Physical-Mechanical Model of Fibrous Polymerized Bar Deformation at the Neck Zone

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ABSTRACT

In the paper the author suggests a model of mechanical deformation of a polymerized bar made of fibres in the neck with regard to forces interacting with a matrix. In the paper, the author suggests an experimental-theoretical method that allows one to define the character of deformation non-homogeneity in the neck area. In particular, analytic dependence of mechanical properties of the material on longitudinal coordinate of the sample is established. Numerical comparison of theoretical results with experimental ones is given.

Keywords: *Adhesive Strength, Cross-Sections of Neck Along The Length, Neck in Composite Material, Non-Homogeneity, Polymerized Bar Made of Fibres*

INTRODUCTION

Adhesive strength of a composite material is connected with interacting forces in the contact area between reinforcing force fibrous structure and a binding material. The binding medium itself also renders essential influence on the character of mechanical deformation of fibrous structure in a binding medium. Therefore, it is necessary to know the model of mechanical deformation of a polymerized bundle-like body with respect to forces interacting with a matrix, in particular if a bundle has necking.

The necking arising in structural elements made of visco-plastic, elastico-plastic or visco-elastic materials is as follows. The experiments on simple tension of samples made of plastic metals show that starting with some moment, the deformed state of the smooth part of the sample stops to be non-homogeneous. The arising deformation is concentrated on some not large area of the sample. This narrowed area is called “a neck”. The following questions are of scientific interest: 1) in what form will be the physico-mechanical dependence between the stress and deformation at the neck zone points; 2) in what form will be the stress-strain state at the points of the area of the neck zone. Up to day, the scientific investigations were carried out mainly on the second theme; i.e. on studying the stress-strain state of the structural elements at the necking zone points. These investigations were carried out at the initial period of necking. In other words, at the moment of collapse of

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homogeneity, the model of mechanical deformation of the material is accepted as before necking. A.A. Ilyushin first investigated the neck phenomena in such a statement. He first gave statement of the problem on stability of visco-plastic flow of a strip and a bar; differential equations and boundary conditions of the problem for defining visco-plastic plane-parallel flows were composed. Complexity of this problem was connected with the Lagrange method for describing the motion of continuous medium (Ilyushin, 1940; Ilyushin, 1943; Ilyushin, 2011). Using the method based on the Euler way of description of medium motion, A. Yu. Ishlinsky used the visco-plastic flow of a strip and a bar (Ishlinsky, 1943). Zhukov A.M. considered a problem on stress strain state at the neck zone of a strip and a bar made of elastico-plastic material. They gave numerical comparison of theoretical results with experimental ones (Zhukov, 1949). A number of scientists carried out experimental investigations on finding true stresses at the neck zone, i.e. the stresses with regard to change of the cross section area at the neck zone. They also analyzed the character of the arising deformation inhomogeneity of the tested materials (Siebel, 1944; Davidenko, & Spiridonova, 1945; Sachs & Lubahn, 1946; MacGrefor, 1944).

In 1970-2000 the necking phenomenon got a new heightened scientific interest. This was connected with appearance of laminated and composite materials made on the base of polymers. One of the important problems in strength of laminated and composite materials is the provision of adhesive strength between the layers of the material. This directly depends on the character of necking in the area of contact of heterogeneous materials. A cycle of experimental investigations on studying the interlayer failure mechanism in laminated and composite materials was performed. Creation of some criteria of adhesive strength in laminated and composite materials (Ogibalov & Suvorova Yu, 1965; Korten, 1967; Aliyev, 1987; Ogibalov, Malinin, Netrebko, & Kishkin, 1972). Aliyev G.G. has created a generalized conception of composite materials mechanics with regard to arising interacting forces between the elements of the composite. He has suggested a more precise hypothesis of adhesive relativn between the layers (Aliyev, 1984; Aliyev, 1987; Aliyev, 1998). A new theory of strength, stability and vibration of laminary-reinforced flexible thick-walled pipes and thin-walled shells with regard to arising forces interacting between the elements of the composite has been created on this base.

