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NUMERICAL METHOD OF SOLUTION OF WATER-OIL DISPLACEMENT PROBLEM IN THE PRESENCE OF SEVERAL OIL- DRAINAGE BOUNDARIES IN BOUNDED SPACE

Abstract

In the paper filtration process of two fluids with several moving boundaries in bounded space is considered, numerical method of solution of the given problem is suggested.

The papers [1-3] are devoted to motion equation of boundary surface of two immiscible fluids in porous medium under the spatial filtering where formula of surface S is given in implicit form at any moment of time. As distinct from these papers, in [4] formula of surface S is given in parametric form, and process of space filtering of two fluids is reduced to solving the system of equations. In the paper [5] process of space filtering of two fluids with several moving boundaries is considered.

Let at initial moment of time $t = 0$ there be two fluids- displaced and displacing ones in bounded isotropic bed of thickness H, permeability k and porosity m. The displaced fluid occupies simply connected domains $D_i(0)$ bounded above and below by impermeable horizontal planes- top and subface of the bed and by circumference- by smooth surfaces $S_i(t)$ $(i = 1, ..., N)$, and the displacing fluid occupies the rest part of bed bounded by surface S_0. Displaced fluid viscosity being in domain $D_i(t)$ will be denoted by μ_i and viscosity of displacing fluid- by μ_0. The total amount of wells at the moment of time t will be denoted by $M_2(t)$ $(M_2(t) > 0)$. We will denote the coordinates of the outlet well by (x_j, y_j, z_j) and its well production- by $Q_j(t)$ $(j = 1, 2, ..., M_1(t))$. The coordinates of production wells situated in domain $D_i(t)$ will be denoted by (x_{ij}, y_{ij}, z_{ij}), well production- by $Q_{ij}(t)$ $(i = 1, 2, ..., N_j;\ j = 1, 2, ..., l_i(t))$ ..

If we will assume that rock and fluids saturating it, are incompressible, filtration is laminar and obeys the Darcy's law, the system of equations of isothermal filtration of fluids without taking the capillary gravitational forces into account is reduced to Laplace equation for defining $P(x, y, z)$ in each of domains $D_i(t)$ and D_0 :

$$\Delta P_i = 0;\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (i = 1, 2, ..., N);(1) \tag{1}$$

(x, y, z) are Cartesian coordinates.

This equation holds in a whole filtration domain except of singular points (wells), boundary surfaces of phases $S_i(t)$ and, generally, at infinite point.

Let $\forall t \in [0, T]$ boundary surfaces of phases $S_i(t)$ have not common points, and each of them represents closed oriented non-self- intersecting smooth surface given by parametric equation

$$x_i = x_i(u, v),\ y_i = y_i(u, v),\ z_i = z_i(u, v). \tag{2}$$

For any $t \in [0, T]$, the following condition is satisfied on the boundary surface of phases $S_i(t)$:

I. By passing through the surface $S_i(t)$, pressure continuously varies

$$P^+(x, y, z, t) = P^-(x, y, z, t), \tag{3}$$

P^+ is a limiting pressure by approaching to $S_i(t)$ from within, and P^-- by approaching to $S_i(t)$ from the outside.

II. The flow continuously varies by passing through the surface $S_i(t)$

$$c_i \frac{\partial P^+(x, y, z, t)}{\partial n} = c_0 \frac{\partial P^-(x, y, z, t)}{\partial n}. \tag{4}$$

Here n is a normal to the boundary surface of phases by passing from domain $D_i(t)$ into domain D_0, $c_i = \frac{k}{\mu_i}$, μ_i is a viscosity of displaced phase, $c_0 = \frac{k}{\mu_0}$, μ_0 is a viscosity of displacing phase.

