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COERCIVE ESTIMATE FOR DEGENERATED ELLIPTIC-PARABOLIC OPERATORS OF THE SECOND ORDER

Abstract

In this work a class of degenerated elliptic-parabolic operators of the second order of non-divergent structure with, generally speaking, discontinuous coefficients, is considered. For these operators the coercive estimate in appropriate Sobolev space is established.

Let $\mathbb{E}_n$ be an n-dimensional Euclidean space of points $x=(x_1,...,x_n)$, Ω be a bounded domain in $\mathbb{E}_n$ with a boundary $\partial\Omega$, $\partial\Omega\in C^2$, Q_T be a cylinder $\Omega\times(0,T)$, where $T\in(0,\infty)$.

Let's consider in Q_T the first boundary- value problem

$$Lu=\sum_{i,j=1}^{n}a_{ij}(x,t)\,u_{ij}+\psi(x,t)\,u_{tt}-u_t=f(x,t)\,, \tag{1}$$

$$u|_{\Gamma(Q_T)}=0, \tag{2}$$

where for $i,j=\overline{1,n}$ $u_{ij}=\frac{\partial^2u}{\partial x_i\partial x_j}$, $u_i=\frac{\partial u}{\partial x_i}$, $u_{it}=\frac{\partial^2u}{\partial x_i\partial t}$, $\Gamma(Q_T)==(\partial\Omega\times[0,T])\cup(\Omega\times\{(x,t):t=0\})-$ parabolic boundary of Q_T, and

$$\psi(x,t)=\lambda(\rho)\,\omega(t)\,\varphi(T-t)\,,\quad \rho=\rho(x)=dist\,(x,\partial\Omega)\,.$$

Assume that for coefficients of the operator L the following conditions are fulfilled: $\|a_{ij}(x,t)\|$ is a real symmetrical matrix with elements measurable in Q_T, and for every $(x,t)\in Q_T$ and $\xi\in\mathbb{E}_n$ the inequalities are true:

$$\gamma\,|\xi|^2\le\sum_{i,j=1}^{n}a_{ij}(x,t)\,\xi_i\xi_j\le\gamma^{-1}\,|\xi|^2\,, \tag{3}$$

where γ is a constant from a semiinterval $(0,1]\,,$

$$\sigma=\sup_{Q_T}\sum_{i,j=1}^{n}a_{ij}^2(x,t)\Big/\inf_{Q_T}\left(\sum_{i=1}^{n}a_{ii}(x,t)\right)^2<\frac{1}{n-1}; \tag{4}$$

$$\lambda(\rho)\ge0,\quad\lambda(\rho)\in C^1[0,diam\;\Omega]\,,\quad\text{where}\quad\left|\lambda'(\rho)\right|\le p\sqrt{\lambda(\rho)}; \tag{5}$$

$$\omega(t)\ge0,\qquad\omega(t)\in C^1[0,T]\,; \tag{6}$$

$$\varphi(z)\ge0,\quad\varphi'(z)\ge0,\quad\varphi(z)\in C^1[0,T]\,,\ \varphi(0)=\varphi'(0)=0, \tag{7}$$

here p is a positive constant.

Condition (4) is called Cordes condition and is understood to within non-degenerated linear transformation in the following sense: the domain Q_T can be covered by a finite number of domains $Q^1, ..., Q^M$ so that in every Q^i such a non-degenerated linear transformation of coordinates exists that a matrix of senior coefficients of the operator L's image satisfies the condition (4) in the image of Q^i, $i = \overline{1, M}$.

The purpose of this work is to obtain a coercive estimate for the operator L in appropriate Sobolev space.

The obtained estimate can be used when proving a unique strong (almost everywhere) solvability of the first boundary-value problem (1)-(2) at every $f(x,t) \in L_2(Q_T)$.

The researches under the theory of degenerated elliptic-parabolic equations ascend to classical work by Keldysh [1], in which the correct statements of boundary-value problems for the equations of a kind (1) with one space variable were found. G.Fickera [2] has established a weak solvability of the first boundary-value problem for a wide class second order equations with the nonnegative characteristic form (see also [3]). As to a strong solvability of the first boundary-value problem for elliptic-parabolic equations in the nondivergent form with smooth coefficients, we shall note in this connection the works [4-6]. The similar result for the equations of a kind (1) in case of $\psi(x,t) = \varphi(T-t)$ under the condition close to Cordes condition is obtained in [7]. Let's also specify the works [8-11], where a unique strong solvability of the first boundary-value problem for elliptic and parabolic equations of the second order in nondivergent form with discontinuous coefficients is proved under the conditions of Cordes kind. Let's mention the works [12-13] where a weak solvability of the first boundary-value problem was investigated for elliptic-parabolic equations in divergent form. Note that the coercive estimate for $\lambda \equiv \omega \equiv 1$ was obtained in [7].

Let's introduce some denotation.

Let $W_2^{1,0}(Q_T)$, $W_2^{2,0}(Q_T)$, $W_2^{2,1}(Q_T)$ and $W_{2,\psi}^{2,2}(Q_T)$ be Banach spaces of functions $u(x,t)$ given on Q_T with finite norms

$$\|u\|_{W_2^{1,0}(Q_T)} = \left(\int\limits_{Q_T} \left(u^2 + \sum_{i=1}^n u_i^2 \right) dxdt \right)^{\frac{1}{2}},$$

$$\|u\|_{W_2^{2,0}(Q_t)} = \left(\int\limits_{Q_T} \left(u^2 + \sum_{i=1}^n u_i^2 + \sum_{i,j=1}^n u_{ij}^2 \right) dxdt \right)^{\frac{1}{2}},$$

$$\|u\|_{W_2^{2,1}(Q_T)} = \|u\|_{W_2^{2,0}(Q_T)} + \|u_t\|_{L_2(Q_T)},$$

$$\|u\|_{W_{2,\psi}^{2,2}(Q_T)} = \left(\int\limits_{Q_T} \left(u^2 + \sum_{i=1}^n u_i^2 + \sum_{i,j=1}^n u_{ij}^2 + u_t^2 + \psi^2(x,t)\, u_{tt}^2 + \right.\right.$$

$$+\psi(x,t)\sum_{i=1}^{n}u_{it}^2\Bigg)dxdt\Bigg)^{\frac{1}{2}},$$

respectively, $\dot{W}^{2,2}_{2,\psi}(Q_T)$ is a subspace of $W^{2,2}_{2,\psi}(Q_T)$ that has a set of all functions from $C^\infty(\bar{Q}_T)$, vanishing on parabolic boundary $\Gamma(Q_T)$ as a dense set.