However, the problem on definition of the mechanism of appearance of deformation non-homogeneity at the neck zone and also establishment of physico-mechanical dependence between the stress and deformation at the neck zone points has not been solved to day because of its complexity. At present this problem belongs to the unsolved problems of continuum mechanics.

Aliyev G.G. (2011) first suggested the appearance of deformation non-homogeneity mechanism at the neck zone, and representation one of the models of physico-mechanical dependence between stress and deformation at the neck zone points. He suggested an experimental-theoretical method on the base of which it was established that the character of deformation non-homogeneity at the neck zone is linear with respect to longitudinal coordinate.

In the considered paper, an experimental-theoretical method that allows to establish the character of deformation non-homogeneity at the neck zone is suggested. In particular, the analytic dependence of mechanical characters of the material on longitudinal coordinate of the sample is established.

METHOD

In the paper, we suggest a physical-mechanical model of deformation of a polymerized bar made of fibres at the neck area with regard to arising forces interacting with a binder under the following assumptions:

- The cross-section area of a bar made of fibres at the neck zone is a function of longitudinal coordinate x. The cross-section area has the form of the area of a circle,

- The equation of neck's contour is known,
- The thickness of the polymerized bar is such that radial stress σ_r in the interval $|r| \leq r_{bundle}$ changes negligibly.

THEORETICAL BACKGROUND

Under these assumptions, the normal $\vec{\varepsilon}_\nu$ and tangential $\vec{\varepsilon}_\tau$ deformation vectors arising on the boundary of lateral surface of a polymerized bar with a binder, will characterize deformed state of each inner point of a polymerized bar made of fibres. On the other hand, the deformation vectors $\vec{\varepsilon}_\nu$ and $\vec{\varepsilon}_\tau$ at the points of the boundary of a polymerized bar with a binder will create appropriate stress vectors $\vec{\sigma}_\nu = E_F(x)\lambda_\nu(x)\vec{\varepsilon}_\nu$ and $\vec{\sigma}_\tau = E_F(x)\lambda_\tau(x)\vec{\varepsilon}_\tau$ that will be the functions in the form of neck's contour. Note that the deformation vector $\vec{\varepsilon}$ at the lateral surface points of the neck is expressed by normal $\vec{\nu}$ and tangential $\vec{\tau}$ unit vectors and also Cartesian coordinates $(\vec{i}, \vec{j}, \vec{k})$ by the following formulae:

$$\vec{\varepsilon} = \vec{\varepsilon}_\nu + \vec{\varepsilon}_\tau = \varepsilon_\nu \vec{\nu} + \varepsilon_\tau \vec{\tau} \quad (1)$$

$$\vec{\varepsilon} = \vec{\varepsilon}_x \ell + \vec{\varepsilon}_y m + \vec{\varepsilon}_z n, \quad (2)$$

where

$$\varepsilon_\nu = \vec{\varepsilon}\vec{\nu},$$

$$\varepsilon_\tau = \vec{\varepsilon}\vec{\tau} \quad (3)$$

$$\vec{\nu} = \vec{i}Cos\alpha + \vec{j}Sin\alpha = -\vec{i}\,\frac{dy}{ds} + \vec{j}\,\frac{dx}{ds}$$

$$\vec{\tau} = -\vec{i}Sin\alpha + \vec{j}Cos\alpha = -\vec{i}\,\frac{dx}{ds} - \vec{j}\,\frac{dy}{ds} \quad (4)$$

The vectors $\vec{\varepsilon}_x, \vec{\varepsilon}_y, \vec{\varepsilon}_z,$ are expressed by the known way through Cartesian coordinates ε_{ij}.