The following condition is given on the surface S_0

$$P(x, y, z, t) = P_0(x, y, z, t), \ (x, y, z) \in S_0, \ t \in [0, T]. \tag{5}$$

We will seek the pressure distribution $P(x, y, z, t)$ in the form

$$\begin{gathered} P(x, y, z, t) = \varphi(x, y, z, t) + \sum_{i=1}^{N} \int_{S_i(t)} \rho(\xi_i(\delta), \ \eta_i(\delta), \ \varsigma_i(\delta), t) R_i^{-1} dS_{\theta_i} + \\ + \int_{S_0} \mu(\xi_i'(\delta), \ \eta_i'(\delta), \ \varsigma_i'(\delta), t) R_1^{-1} dS_{\theta_0'}, \end{gathered} \tag{6}$$

where ρ_i is Newtonian potential of simple fiber at the point $\theta_i(\xi_i(\delta), \ \eta_i(\delta), \ \varsigma_i(\delta))$, being continuously distributed on $S_i(t)$, μ_i is a density of Newtonian potential of simple fiber being continuously distributed on S_0,

$$R_i = \left((x - \xi_i)^2 + (y - \eta_i)^2 + (z - \varsigma_i)^2 \right)^{\frac{1}{2}},$$

$$\varphi(x, y, z, t) = (2\pi H c_0)^{-1} \sum_{j=1}^{M_1} \theta_j(t) R_j^{-1} + (2\pi H)^{-1} \sum_{i=1}^{N} c_i^{-1} \sum_{j=1}^{l_i(t)} \theta_{ij} R_{ij}^{-1},$$

$$R_{ij} = \left((x - x_{ij})^2 + (y - y_{ij})^2 + (z - z_{ij})^2 \right)^{\frac{1}{2}},$$

(x_{ij}, y_{ij}) are coordinates of the j-th well, situated on domain $D_i(t)$.

It is obvious that function $P(x, y, z, t)$ satisfies the Laplace equation, and it is continuous on the boundary.

By using Darcy's law and condition (4) we can write the traverse speed of surface $S_i(t)$ along a normal line to it in the form

$$v_{in} = -\frac{c_i}{m} \frac{\partial P^+}{\partial n} = \frac{c_0}{m} \frac{\partial P^-}{\partial n}.$$

Based on the property of potential of simple fiber, (5) implies

$$\rho = \left(\frac{1}{c_i} - \frac{1}{c_0} \right) \frac{m}{2\pi} v_{in}. \tag{7}$$

Limiting value of normal derivative of pressure by approaching to $S_i(t)$ from within is equal to

$$\frac{\partial P^+}{\partial n}=\frac{\partial \varphi}{\partial n}-\pi\rho+\sum_{i=1}^{N}\int\limits_{S_i(t)}\rho\frac{\partial}{\partial n}R_i^{-1}dS_Q+\frac{\partial}{\partial n}\int\limits_{S_0}\mu R_i^{-1}dS_{Q'}, \tag{8}$$

then using (6), (7) for defining v_{in}, we will obtain the following integral equation

$$v_{in}-a\frac{c_0-c_i}{2\pi c_0}\sum_{i=1}^{N}\int\limits_{S_i(t)}v_{in}\frac{\partial}{\partial n}R_i^{-1}dS_{\theta_i}-a\frac{c_i}{m}\frac{\partial}{\partial n}\int\limits_{S_0}\mu R_1^{-1}dS_{\theta'}=a\frac{c_i}{m}\frac{\partial\varphi}{\partial n}, \tag{9}$$

where $a=\frac{c_ic_o}{(c_0-c_i)m-c_ic_0}$.

Using the property of Newtonian potential of a simple fiber and condition (5) for calculating μ, we will obtain

$$\mu=\frac{1}{\pi}\left(P_0-\varphi-\frac{m}{2\pi}\frac{c_0-c_i}{c_ic_0}\sum_{i=1}^{N}\int\limits_{S_i(t)}v_{in}R_i^{-1}dS_{\theta_i}-\int\limits_{S_0}\mu\frac{\partial}{\partial n_1}R_1^{-1}dS_{\theta'}\right). \tag{10}$$