For $R>0$, $x^0\in\mathbb{E}_n$ we denote a ball $\{x:|x-x^0|<R\}$ by $B_R(x^0)$, a cylinder $B_R(x^0)\cap(0,T)$ by $Q^R_T(x^0)$. Let $\bar{B}_R(x^0)\subset\Omega$. We say that $u(x,t)\in A(Q^R_T(x^0))$ if $u(x,t)\in C^\infty(\bar{Q}^R_T(x^0))$, $u|_{t=0}=0$ and supp $u\subset\bar{Q}^\rho_T(x^0)$ for some $\rho\in(0,R)$.

Everywhere further a notation $C(\cdot)$ shows that a positive constant C depends only on the contents of brackets.

Let's consider a model operator

$$L_0=\Delta+\psi(x,t)\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t},$$

where $\Delta=\sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}$ is Laplace operator.

Lemma 1. *If the conditions (5)-(7) are fulfilled for the function $\psi(x,t)$ then such $T_1(\psi,n)$ exists at $T\le T_1$ for any function $u(x,t)\in A(Q^R_T(x^0))$ the estimate is true:*

$$\int\limits_{Q^R_T(x^0)}\left(\sum_{i,j=1}^{n}u_{ij}^2+u_t^2+\psi^2(x,t)u_{tt}^2+\psi(x,t)\sum_{i=1}^{n}u_{it}^2\right)dxdt\le$$

$$\le(1+2D\cdot S)\int\limits_{Q^R_T(x^0)}(L_0u)^2dxdt, \qquad (8)$$

where $S=S(\psi,n)$ is some constant,

$$D=D(T)=q(T)+q_1(T),$$

$$q(T)=\sup_{t\in[0,T]}\varphi(t),\qquad q_1(T)=\sup_{t\in[0,T]}\varphi'(t).$$

Proof. Let's denote for simplicity $B_R(x^0)$ and $Q^R_T(x^0)$ by B and Q respectively, and $dxdt$ by $d\nu$. We have

$$I=\int\limits_Q(L_0u)^2d\nu=\int\limits_Q(\Delta u+\psi(x,t)u_{tt}-u_t)^2d\nu=$$

$$=\int\limits_Q(\Delta u)^2d\nu+\int\limits_Q\psi^2(x,t)u_{tt}^2d\nu+\int\limits_Q u_t^2d\nu+2\int\limits_Q\psi(x,t)\Delta u\cdot u_{tt}d\nu-$$

$$-2\int\limits_Q\Delta u\cdot u_td\nu-2\psi(x,t)u_{tt}u_td\nu=i_1+i_2+i_3+i_4+i_5+i_6. \qquad (9)$$

We will consider each addend separately. We have

$$i_1=\int\limits_Q(\Delta u)^2\,d\nu=\sum_{i,j=1}^n\int\limits_Q u_{ii}u_{jj}d\nu.$$

Here, successively applying integration by parts with respect to variables $x_j,\ x_i$ and taking into account that $\left.\dfrac{\partial u}{\partial x_j}\right|_{\partial B}=0$, we obtain

$$\sum_{i,j=1}^n\int\limits_Q u_{ii}u_{jj}d\nu=-\sum_{i,j=1}^n\int\limits_Q u_{iij}u_jd\nu=\sum_{i,j=1}^n\int\limits_Q u_{ij}^2d\nu.$$

Therefore, $i_1=\sum\limits_{i,j=1}^n\int\limits_Q u_{ij}^2d\nu$.

The integrals $i_2,\ i_3$ do not change. In the integral i_4, integrating by parts with respect to a variable t, then- x_i and taking into account that $\psi\,(x,T)=u_{ii}|_{t=0}=0$, we get

$$i_4=2\int\limits_Q\psi\,(x,t)\,\Delta u\cdot u_{tt}d\nu=2\sum_{i=1}^n\int\limits_Q\psi\,(x,t)\,u_{ii}u_{tt}d\nu=$$

$$=-2\sum_{i=1}^n\int\limits_Q\left(\psi\,(x,t)\,u_{ii}\right)_t u_td\nu=-2\sum_{i=1}^n\int\limits_Q\psi_t\,(x,t)\,u_{ii}u_td\nu-$$

$$-2\sum_{i=1}^n\int\limits_Q\psi\,(x,t)\,u_{iit}u_td\nu=-2\sum_{i=1}^n\int\limits_Q\psi_t\,(x,t)\,u_{ii}u_td\nu+$$

$$+2\sum_{i=1}^n\int\limits_Q\left(\psi\,(x,t)\,u_t\right)_i u_{it}d\nu=-2\sum_{i=1}^n\int\limits_Q\psi_t\,(x,t)\,u_{ii}u_td\nu+$$

$$+\,2\sum_{i=1}^n\int\limits_Q\psi_i\,(x,t)\,u_tu_{it}d\nu+2\sum_{i=1}^n\int\limits_Q\psi\,(x,t)\,u_{it}^2d\nu. \tag{10}$$

Recollecting that $\psi\,(x,t)=\lambda\,(\rho)\,\omega\,(t)\,\varphi\,(T-t)$ we rewrite (10) in the following form

$$i_4=-2\sum_{i=1}^n\int\limits_Q\lambda\,(\rho)\,\omega'\,(t)\,\varphi\,(T-t)\,u_{ii}u_td\nu+$$

$$+2\sum_{i=1}^n\int\limits_Q\lambda\,(\rho)\,\omega\,(t)\,\varphi'\,(T-t)\,u_{ii}u_td\nu+2\sum_{i=1}^n\int\limits_Q\frac{\partial\lambda}{\partial x_i}\omega\,(t)\,\varphi\,(T-t)\,u_tu_{it}d\nu+$$

$$+\,2\sum_{i=1}^n\int\limits_Q\lambda\,(\rho)\,\omega\,(t)\,\varphi\,(T-t)\,u_{it}^2d\nu. \tag{11}$$

For convenience we denote the expression $\lambda(\rho)\left[\left|\omega'(t)\right|\varphi(T-t)+\omega(t)\varphi'(T-t)\right]$ by A.