From the equations equilibrium of forces per the element length of the dx at the neck zone with the area of annular cross-section $dF(x)$ and lateral surface of the form of a hyperboloid $dL(x)$ of one sheet we suggest the dependence of longitudinal stress vector $\vec{\sigma}_F(\chi)$ on longitudinal deformation vector $\vec{\varepsilon}_\chi = \varepsilon_\chi \vec{i}$ and vectors of deformations $\vec{\varepsilon}_\nu$ and $\vec{\varepsilon}_\tau$ acting on lateral surface of a polymerized bar at the neck zone, in the form (Figure1):

$$\vec{\sigma}_F(x) = E_F(x)[\vec{\varepsilon}_x + \lambda_\nu(x)\vec{\varepsilon}_\nu + \lambda_\tau(x)\vec{\varepsilon}_\tau] \quad (5)$$

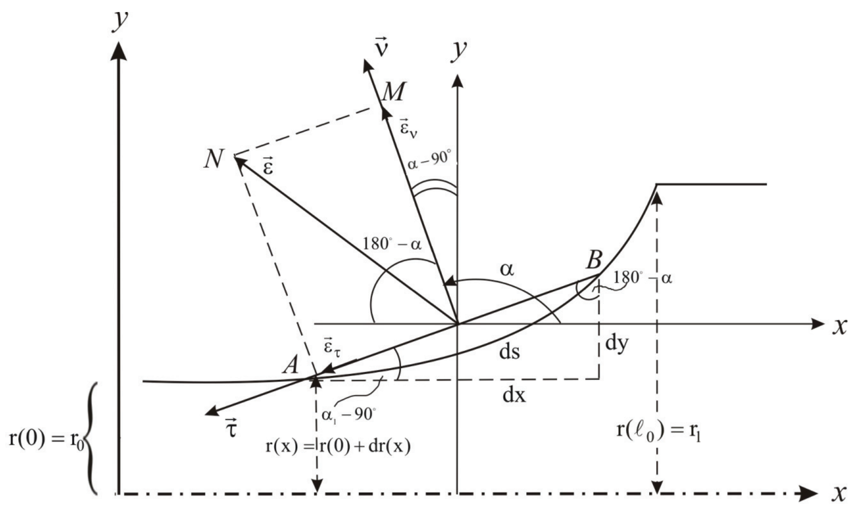
Here $E_F(x) = E_{Filament}(x),\ \lambda_\nu(x),\ \lambda_\tau(x)$ are mechanical characteristics of a polymerized bar made of fibres at the sections of the neck zone. They will depend on a longitudinal coordinate x of the bar. These mechanical characteristics will depend on the ratio of elementary area of the lateral surface $dL(x)$ to the cross section area $dF(x)$ of a polymerized bar in the form $f(x) = \dfrac{dL(x)}{dF(x)} = \dfrac{\ell}{2\eta}\dfrac{1}{x}$, and also on the parameter $\eta = \dfrac{\delta}{\ell}$, i.e. the ratio of the depth of the neck δ to its length ℓ. The mechanical characteristics stated above will be defined by the following form special experiments.

THEORETICAL FRAMEWORK

Experimental theoretical method for defining mechanical characteristics of a polymerized bar made of fibres $E_F(x),\ \lambda_\nu(x),\ \lambda_\tau(x)$ at the neck zone.

Consider a bar made on the base of a fibrous structure, situated under the action of longitudinally applied tensile force P and polymerized into a matrix. Moreover, under tension there happens necking in the bar (Figure 1). Arrange the system of coordinates in the narrow place of the neck with the following geometrical characteristics: for $x = 0$ a neck of length $2\ell_0$ has the depth δ_0. It is also supposed that

Figure 1. Dependence of longitudinal stress vector $\vec{\sigma}_F(x)$ *on longitudinal deformation vector* $\vec{\varepsilon}_x = \varepsilon_x \vec{i}$ *and vectors of deformations* $\vec{\varepsilon}_\nu$ *and* $\vec{\varepsilon}_\tau$ *acting on lateral surface of a polymerized bar at the neck zone*



the form of the neck's contour is known and it is a part of a circle.