Motion equation of boundary surface of two fluids $S_i\ (t)$ will be defined from the system

$$\frac{\partial x_i}{\partial t}=\pm v_{in}\frac{\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial y_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial y_i}{\partial v}\end{vmatrix}}{W}, \tag{11}$$

$$\frac{\partial y_i}{\partial t}=\pm v_{in}\frac{\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial x_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial x_i}{\partial v}\end{vmatrix}}{W}, \tag{12}$$

$$\frac{\partial z_i}{\partial t}=\pm v_{in}\frac{\begin{vmatrix}\frac{\partial y_i}{\partial u} & \frac{\partial x_i}{\partial u}\\ \frac{\partial y_i}{\partial v} & \frac{\partial x_i}{\partial v}\end{vmatrix}}{W}, \tag{13}$$

where

$$W=\left(\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial x_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial x_i}{\partial v}\end{vmatrix}^2+\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial y_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial y_i}{\partial v}\end{vmatrix}^2+\begin{vmatrix}\frac{\partial y_i}{\partial u} & \frac{\partial x_i}{\partial u}\\ \frac{\partial y_i}{\partial v} & \frac{\partial x_i}{\partial v}\end{vmatrix}^2\right)^{\frac{1}{2}}.$$

Development of the motion equation of surface $S_i(t)$ in more detail was given in the paper [4].

Thus, problem on motion of several boundary surfaces of fluid in bounded space is reduced to solving system of equations (9)- (13).

Consider operator

$$\overline{f}(\theta)=\int\limits_{S(t)}v_n\frac{\partial}{\partial n}R^{-1}dS_\theta.) \tag{14}$$

In order to construct a cubature formula for Newtonian potential of simple fiber, the technique of investigation of singular integral operators with continuous density is used.

We choose a positive direction on surface $S(t)$. Suppose that on the surface $S(t)$ a finite number τ of partial surfaces is given. Using the finite- difference method, these surfaces may be curvilinear triangles. Let us enumerate curvilinear triangles on the surface $S(t)$ starting with any of them.

The obtained set

$$\tau = \{\theta_0, \theta_1, ..., \theta_N\}, \ \theta_i \in S(t), \ (i = 0, 1, ..., N)$$

will be called τ partition of the surface $S(t)$.

By using the first order interpolation polynomial for each of curvilinear triangles of surface $S(t)$, we will obtain the following two- parameter cubature formula for operator (14)

$$\overline{f}(\theta) = L(v_n, \theta, \varepsilon) \approx$$

$$\approx \frac{1}{\pi}\sum_{i=1}^{N}\int_{\theta} v_n\left(\theta\left(x_i, y_i, z_i; x_{i+1}, y_{i+1}, z_{i+1}; x_{i+2}, y_{i+2}; z_{i+2}\right)\right)\frac{\partial}{\partial n}R_i^{-1}d\theta_i, \tag{15}$$

where $R_i = \left((x - x_{i+1})^2 + (y - y_{i+1})^2 + (z - z_{i+1})^2\right)^{\frac{1}{2}}$, $d\theta_i$ is area of θ_i- th curvilinear triangle. Let us split the segment $[0, T]$ by the points $0 = t_0 < t_1 < ... < t_{q-1} < t_q = T$. By using cubature formula (15) and also implicit difference scheme for approximate solving of problem (9)- (13) at the moment $t = t_k$ $(k = 1, 2, ..., q)$, we will obtain the nonlinear system for which the following iteration process is used for each $t = t_k$

$$v_{jk} - a\sum_{j=1}^{N}\sum_{i=1}^{N_1}\tfrac{c_0 - c_{ik}}{2\pi c_0}\delta_{ik}\left(v_{jk} - v_{ik}\right)\left((x_{i+lk} - x_{ik}) + (y_{i+lk} - y_{ik}) + (z_{i+lk} - z_{ik})\right)\times$$

$$\times\left((x_{i+lk} - x_{ik})^2 + (y_{i+lk} - y_{ik})^2 + (z_{i+lk} - z_{ik})^2\right)^{\frac{1}{2}}\Delta\theta_{ik} - \quad ,$$

$$-a\tfrac{c_{ik}}{m}\sum_{l=1}^{N_2}\left(\mu_{jk} - \mu_{lk}\right)\left(\left(x'_{i+lk} - x'_{lk}\right)^2 +\right.$$

$$\left.+\left(y'_{i+lk} - y'_{lk}\right)^2 + \left(z'_{i+lk} - z'_{lk}\right)^2\right)^{\frac{1}{2}}\Delta\theta'_{lk} = a\frac{c_{ik}}{m}\frac{\partial\varphi}{\partial n} \tag{16}$$