Then first two terms in the right hand side of i_4 can be estimated as follows

$$-2\sum_{i=1}^{n}\int_{Q}A\left|u_{ii}\right|\left|u_t\right|d\nu\geq-\sum_{i=1}^{n}\int_{Q}Au_{ii}^2d\nu-\sum_{i=1}^{n}\int_{Q}Au_t^2d\nu\geq$$

$$\geq-\sum_{i,j=1}^{n}\int_{Q}Au_{ij}^2d\nu-n\int_{Q}Au_t^2d\nu. \tag{12}$$

Note that $\dfrac{\partial\lambda}{\partial x_i}=\lambda'(\rho)\dfrac{\partial\rho}{\partial x_i}$. Taking into account that $\left|\dfrac{\partial\rho}{\partial x_i}\right|\leq const$ and condition (5), we get $\left|\dfrac{\partial\lambda}{\partial x_i}\right|\leq p\sqrt{\lambda(\rho)},\ i=\overline{1,n}$, where p is some constant.

Then the third term in (11) can be estimated from below by

$$-2p\sum_{i=1}^{n}\int_{Q}\sqrt{\lambda(\rho)}\omega(t)\varphi(T-t)\left|u_t\right|\left|u_{it}\right|d\nu,$$

and taking into account that for any $\varepsilon>0$ and arbitrary a and b the inequality $2|ab|\leq\varepsilon a^2+\dfrac{1}{\varepsilon}b^2$ is true, we conclude that

$$2\sum_{i=1}^{n}\int_{Q}\frac{\partial\lambda}{\partial x_i}\omega(t)\varphi(T-t)u_tu_{it}d\nu\geq-2p\sum_{i=1}^{n}\int_{Q}\sqrt{\lambda(\rho)}\omega(t)\varphi(T-t)\times$$

$$\times\left|u_t\right|\left|u_{it}\right|d\nu\geq-p\varepsilon\sum_{i=1}^{n}\int_{Q}\lambda(\rho)\omega(t)\varphi(T-t)u_{it}^2d\nu-\frac{pn}{\varepsilon}\int_{Q}\omega(t)\varphi(T-t)u_t^2d\nu\geq$$

$$\geq-p\varepsilon\sum_{i=1}^{n}\lambda(\rho)\omega(t)\varphi(T-t)u_{it}^2d\nu-\frac{pnq(T)k}{\varepsilon}\int_{Q}u_t^2d\nu, \tag{13}$$

where $k=\sup\limits_{t\in[0,T]}\omega(t)$, and $\varepsilon>0$ will be chosen later.

Then from (10)-(13) we obtain an estimate for i_4

$$i_4\geq-\sum_{i,j=1}^{n}\int_{Q}Au_{ij}^2d\nu-n\int_{Q}Au_t^2d\nu-p\varepsilon\sum_{i=1}^{n}\int_{Q}\lambda(\rho)\omega(t)\times$$

$$\times\varphi(T-t)u_{it}^2d\nu-\frac{pnq(T)k}{\varepsilon}\int_{Q}u_t^2d\nu+2\sum_{i=1}^{n}\int_{Q}\lambda(\rho)\omega(t)\varphi(T-t)u_{it}^2d\nu\geq$$

$$\geq-\sum_{i,j=1}^{n}\int_{Q}Au_{ij}^2d\nu-n\int_{Q}Au_t^2d\nu+(2-p\varepsilon)\sum_{i=1}^{n}\int_{Q}\lambda(\rho)\omega(t)\times$$

$$\times\varphi\left(T-t\right)u_{it}^{2}d\nu-\frac{pnq\left(T\right)k}{\varepsilon}\int\limits_{Q}u_{t}^{2}d\nu.$$

Then, $i_5=-2\int\limits_{Q}\Delta u\cdot u_t d\nu=-2\sum\limits_{i=1}^{n}\int\limits_{Q}u_{ii}u_t d\nu.$

Integrating by parts with respect to a variable x_i and taking into account that $u_i|_{t=0}=0$, we obtain

$$i_5=2\sum_{i=1}^{n}\int\limits_{Q}u_i u_{it}d\nu=\sum_{i=1}^{n}\int\limits_{Q}\left(u_i^2\right)_t d\nu=\sum_{i=1}^{n}\int\limits_{B}u_i^2\left(x,T\right)dx\geq 0.$$

And finally,

$$i_6=-2\int\limits_{Q}\psi\left(x,t\right)u_{tt}u_t d\nu=-\int\limits_{Q}\psi\left(x,t\right)\left(u_t^2\right)_t d\nu=$$

$$=-\int\limits_{Q}\lambda\left(\rho\right)\omega\left(t\right)\varphi\left(T-t\right)\left(u_t^2\right)_t d\nu.$$

Integrating by parts with respect to t, and taking into account that $\varphi\left(T-t\right)|_{t=T}=0$, we get

$$i_6\geq\int\limits_{Q}\lambda\left(\rho\right)\left(\omega\left(t\right)\varphi\left(T-t\right)\right)_t u_t^2 d\nu\geq\int\limits_{Q}\lambda\left(\rho\right)\omega'\left(t\right)\varphi\left(T-t\right)u_t^2 d\nu-$$

$$-\int\limits_{Q}\lambda\left(\rho\right)\omega\left(t\right)\varphi'\left(T-t\right)u_t^2 d\nu\geq-\int\limits_{Q}Au_t^2 d\nu.$$