Suppose that at the points $x = 0$ and $x = \ell_0$ of the neck the values of a longitudinal module of elasticity $E_F(0)$ and $E_F(\ell_0)$ of a polymerized bar are known. Is this case we represent the dependence of the modulus of elasticity $E_F(x)$ at any x section of the neck of a polymerized bar made of fibers on longitudinal coordinate x in the following linear form:

$$E_F(x) = E_F(\ell_0)[1 + (\frac{E_F(0)}{E_F(\ell_0)} - 1)(1 - \frac{x}{\ell_0})]$$

for

$$0 \le x \le \ell_0 \tag{6}$$

Here, $E_F(\ell_0)$ is the value of the modulus of elasticity of a polymerized bar at the section $x = \ell_0$ of the neck, i.e. at the end of the neck. The value of the modulus of elasticity $E_F(\ell_0)$ on the end of the neck will coincide with the value of the modulus of elasticity of a polymerized bar outside of the neck whose definition method was suggested in the paper (Aliyev, 1987) and equals:

$$E_F(\ell_0) = k_\sigma E_0 \tag{7}$$

where E_0 is a modulus of elasticity of a bundle without the influence of binding medium.

Procedure

As the influence of lateral deformation of a binder in the narrowest part of the neck $x = 0$ of the bar on a lateral deformation of a polymerized bar is small, we can assume that in the narrow part of the neck $x = 0$ only the fibres directed strictly in longitudinal direction work. Therefore, as a modulus of elasticity $E_F(0)$ at the section $x = 0$ of the neck we can accept the modulus of elasticity of one fibre, i.e.

$$E_F(0) = E_{fibre} \tag{8}$$

Then, allowing for (8), formula (6) will take the following terminal form:

$$E_F(x) = E_F(\ell_0)[1 + (\frac{E_f}{E_F(\ell_0)} - 1)(1 - \frac{x}{\ell_0})]$$

For

$$0 \le x \le \ell_0 \quad (9)$$

Here $E_f(0) = E_{fibre}$. Thus, dependence of the modulus of elasticity $E_F(x)$ on the longitudinal coordinate x in the interval $0 \le x \le \ell_0$ will characterize non-homogeneity of the points of sections along the neck's length that will change in the interval:

$$E_F(\ell_0) \le E_F(x) \le E_f$$

For

$$0 \le x \le \ell_0 \quad (10)$$

By formula (5) the coefficient $\lambda_\nu(x)$ characterizes effect of lateral contraction of a polymerized bar made of fibres at its lateral surface in the direction of the normal $\vec{\nu}$ to the contour of the curved neck with regard to influence of all arising deformations of the form:

$$\lambda_\nu(x) = -\frac{\varepsilon_\nu}{\varepsilon_x} = -\frac{\vec{\varepsilon}_\nu \vec{\nu}}{\vec{\varepsilon}_x \vec{i}} \quad (11)$$

RESULTS

For establishing its mathematical dependence, we behave as follows. Let the numerical values of this parameter in the sections $x = 0$ and $x = \ell_0$ of the neck a polymerized bar be given, i.e. the coefficients $\lambda_\nu(0)$ and $\lambda_\nu(\ell_0)$ be given. Here $\lambda_\nu(\ell_0)$ is the value of the coefficient $\lambda_\nu(x)$ in the section $x = \ell_0$ of the neck, i.e. at the end of the neck. But the value of the coefficient $\lambda_\nu(0)$ is given in the narrow place of the neck. As the influence of lateral deformation of a binding material in the narrowest part of the neck $x = 0$ of the bar on lateral deformation of a polymerized bar is small, we can assume that in the narrow part of the neck $x = 0$ only the fibers directed strongly in longitudinal direction work. Therefore, as the value of the coefficient $\lambda_\nu(0)$ in the narrow part of the neck we can accept the Poisson ratio of only one fibre $\lambda_\nu(fibre)$. Under these conditions, dependence of the coefficient $\lambda_\nu(x)$ at any section of the neck on the longitudinal coordinate x may be represented in the following linear form:

$$\lambda_\nu(x) = \lambda_\nu(fib.) + \\ + \frac{x}{\ell_0}[\lambda_\nu(\ell_0) - \lambda_\nu(fib.)] \quad (12)$$