$$\mu_{jk} = \tfrac{1}{\pi}\left(P_0 - \varphi_{jk} - \tfrac{m}{2\pi}\sum_{i=1}^{N}\tfrac{c_0 - c_{ik}}{c_{ik}c_0}\sum_{i=1}^{N_1}v_{lk}R_{lk}^{-1}\Delta\theta_{lk} +\right.$$

$$+\sum_{q=1}^{N_2}\left(\mu_{jk} - \mu_{qk}\right)\left(\left(x'_{q+lk} - x'_{qk}\right)^2 + \left(y'_{q+lk} - y'_{qk}\right)^2 + \left(z'_{q+lk} - z'_{qk}\right)^2\right)\times \quad , \tag{17}$$

$$\left.\left(\left(x'_{q+lk} - x'_{qk}\right)^2 + \left(y'_{q+lk} - y'_{qk}\right)^2 + \left(z'_{q+lk} - z'_{qk}\right)^2\right)^{\frac{1}{2}}\Delta\theta'_{qk}\right)$$

$$x_{jk}=x_{jk-1}\pm v_{jk}\frac{\begin{vmatrix}\frac{\partial z_{jk}}{\partial u} & \frac{\partial y_{jk}}{\partial u}\\ \frac{\partial z_{jk}}{\partial v} & \frac{\partial y_{jk}}{\partial v}\end{vmatrix}}{W}\Delta t_k, \qquad (18)$$

$$y_{jk}=y_{jk-1}\pm v_{jk}\frac{\begin{vmatrix}\frac{\partial z_{jk}}{\partial u} & \frac{\partial x_{jk}}{\partial u}\\ \frac{\partial z_{jk}}{\partial v} & \frac{\partial x_{jk}}{\partial v}\end{vmatrix}}{W}\Delta t_k, \qquad (19)$$

$$z_{jk}=z_{jk-1}\mp v_{jk}\frac{\begin{vmatrix}\frac{\partial y_{jk}}{\partial u} & \frac{\partial x_{jk}}{\partial u}\\ \frac{\partial y_{jk}}{\partial v} & \frac{\partial x_{jk}}{\partial v}\end{vmatrix}}{W}\Delta t_k. \qquad (20)$$

Let we have r-th approximation of unknown functions $x_{jk}, y_{jk},\ z_{jk},\ v_{jk}, \mu_{jk}$ $(j=0,1,...,n_k, r>0)$. As an initial approximation of unknown functions their values on previous time layer are taken. If we substitute instead of unknown functions their values at the r-th approximation into the right- hand side of equations of the system, we will define $x_{jk}, y_{jk},\ z_{jk},$ $(r+1)$- th approximation. Then by using the obtained approximation $x_{jk}, y_{jk},\ z_{jk}$, we define v_{jk}, μ_{jk} by the method of successive approximations, moreover, those values of v_{jk}, μ_{jk} are recognized as the $(r+1)$- th approximation for which the conditions

$$\max_j \left| v_{jk}^{(P_r)} - v_{jk}^{(P_{r-1})} \right| < \bar{\varepsilon}_1,$$

$$\max_j \left| \mu_{jk}^{(P_r)} - \mu_{jk}^{(P_{r-1})} \right| < \bar{\varepsilon}_2,$$

are fulfilled, where $v_{jk}^{(P_r)}, \mu_{jk}^{(P_r)}$ are values of v_{jk}, μ_{jk} on the p-th iteration calculated for $x_{jk}, y_{jk},\ z_{jk}$; $\bar{\varepsilon}_1, \bar{\varepsilon}_2$ are given positive numbers.

If

$$\max_j \left| x_{jk}^{(r+1)} - x_{jk}^{(r)} \right| < \bar{\varepsilon}_3,$$

$$\max_j \left| y_{jk}^{(r+1)} - y_{jk}^{(r)} \right| < \bar{\varepsilon}_4,$$

$$\max_j \left| z_{jk}^{(r+1)} - z_{jk}^{(r)} \right| < \bar{\varepsilon}_5$$

then $x_{jk}^{(r+1)}, y_{jk}^{(r+1)}, z_{jk}^{(r+1)}, v_{jk}^{(r+1)}, \mu_{jk}^{(r+1)}$ are recognized as an approximate value of function at the moment t_k. Otherwise, the iteration process continues.

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