So,

$$I\geq\sum_{i,j=1}^{n}\int\limits_{Q}u_{ij}^2 d\nu+\int\limits_{Q}\psi^2\left(x,t\right)u_{tt}^2 d\nu+\int\limits_{Q}u_t^2 d\nu+$$

$$+\left(2-p\varepsilon\right)\sum_{i=1}^{n}\int\limits_{Q}\psi\left(x,t\right)u_{it}^2 d\nu-\frac{pnq\left(T\right)k}{\varepsilon}\int\limits_{Q}u_t^2 d\nu-\sum_{i,j=1}^{n}\int\limits_{Q}Au_{ij}^2 d\nu-$$

$$-n\int\limits_{Q}Au_t^2 d\nu-\int\limits_{Q}Au_t^2 d\nu\geq\left[1-S_1D\left(T\right)\right]\sum_{i,j=1}^{n}\int\limits_{Q}u_{ij}^2 d\nu+$$

$$+\int\limits_{Q}\psi^2\left(x,t\right)u_{tt}^2 d\nu+\left[1-S_2D\left(T\right)-\frac{pnq\left(T\right)k}{\varepsilon}\right]\int\limits_{Q}u_t^2 d\nu+$$

$$+\left(2-p\varepsilon\right)\sum_{i=1}^{n}\int\limits_{Q}\psi\left(x,t\right)u_{it}^2 d\nu.$$

Assume that $\varepsilon = \frac{1}{p}$, then

$$I \geq [1 - S_3 D(T)] \sum_{i,j=1}^{n} \int_Q u_{ij}^2 d\nu + \int_Q \psi^2(x,t) u_{tt}^2 d\nu +$$

$$+ \left[1 - S_4 D(T) - p^2 n q(T) K\right] \int_Q u_t^2 d\nu + \sum_{i,j=1}^{n} \int_Q \psi(x,t) u_{it}^2 d\nu \geq$$

$$\geq [1 - SD(T)] \left(\sum_{i,j=1}^{n} \int_Q u_{ij}^2 d\nu + \int_Q \psi^2(x,t) u_{tt}^2 d\nu + \int_Q u_t^2 d\nu + \sum_{i=1}^{n} \int_Q \psi u_{it}^2 d\nu \right),$$

where $S_1,\ S_2,\ S_3,\ S_4,\ S$ are some constants dependent on ψ and n.

Therefore,

$$I \geq [1 - SD(T)] \left(\sum_{i,j=1}^{n} \int_Q u_{ij}^2 d\nu + \int_Q \psi^2(x,t) u_{tt}^2 d\nu + \right.$$

$$\left. + \int_Q u_t^2 d\nu + \sum_{i=1}^{n} \int_Q \psi(x,t) u_{it}^2 d\nu \right). \tag{14}$$

Considering T_1 so small that $SD(T_1) \leq \frac{1}{2}$ and taking into account that at $T \leq T_1$

$$\frac{1}{1 - SD(T)} = 1 + \frac{SD(T)}{1 - SD(T)} \leq 1 + 2SD(T),$$

we conclude that

$$\int_Q \left(\sum_{i,j=1}^{n} u_{ij}^2 + u_t^2 + \psi^2(x,t) u_{tt}^2 + \psi(x,t) \sum_{i=1}^{n} u_{it}^2 \right) d\nu \geq$$

$$\geq [1 + 2SD(T)] \int_Q (L_0 u)^2 d\nu.$$

Lemma is proved.

Let $\delta = \sup_{Q_T} \left(\sum_{i,j=1}^{n} (a_{ij}(x,t) - \delta_{ij})^2 \right)^{\frac{1}{2}}$, δ_{ij} is Cronecker's symbol.

Lemma 2. *If the conditions of the previous lemma are fulfilled and besides $\delta < 1$, then for any function $u(x,t) \in A\left(Q_T^R(x^0)\right)$ at $T \leq T_2(\psi, \delta, n)$ the following estimate is true*

$$I = \int_{Q_T^R(x^0)} \left(\sum_{i,j=1}^{n} u_{ij}^2 + u_t^2 + \psi^2(x,t) u_{tt}^2 + \psi(x,t) \sum_{i=1}^{n} u_{it}^2 \right) dxdt \leq$$

$$\leq c_1 \int\limits_{Q_T^R(x^0)} (Lu)^2\, dxdt, \tag{15}$$

where $c_1 = c_1(\psi, \delta, n)$.

Proof. Again denote $Q_T^R(x^0)$ by Q for simplicity. Assuming $T \leq T_1$ we get from the previous lemma

$$I^{\frac{1}{2}} \leq c\,\|L_0 u\|_{L_2(Q)} \leq c\,\|Lu\|_{L_2(Q)} + c\,\|(L - L_0)\,u\|_{L_2(Q)}\,, \tag{16}$$

where $c = \sqrt{1 + 2SD(T)}$.

On the other hand

$$\|(L - L_0)\,u\|_{L_2(Q)} = \left(\int\limits_Q \sum_{i,j=1}^{n} \left((a_{ij}(x,t) - \delta_{ij})\,u_{ij}\right)^2 dxdt\right)^{\frac{1}{2}} \leq$$

$$\leq \delta \left(\int\limits_Q \sum_{i,j=1}^{n} u_{ij}^2 dxdt\right)^{\frac{1}{2}}.$$

Thus from (16) we conclude

$$I^{\frac{1}{2}} \leq c\,\|Lu\|_{L_2(Q)} + \delta c \left(\int\limits_Q \sum_{i,j=1}^{n} u_{ij}^2 dxdt\right)^{\frac{1}{2}}. \tag{17}$$

As $q(T) \to 0$, $q_1(T) \to 0$ at $T \to 0$, then $D(T)$ tends to zero as $T \to 0$.

Taking into account the above-stated and the fact that at $\delta < 1$ such $T'(\psi, \delta, n)$ exists that $\delta\,(1 + 2SD(T'))^{\frac{1}{2}} \leq \dfrac{1+\delta}{2} < 1$ and assuming that $T_2 = \min\{T_1, T'\}$ we get the needed estimate (15) at $T \leq T_2$ from (17). The lemma is proved.