For $\lambda_\nu(fib.) = 0$ we get the following simplified dependence:

$$\lambda_\nu(x) = \lambda_\nu(\ell_0)\frac{x}{\ell_0} \quad (13)$$

Note that the value of the coefficient at the end of the neck $\lambda_\nu(\ell_0)$ will be the value of the coefficient $\lambda_\nu(x)$ outside of the neck whose mathematical expression is given in the paper (Aliyev, 1987) and in conformity to our paper will have the form:

$$\lambda_\nu(\ell_0) = \frac{1}{\nu_{bind.}}[1 - \frac{1}{n}\frac{E_{aver.} - E_{bind.}(1 - nk_F)}{E_0 k_F k_\sigma}] \quad (14)$$

Here $E_{aver.}$ and $E_{bind.}$ are the module of elasticity of a sample and a binder, $\nu_{bind.}$ is a Poisson ratio of a binder, n is the amount of bundles in the sample, $k_F = F_F / F$ is the ratio of the cross section area of a polymerized bundle to the cross section area of a sample

$k_\sigma = \dfrac{\sigma_F^{in\ medium}}{\sigma_F^{free}}$ is the coefficient characterizing the ratio of ultimate strength of a fibrous bundle in medium to ultimate strength of a bundle tested in air, $F_{bind.} = F - nF_F$ is the cross section area of sample's binder.

Thus, dependence of the $\lambda_\nu(x)$ coefficient on the longitudinal coordinate x in the interval $0 \le x \le \ell_0$ will change in the interval:

$$0 \le \lambda_\nu(x) \le \lambda_\nu(\ell_0)$$

For

$$0 \le x \le \ell_0 \tag{15}$$

In formula (5) the $\lambda_\tau(x)$ coefficient characterizes shifting effect arising between the reinforcing fibrous structure of a bar and a binder at the neck zone. Therefore, we can represent mechanical characteristic of the coefficient $\lambda_\tau(x)$ at the points of lateral surface of the neck as the ratio of the shear modulus of the binder G_c to longitudinal modulus of elasticity $E_F(x)$ of a polymerized bar in the form:

$$\lambda_\tau(x) = \frac{G_{bind.}}{E_F(x)} = \frac{G_{bind.}}{E_F(\ell_0)[1 + (\frac{E_f}{E_F(\ell_0)} - 1)(1 - \frac{x}{\ell_0})]} \tag{16}$$

Thus, the dependence of the coefficient $\lambda_\tau(x)$ on the longitudinal coordinate x in the interval $0 \le x \le \ell_0$ will change in the interval:

$$\frac{G_{bind.}}{E_f} \le \lambda_\tau(x) \le \frac{G_{bind.}}{E_F(\ell_0)}$$

for

$$0 \le x \le \ell_0 \tag{17}$$

Thus, having experimentally determined the mechanical characteristics $E_F(x)$, $\lambda_\nu(x)$ and $\lambda_\tau(x)$ of a polymerized bar at the cross section points of the neck by formulae (6), (13) and (16), we can suggest the dependence of a longitudinal stress vector $\vec{\sigma}_F(\chi)$ in a polymerized bar on a longitudinal deformation vector $\vec{\varepsilon}_\chi = \varepsilon_\chi \vec{i}$ and deformation vectors $\vec{\varepsilon}_\nu$ and $\vec{\varepsilon}_\tau$ acting on lateral surface of the bundle in the sections of the neck in the form of formula (5).

DISCUSSION

On an example of a polymerized bar made on the base of a filamentary structure from a fibre glass and rubber, state the methodics as experimental definition of mechanical characteristics $E_F(x)$, $\lambda_\nu(x)$ and $\lambda_\tau(x)$ of a polymerized glass bar at the neck zone and suggest concrete mechanical deformation models of glass rubber bars at the neck zone.

Show numerically the degree of influence of stress state of a binder (rubber) on mechanical characteristics of a bundle shaped structure at the neck zone. To this end, glass ropes of marks БС6-34 × 1 × 3 × n are used. Here, the glass fibre represents itself as 400 elementary glass fibres of the mark БС6-34 × 1.

