[m-1,\delta]\right), \tag{15}$$

$$\sum_{m=0}^{n}\left(p[n-m,\delta]\,k_1[m] + k_1'[n-m+1]\,p[m-1,\delta]\right) = \\ = \sum_{m=1}^{n}\left(p[n-m,\delta]\,k_1'[m] + k_1'[n-m+1]\,p[m-1,\delta]\right), \tag{16}$$

Since at $m=0$ $P[m-1,\delta]\equiv 0,\ k_1'[0] = e\ r\ f\ c(\infty) = 1 - e\ r\ f(\infty) = 1-1 = 0.$ The expression (13) subject to (15), (16) will be

$$P[n,\delta] + \sum_{m=1}^{n}\left(P[n-m,\delta]\,l[m] + l[n-m+1]\,P[m-1,\delta]\right) + \\ + \sum_{m=1}^{n}\left(p[n-m,\delta]\,k_1'[m] + k_1'[n-m+1]\,p[m-1,\delta]\right) = \\ = \sum_{m=0}^{n}\left(P_H[n-m]\,k_2'[m] + k_2'[n-m+1]\,P_H[m-1]\right) + \\ + \sum_{m=0}^{n}\left(P_H[n-m]\,k_3'[m] + k_3'[n-m+1]\,P_H[m-1]\right) - \\ - k_1'\sum_{m=0}^{n} k_4'[m]\,\omega_K\left([n-m]\,k_4'[n-m+1]\,\omega_k[m-1]\right) + \\ + k_1'\sum_{m=0}^{n}\left(\omega_k[n-m]\,k_5'[m] + k_5'[n-m+1]\,\omega_k[m-1]\right) \tag{17}$$

Having solved the equation (17) with respect to the lattice function $P[n,\delta]$ we obtain the following recurrence relation allowing to calculate the function $P[n,\delta]$ sequentially

$$\begin{aligned}P[n,\delta] = \sum_{m=1}^{n} \left(P_H[n-m]\,k_2'[m] + k_2'[n-m+1]\,P_H[m-1]\right) + \\ + \sum_{m=0}^{n} \left(P_H[n-m]\,k_3'[m] + k_3'[n-m+1]\,P_H[m-1]\right) - \\ -k_1' \sum_{m=0}^{n} k_4'[m]\,\omega_K\left([n-m] + k_4'[n-m+1]\,\omega_k[m-1]\right) + \\ +k_1' \sum_{m=0}^{n} \left(\omega_k[n-m]\,k_5'[m] + k_5'[n-m+1]\,\omega_k[m-1]\right) - \\ - \sum_{m=1}^{n} \left(P[n-m,\delta]\,l[m] + l[n-m+1]\,P[m-1,\delta]\right) - \\ - \sum_{m=1}^{n} \left(p[n-m,\delta]\,k_1'[m] + k_1'[n-m+1]\,p[m-1,\delta]\right)\ .\end{aligned} \tag{18}$$

The error of analysises is connected with the quantity λ. Then the more of number λ is chosen, the less the characteristics of the continuos function differ from corresponding characteristics of lattice functions.

Analogously, the expression for the function $\omega(\chi,t)$ accept the following form

$$\begin{aligned}+ \sum_{m=0}^{n} \left(k_8'[m]\,\omega_K[n-m] + k_8'[n-m+1]\,\omega_k[m-1]\right) + \\ + \sum_{m=0}^{n} \left(\omega_k[n-m]\,k_9'[m] + k_9'[n-m+1]\,\omega_k[m-1]\right) - \\ - \sum_{m=1}^{n} \left(\omega[n-m,\delta]\,l[m] + l[n-m+1]\,\omega[m-1,\delta]\right) + \\ + \sum_{m=1}^{n} \left(\omega[n-m,\delta]\,k_1'[m] + k_1'[n-m+1]\,\omega[m-1,\delta]\right)\end{aligned} \tag{19}$$

The obtained recurrence relations (18), (19) as the final result allow significantly to raise the accuracy of analysises by defining the change of pressure velocity at any point of gas-main pipeline at any moment and it easily realized in computer.

According to the obtained algorithms the analysis of nonstationary process in gas main pipelines, when at $t = 0$ at the beginning of the pipeline the pressure jumps, while the velocity at the end of the pipeline is remained fixed.

The parameters of gas pipeline

$$P_H(t) = 0,1\ MPa, \quad l = 100\ km, \quad D = 1m, \quad \lambda_0 = 0,01,$$
$$\omega_{cp} = 10\ m/sek, \quad c = 362\ m/sek, \quad \rho = 0,75\ kg/m^3\ .$$

The analyses were led at $\lambda = 20$.

The results of analyses with respect to pressure for the section $\chi = 50\ km$ are cited in Fig.1 (curve 1), in the same place for comparison of results of the given example (curve 2) obtained by the analytical way [1] are cited. It is obvious from comparison of cited results that the suggested numerical method at chosen step gets good coincidence with exact solution.

The results of the given paper may be used both in projection and in exploitation of gas-main pipelines [2,6].

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