Remark. Condition (3) is not used in our proof. Actually it follows from the fact that $\delta < 1$.

Replace now condition (3) by the weaker one

$$\inf_{Q_T} \sum_{i=1}^{n} a_{ii}(x,t) = \gamma' > 0. \tag{3'}$$

Lemma 3. *If condition (3') is fulfilled then the condition $\delta < 1$ follows from Cordes condition (4) to within non-degenerated linear transformation.*

Proof. Assume that condition (4) is fulfilled. We rewrite the condition $\delta < 1$ in the equivalent form

$$\sup_{Q_T} \left(\sum_{i,j=1}^{n} a_{ij}^2(x,t) - 2\sum_{i=1}^{n} a_{ii}(x,t) < 1 - n\right). \tag{18}$$

Let's make a linear transformation of coordinates $y_i = \sqrt{\mu} x_i, \quad i = \overline{1,n}$, where a positive constant μ will be chosen later. Then condition (18) for the operator L's image takes a form

$$\sup_{Q_T} \left(\mu^2 \sum_{i,j=1}^{n} a_{ij}^2 (x,t) - 2\mu \sum_{i=1}^{n} a_{ii} (x,t) \right) < 1 - n. \tag{19}$$

We choose $\mu = \inf\limits_{Q_T} \sum\limits_{i=1}^{n} a_{ii} (x,t) \big/ \sup\limits_{Q_T} \sum\limits_{i,j=1}^{n} a_{ij}^2 (x,t)$. It's clear that $\mu > 0$ due to condition (3'). Then from (19) we obtain

$$\sup_{Q_T} \left(\frac{\inf\limits_{Q_T} \left(\sum\limits_{i=1}^{n} a_{ii} (x,t) \right)^2}{\left(\sup\limits_{Q_T} \sum\limits_{i,j=1}^{n} a_{ij}^2 (x,t) \right)^2} \sum_{i,j=1}^{n} a_{ij}^2 (x,t) - \right.$$

$$\left. - \frac{2 \inf\limits_{Q_T} \sum\limits_{i=1}^{n} a_{ii} (x,t)}{\sup\limits_{Q_T} \sum\limits_{i,j=1}^{n} a_{ij}^2 (x,t)} \sum_{i=1}^{n} a_{ii} (x,t) \right)^2 < 1 - n. \tag{20}$$

But (20) will be fulfilled if

$$- \frac{\inf\limits_{Q_T} \left(\sum\limits_{i=1}^{n} a_{ii} (x,t) \right)^2}{\sup\limits_{Q_T} \sum\limits_{i,j=1}^{n} a_{ij}^2 (x,t)} < 1 - n.$$

The last condition is equivalent to condition (4). The lemma is proved. Everywhere further, without losing generality, we will consider $R \geq 1$.

Lemma 4. *If the conditions (3'), (4)-(7) are fulfilled for coefficients of the operator L then at $T \leq T_2$ the following estimate holds true for any function $u(x,t) \in A\left(Q_T^R\left(x^0\right)\right)$*

$$\|u\|_{W_{2,\psi}^{2,2}(Q_T^R)} \leq c_2 (\psi, \delta, n) \|Lu\|_{L_2(Q_T^R)} . \tag{21}$$

The proof follows from lemma 2 and Friedrichs inequality.

We say that $u(x,t) \in A_1 \left(Q_T^R\left(x^0\right)\right)$ if

$$u(x,t) \in C^{\infty}\left(\bar{Q}_T^R\left(x^0\right)\right), \quad u|_{t=0} = 0.$$

Lemma 5. *If the conditions (3′), (4)-(7) are fulfilled for coefficients of the operator L then at $T \le T_2$ and at any $\varepsilon > 0$ the following estimate is true for any function $u(x,t) \in A_1\left(Q_T^R\left(x^0\right)\right)$*

$$\|u\|_{W_{2,\psi}^{2,2}\left(Q_T^{R/2}(x^0)\right)} \le c_2 \|Lu\|_{L_2\left(Q_T^R(x^0)\right)} + \varepsilon \|u\|_{W_{2,\psi}^{2,2}\left(Q_T^R(x^0)\right)} +$$

$$+ \frac{c_3(\psi,\sigma,n)}{\varepsilon R^2} \|u\|_{L_2\left(Q_T^R(x^0)\right)} . \qquad (22)$$

Proof. Let Q have the same meaning as before, and $Q_1 = B_{\frac{R}{2}(x^0)} \times (0,T)$. Let's consider a function $\eta(x) \in C_0^\infty\left(B_R\left(x^0\right)\right)$, such that $\eta(x) = 1$ at $x \in B_{\frac{R}{2}(x^0)}$, $0 \le \eta(x) \le 1$ and

$$|\eta_i| \le \frac{c_4(n)}{R}, \qquad |\eta_{ij}| \le \frac{c_4}{R^2}, \qquad i,j = \overline{1,n}. \qquad (23)$$

Applying lemma 4 to a function $u(x,t)\,\eta(x)$ we get

$$\|u\|_{W_{2,\psi}^{2,2}(Q_1)} = \|u\eta\|_{W_{2,\psi}^{2,2}(Q_1)} \le \|u\eta\|_{W_{2,\psi}^{2,2}(Q)} \le c_2 \|L(u\eta)\|_{L_2(Q)} . \qquad (24)$$

But on the other hand

$$L(u\eta) = \eta Lu + uL\eta + 2\sum_{i,j=1}^{n} a_{ij}(x,t)\, u_i \eta_j .$$

Therefore, using (23) in (24) we conclude

$$\|u\|_{W_{2,\psi}^{2,2}(Q_1)} \le c_2 \|Lu\|_{L_2(Q)} + \frac{c_5(\psi,\sigma,n)}{R^2} \|u\|_{L_2(Q)} +$$

$$+ \frac{c_6(\sigma,n)}{R} \sum_{i=1}^{n} \|u_i\|_{L_2(Q)} .$$

The last estimate together with (24) gives us

$$\|u\|_{W_{2,\psi}^{2,2}(Q_1)} \le c_2 \|Lu\|_{L_2(Q)} + \frac{c_7(\psi,\sigma,n)}{R^2} \|u\|_{W_2^{1,0}(Q)} . \qquad (25)$$