The influence of the form of stress state of a binder (rubber) on mechanical properties of a glass bundle of the given structure wherein they are reinforced, is studied. Thick-shelled strips of cross-section area 1, 33 cm^2 along of which the glass ropes of amount 1, 3, 5 were reinforced, were tested.

The modulus of elasticity $E_F(\ell_0)$, average coefficient $\lambda_\nu(\ell_0)$ of lateral compression of a glass bundle in the rubber and ultimate strength of a reinforced glass rope with regard to the form of stress state of a binder (rubber) $\sigma_F^{in\ medium}$ outside of the neck are determined on the base of experimental data. The test results are given in Table 1. The following models of mechanical deformation of a polymerized filament outside of the neck were constructed on the base of these experimental data:

$$\sigma_F^5(\ell_0) = 0,655 \cdot 10^6 (\varepsilon_x + 0,684\varepsilon_\perp),$$

for the mark

$$БС\,6 \times 34 \times 1 \times 3 \times 5$$
$$\sigma_F^{10}(\ell_0) = 0,561 \cdot 10^6 (\varepsilon_x + 0,585\varepsilon_\perp),$$

for the mark

$$БС 6 \times 34 \times 1 \times 3 \times 10$$
$$\sigma_F^{15}(\ell_0) = 0,551 \cdot 10^6 (\varepsilon_x + 0,512\varepsilon_\perp),$$

for the mark

$$БС 6 \times 34 \times 1 \times 3 \times 15 \quad (18)$$

By formulae (6), (13) and (16), allowing for experimental data of Table 1 we establish the mathematical dependence of mechanical characteristics $E_F(x)$, $\lambda_\nu(x)$ and $\lambda_\tau(x)$ of glass ropes polymerized in rubber in the neck on longitudinal coordinate x that are given in Table 2.

The peculiarity of these mechanical characteristics is the following:

- The modulus of elasticity $E_F(x)$ of a glass bundle polymerized in rubber in a narrow part of the neck is independent on the quantity of elementary fibres contained in it and equals:

$$E_F^m(x)\Big|_{x=0} = 0,8 \cdot 10^6 \, {kg}/{cm^2} \quad (19)$$

- At the neck zone, the longitudinal modulus of elasticity $E_F(x)$ depending on x changes in the following form:

$$0,7999 \cdot 10^6 \frac{kg}{cm^2} \geq E_F^5(x) \geq 0,655 \cdot 10^6 \frac{kg}{cm^2}$$

$$0,8 \cdot 10^6 \frac{kg}{cm^2} \geq E_F^{10}(x) \geq 0,561 \cdot 10^6 \frac{kg}{cm^2},$$

For

$$0 \leq x \leq \ell_0$$

$$0,8 \cdot 10^6 \frac{kg}{cm^2} \geq E_F^{15}(x) \geq 0,551 \cdot 10^6 \frac{kg}{cm^2} \quad (20)$$

CONCLUSION

Hence it is seen that at the neck zone, variability of the modulus of elasticity of a rubber bundle polymerized in rubber is essential depending on x. Comparing the values of the modulus of elasticity of a polymerized glass bundle in the narrowest place with modulus of elasticity at the end of the neck (outside of the neck), they will be as follows: for a glass bundle in 5 fold 22,12%, in 10 fold 42,6% and in 15 fold 45,2% more than appropriate modulus of elasticity at the end of the neck.