By interpolation inequality for any $\varepsilon_1 > 0$

$$\|u\|_{W_2^{1,0}(Q)} \le \varepsilon_1 \|u\|_{W_2^{2,0}(Q)} + \frac{c_8(n)}{\varepsilon_1} \|u\|_{L_2(Q)} . \qquad (26)$$

Let's fix an arbitrary $\varepsilon > 0$ and assume that $\varepsilon_1 = \dfrac{\varepsilon R^2}{c_7}$. Then from (25)-(26), taking into account that $\|u\|_{W_2^{2,0}(Q)} \le \|u\|_{W_{2,\psi}^{2,2}(Q)}$, we obtain

$$\|u\|_{W_{2,\psi}^{2,2}(Q_1)} \le c_2 \|Lu\|_{L_2(Q)} + \varepsilon \|u\|_{W_{2,\psi}^{2,2}(Q)} + \frac{c_8 \cdot c_7}{\varepsilon R^2} \|u\|_{L_2(Q)} ,$$

whence the needed estimate (22) follows with $c_3 = c_8 \cdot c_7$. The lemma is proved.

Let's denote the set $\{x : x \in \Omega,\ dist(x,\partial\Omega) > \rho\}$ for $\rho > 0$ by Ω_ρ and let $Q_T(\rho) = \Omega_\rho \times (0,T)$.

Consequence. *If coefficients of the operator L satisfy the conditions (3′), (4)-(7) then at $T \le T_2$ and any $\varepsilon > 0$ the following estimate is true for any function $u(x,t) \in C^\infty\left(\bar{Q}_T\right)$, $u|_{t=0} = 0$*

$$\|u\|_{W^{2,2}_{2,\psi}(Q_T(\rho))} \le c_9(\psi,\sigma,n,\rho,\Omega)\|Lu\|_{L_2(Q_T)} + \varepsilon\|u\|_{W^{2,2}_{2,\psi}(Q_T)} +$$

$$+\frac{c_{10}(\psi,\sigma,n,\rho,\Omega)}{\varepsilon}\|u\|_{L_2(Q_T)}.$$

Lemma 6. *If coefficients of the operator L satisfy the conditions (3′), (4)-(7), then such $\rho_1(n,\sigma,\Omega)$ exists that if $T \le T_2$ then for any $\varepsilon > 0$ the following estimate is true for any function $u(x,t) \in C^\infty\left(\bar{Q}_T\right)$, $u|_{\Gamma(Q_T)} = 0$*

$$\|u\|_{W^{2,2}_{2,\psi}(Q'_T(\rho_1))} \le c_{11}(\psi,\sigma,n,\rho_1,\Omega)\|Lu\|_{L_2(Q_T)} + \varepsilon\|u\|_{W^{2,2}_{2,\psi}(Q_T)} +$$

$$+\frac{c_{12}(\psi,\sigma,n,\rho_1,\Omega)}{\varepsilon}\|u\|_{L_2(Q_T)}, \qquad (27)$$

where $Q'_T(\rho_1) = Q_T \backslash Q_T(\rho_1)$.

Proof. Let's fix arbitrary point $x^0 \in \partial\Omega$ and $\varepsilon > 0$. Let's make an orthogonal transformation of coordinates $x \to y$ so that the tangent hyperplane to $\partial\tilde{\Omega}$ at the point y^0 is perpendicular to y_n- axis. (Here $\tilde{\Omega}$, $\partial\tilde{\Omega}$ and y^0 are images of Ω, $\partial\Omega$ and x^0 under such a transformation). By condition $\partial\Omega \in C^2$. For simplicity we take that the equation of $\partial\tilde{\Omega}$ is given by $y_n = \nu(y_1,..,y_{n-1})$ in its intersection with some neighbourhood O_h of the point y^0, and $\nu \in C^2$, and the part of $\tilde{\Omega}$ adjacent to $\partial\tilde{\Omega}\cap 0_h$ is situated on the set $\{y : y_n > 0\}$.

It can be shown that Cordes condition for the operator L's image holds true under our transformation.

Let's make another transformation of coordinates $y \to z$ in the following way: $z_1 = y_1, ..., z_{n-1} = y_{n-1}$, $z_n = y_n - \nu(y_1,...,y_{n-1})$.

Denote by $\bar{a}_{ij}(z,t)$ the images of $\tilde{a}_{ij}(y)$ under such transformation of space coordinates, $i,j = \overline{1,n}$ and by z^0- an image of the point y^0. We have

$$\bar{a}_{ij}(z,t) = \sum_{k,l=1}^{n} \tilde{a}_{kl}(y,t)\frac{\partial z_i}{\partial y_k}\frac{\partial z_j}{\partial y_l}, \quad i,j,k,l = \overline{1,n}.$$

It's clear that

$$\frac{\partial y_i}{\partial z_k} = \begin{cases} \delta_{ik} & \text{at} \quad i < n, \\ -\nu_{y_k} & \text{at} \ \ i = n,\ k < n, \\ 1 & \text{at} \quad i = k = n. \end{cases} .$$

Let $1 \le i,\ j \le n-1$. Then

$$\bar{a}_{ij}(z,t) = \sum_{k,l=1}^{n} \tilde{a}_{kl}(y,t)\,\delta_{ik}\delta_{jl} = \tilde{a}_{ij}(y,t).$$

Let $i = n$, $1 \le j \le n-1$. Then

$$\bar{a}_{nj}(z,t) = \sum_{k,l=1}^{n-1} \tilde{a}_{kl}(y,t)\left(-\nu_{y_k}\right)\delta_{jl} + \sum_{l=1}^{n-1} \tilde{a}_{nl}(y,t)\,\delta_{jl} +$$

$$+\sum_{k=1}^{n-1}\tilde{a}_{kn}(y,t)(-\nu_{y_k})\delta_{jn}+\tilde{a}_{nn}(y,t)\delta_{jn}=\sum_{k=1}^{n-1}\tilde{a}_{kj}(y,t)(-\nu_{y_k})+\tilde{a}_{nj}(y,t);$$

$$\bar{a}_{nn}(z,t)=\sum_{k,l=1}^{n-1}\tilde{a}_{kl}(y,t)(-\nu_{y_k})(-\nu_{y_l})+\sum_{l=1}^{n-1}\tilde{a}_{nl}(y,t)(-\nu_{y_l})+$$

$$+\sum_{k=1}^{n-1}\tilde{a}_{kn}(y,t)(-\nu_{y_k})+\tilde{a}_{nn}(y,t).$$

As $\nu_{y_i}\left(y^0\right)=0,\ i=\overline{1,n-1}$, there exists such $h=h\left(y^0\right)$ that in $\bar{\Omega}\cap O_h\left(z^0\right)$ $|\nu_{y_i}(z)|$ are small enough. Hence

$$a_{ij}(z,t)=\begin{cases}\tilde{a}_{ij}(y,t) & \text{at } 1\le i,\ j\le n-1,\\ \tilde{a}_{nj}(y,t)-\sum\limits_{k=1}^{n-1}\tilde{a}_{kj}(y,t)\nu_{y_k} & \text{at } i=n,\ \ 1\le j\le n-1,\\ \tilde{a}_{nn}(y,t)+\sum\limits_{k,l=1}^{n-1}\tilde{a}_{kl}(y,t)\nu_{y_k}\nu_{y_l}-2\sum\limits_{k=1}^{n}\tilde{a}_{kn}(y,t)\nu_{y_k} & \text{at } i=j=n.\end{cases}$$

As $\nu_{y_i}\left(y^0\right)=0,\ i=\overline{1,n-1}$ for any $\varepsilon_2>0$ there exists such $h\left(y^0,\varepsilon_2\right)\le R_1$ that $|\nu_{y_i}(z)|<\varepsilon_2$ for $z\in\bar{\Omega}\cap O_h\left(z^0\right)$, where $\bar{\Omega}$ is an image of $\tilde{\Omega}$ under our transformation, and as a neighbourhood $O_h\left(z^0\right)$ we will take an image of $B_{h(y^0,\varepsilon_2)}\left(z^0\right)$. Whence, if the condition (4) is fulfilled for the matrix $\|\tilde{a}_{ij}(y,t)\|$ then it's fulfilled for the matrix $\|\bar{a}_{ij}(z,t)\|$ too, if only $(z,t)\in\left(\bar{\Omega}\cap B_{h(y^0,\varepsilon_2)}\left(z^0\right)\right)\times(0,T)$ and ε_2 is small enough.

We will choose and fix so small ε_2 that condition (4) is fulfilled for the matrix $\|\bar{a}_{ij}(z,t)\|$ with a constant $\sigma_1=\dfrac{\sigma+\frac{1}{n-1}}{2}$. Then the correspondent h will depend only on y^0,σ,n,ν. For simplicity we denote the obtained $h\left(y^0,\sigma,n,\nu\right)$ by $h\left(y^0\right)$.

Now let $\bar{u}(z,t)$ be an image of a function $u(x,t)$ after the transformations of space coordinates $x\to y$ and $y\to z$. It's clear that $C^+_{h(y^0)}=\left(\bar{\Omega}\cap B_{h(y^0)}\right)\times\times(0,T)$ represents a cylinder with the base $B^+_{h(y^0)}=\left\{z:\left|z-z^0\right|<h\left(y^0\right),z_n\ge0\right\}$. For any $t\in(0,T)$ we continue the function $\bar{u}(z,t)$ in an odd way and the coefficients of the operator $\bar{L}$ in an even way through the hyperplane $z_n=0$ into a semiball $B^-_{h(y^0)}=\left\{z:\left|z-z^0\right|<h\left(y^0\right),z_n<0\right\}$ and denote the function and operator continued again by $\bar{u}(z,t)$ and $\bar{L}$ respectively. Then, if

$$C_{h(y^0)}=\left(B^+_{h(y^0)}\cup B^-_{h(y^0)}\right)\times(0,T),\quad \text{then } \ \bar{u}(z,t)\in W^{2,2}_{2,\psi}\left(C_{h(y^0)}\right).$$

By lemma 5 for any $\varepsilon_3>0$

$$\|\bar{u}\|_{W^{2,2}_{2,\psi}\left(C_{\frac{h(y^0)}{2}}\right)}\le C_{13}\left(\psi,\sigma,n,y^0,\nu\right)\left\|\bar{L}\bar{u}\right\|_{L_2\left(C_{h(y^0)}\right)}+$$

$$+\varepsilon_3\|\bar{u}\|_{W^{2,2}_{2,\psi}\left(C_{h(y^0)}\right)}+\frac{c_{14}\left(\psi,\sigma,n,y^0,\nu\right)}{\varepsilon_3}\|\bar{u}\|_{L_2\left(C_{h(y^0)}\right)},\qquad(28)$$

where $C_{\frac{h(y^0)}{2}} = B_{\frac{h(y^0)}{2}}\left(z^0\right) \times (0,T)$.

But a square of every norm in the last inequality due to our continuation is doubled square of a norm taken with respect to the cylinder $C^+_{\frac{h(y^0)}{2}}$ (in the left hand side) and $C^+_{h(y^0)}$ (in the right hand side) respectively. Hence

$$\|\bar{u}\|_{W^{2,2}_{2,\psi}\left(C^+_{\frac{h(y^0)}{2}}\right)} \leq c_{13}\left\|\bar{L}\bar{u}\right\|_{L_2\left(C^+_{h(y^0)}\right)} + \varepsilon_3\|\bar{u}\|_{W^{2,2}_{2,\psi}\left(C^+_{h(y^0)}\right)} + \frac{c_{14}}{\varepsilon_3}\|\bar{u}\|_{L_2\left(C^+_{h(y^0)}\right)}$$

Let $\Gamma_{\frac{h(x^0)}{2}}$ be a preimage of $C^+_{\frac{h(y^0)}{2}}$ in x-variables. Then

$$\|u\|_{W^{2,2}_{2,\psi}\left(\Gamma_{\frac{h(x^0)}{2}}\right)} \leq c_{15}\|Lu\|_{L_2(Q)} + c_{16}\varepsilon_3\|u\|_{W^{2,2}_{2,\psi}(Q)} + \frac{c_{17}}{\varepsilon_3}\|u\|_{L_2(Q)}, \tag{29}$$

where constants $c_{15},\ c_{16},\ c_{17}$ depend only on $\gamma,\ \sigma,\ n$ and Ω. Let's cover $\partial\bar{\Omega}$ with a system of balls $\left\{B_{\frac{h(z^\nu)}{2}}\right\}$ and choose a finite subcover $\{B_1,...,B_N\}$ from it. It's obvious that N depends only on $\partial\Omega, n$ and σ. Let $\Gamma_1,...,\Gamma_N$ be preimages of cylinders $[B_1\times(0,T)],...,[B_N\times(0,T)]$ in (x,t)- variables.

Taking a square of both sides of (29) and adding up the obtained inequalities with respect to i from 1 to N we get

$$\|u\|_{W^{2,2}_{2,\psi}(Q\backslash Q(\rho_1))} \leq 3N\left(c^2_{15}\|Lu\|^2_{L_2(Q)} + c^2_{16}\varepsilon^2_3\|u\|^2_{W^{2,2}_{2,\psi}(Q)} + \frac{c^2_{17}}{\varepsilon^2_3}\|u\|^2_{L_2(Q)}\right), \tag{30}$$

where ρ_1 is such, that $\bigcup\limits_{i=1}^{N}\Gamma_i \supset Q\backslash Q(\rho_1)$. It's clear that $\rho_1 = \rho_1(n,\sigma,\partial\Omega)$. Now it's sufficient to put $c_{11} = 3Nc_{15},\ c_{12} = 3N^2c_{16}c_{17}$ and $\varepsilon_3 = \dfrac{\varepsilon}{3Nc_{16}}$, and the needed estimate (27) follows from (30).

The lemma is proved.

Lemma 7. *Under the conditions of lemma 6 the following estimate is true for any* $u(x,t) \in \dot{W}^{2,2}_{2,\psi}(Q_T)$ *at* $T \leq T_2$

$$\|u\|_{W^{2,2}_{2,\psi}(Q_T)} \leq c_{18}(\psi,\sigma,n,\Omega)\|Lu\|_{L_2(Q_T)} + c_{19}(\psi,\sigma,n,\Omega)\|u\|_{L_2(Q)}. \tag{31}$$

Proof. By lemma 5 and 6 for any $\varepsilon > 0$

$$\|u\|^2_{W^{2,2}_{2,\psi}(Q(\rho_1))} \leq 3c^2_9\|Lu\|^2_{L_2(Q)} + 3\varepsilon^2\|u\|^2_{W^{2,2}_{2,\psi}(Q)} + \frac{3c^2_{10}}{\varepsilon^2}\|u\|^2_{L_2(Q)},$$

$$\|u\|^2_{W^{2,2}_{2,\psi}(Q\backslash Q(\rho_1))} \leq 3c^2_{11}\|Lu\|^2_{L_2(Q)} + 3\varepsilon^2\|u\|^2_{W^{2,2}_{2,\psi}(Q)} + \frac{3c^2_{12}}{\varepsilon^2}\|u\|^2_{L_2(Q)}.$$

Adding up these inequalities and denoting $c^2_9 + c^2_{11} = c_{20},\ c^2_{10} + c^2_{12} = c_{21}$ we obtain

$$\|u\|^2_{W^{2,2}_{2,\psi}(Q)} \leq 3c_{20}\|Lu\|^2_{L_2(Q)} + 6\varepsilon^2\|u\|^2_{W^{2,2}_{2,\psi}(Q)} + \frac{3c_{21}}{\varepsilon^2}\|u\|^2_{L_2(Q)}.$$

Let's choose and fix $\varepsilon = \dfrac{1}{2\sqrt{3}}$. We get

$$\|u\|^2_{W^{2,2}_{2,\psi}(Q)} \leq 6c_{20} \|Lu\|^2_{L_2(Q)} + 72c_{21} \|u\|^2_{L_2(Q)} .$$

Whence the needed estimate (31) follows with $c_{18} = \sqrt{6c_{20}}$, $c_{19} = 6\sqrt{2c_{21}}$. The lemma is proved.

Theorem. *If the conditions (3′), (4)-(7) are fulfilled then such* $T_0 = T_0(\psi, \sigma, n, \Omega)$ *exists that at* $T \leq T_0$ *the following estimate is true for any function* $u(x,t) \in \dot{W}^{2,2}_{2,\psi}(Q)$

$$\|u\|_{W^{2,2}_{2,\psi}(Q)} \leq c_{22}(\psi, \sigma, n, \Omega) \|Lu\|_{L_2(Q)} . \tag{32}$$

Proof. It's sufficient to prove estimate (32) for smooth funcitons from $\dot{W}^{2,2}_{2,\psi}(Q)$. We have for any $t \in (0,T)$ and $x \in \Omega$

$$u(x,t) = \int_0^t u_t(x,\tau)\, d\tau.$$

Using Cauchy-Bunyakowsky inequality we can write

$$u^2(x,t) \leq T \int_0^T u_t^2(x,\tau)\, d\tau.$$

Then $\int_Q u^2(x,t)\, dxdt \leq T^2 \int_Q u_t^2(x,t)\, dxdt.$.

So, $\|u\|_{L_2(Q)} \leq T \|u_t\|_{L_2(Q)} \leq T \|u\|_{W^{2,2}_{2,\psi}(Q)}$.

Let $T_0 = \min\left\{T_2, \dfrac{1}{2c_{19}}\right\}$. Then from (31) at $T \leq T_0$ the needed estimate (32) follows with $c_{22} = 2c_{18}$.

The theorem is proved.

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