- Average coefficient $\lambda_\nu(\ell_0)$ of lateral contraction of a glass bundle polymerized in rubber at the end of the neck essentially depends on the amount of elementary glass fibres that it contains and that equal:

$$\lambda_\nu^5(\ell_0) = 0,684,$$
$$\lambda_\nu^{10}(\ell_0) = 0,585,$$
$$\lambda_\nu^{15}(\ell_0) = 0,512.$$

- At the neck zone, the average coefficient $\lambda_\nu(x)$ of lateral contraction of a glass bundle polymerized in a rubber depending on x $(0 \leq x \leq \ell_0)$ changes in the following form:

Table 1. For $E_{rubber} = 80 \, {}^{kg}\!/_{cm^2}$, $\nu_{rubber} = 0,47$

Mark of glass bundle	Б $C6 \times 34 \times 1 \times 3 \times 5$	Б $C6 \times 34 \times 1 \times 3 \times 10$	Б $C6 \times 34 \times 1 \times 3 \times 15$
Cross section of glass bundle $F_F (cm^2)$	$237 \cdot 10^{-5}$	$465 \cdot 10^{-5}$	$685 \cdot 10^{-5}$
Modulus of elasticity of bundle $E_0 ({}^{kg}\!/_{cm^2})$	$0,524 \cdot 10^{6}$	$0,442 \cdot 10^{6}$	$0,427 \cdot 10^{6}$
Ultimate strength of glass bundle without influence of binder $\sigma^0_{\epsilon F} ({}^{kg}\!/_{cm^2})$	6751	6451	6268
Sample's cross section $F(cm^2)$	1,33	1,33	1,33
$k_F = \frac{F_F}{F}$	$178 \cdot 10^{-5}$	$350 \cdot 10^{-5}$	$516 \cdot 10^{-5}$
Ultimate strength of glass bundle in rubber $\sigma_{\epsilon F} ({}^{kg}\!/_{cm^2})$	8439	8172	8105
$k_\sigma = \frac{\sigma_{\epsilon F}}{\sigma^0_{\epsilon F}}$	1,2500	1,2668	1,2931
$E_{aver} ({}^{kg}\!/_{cm^2})$	872,38	1501,93	2234,92
Modulus of elasticity of a bundle in rubber $E_F(\ell_0) ({}^{kg}\!/_{cm^2})$	$0,655 \cdot 10^{6}$	$0,561 \cdot 10^{6}$	$0,551 \cdot 10^{6}$
Average coefficient of lateral compression of a glass bundle $\lambda_\nu(\ell_0)$	0,684	0,585	0,512

Table 2. For $E_{fibers} = 0,8 \cdot 10^6 \dfrac{kg}{cm^2}$

	$\sigma_F^m(x)$	$\lambda_\nu^m(x)$	$\lambda_\tau^m(x)$
1	$E_F^5(x) = 0,655 \cdot 10^6[1 + 0,2213(1 - \frac{x}{\ell_0})]$	$\lambda_\nu^5(x) = 0,684 \frac{x}{\ell_0}$	$\lambda_\tau^5(x) = \dfrac{41,542}{1 + 0,2213(1 - \frac{x}{\ell_0})} 10^{-6}$
2	$E_F^{10}(x) = 0,561 \cdot 10^6[1 + 0,4260(1 - \frac{x}{\ell_0})]$	$\lambda_\nu^{10}(x) = 0,585 \frac{x}{\ell_0}$	$\lambda_\tau^{10}(x) = \dfrac{48,503}{1 + 0,4260(1 - \frac{x}{\ell_0})} 10^{-6}$
3	$E_F^{15}(x) = 0,551 \cdot 10^6[1 + 0,4519(1 - \frac{x}{\ell_0})]$	$\lambda_\nu^5(x) = 0,512 \frac{x}{\ell_0}$	$\lambda_\tau^{15}(x) = \dfrac{49,383}{1 + 0,4519(1 - \frac{x}{\ell_0})} 10^{-6}$

$$0 \le \lambda_\nu^5(x) \le 0,684,$$
$$0 \le \lambda_\nu^{10}(x) \le 0,584,$$
$$0 \le \lambda_\nu^{15}(x) \le 0,512 \tag{21}$$

- The shift coefficient $\lambda_\tau(x)$ between the reinforcing fibrous structure of the bar and binding rubber at the neck zone has the following properties.

The shift coefficient $\lambda_\tau(x)$ in the narrow part of the neck is independent on the amount of elementaly glass fibres contained in it and equals:

$$\lambda_\tau^m(x)\Big|_{x=0} = 34 \cdot 10^{-6} \tag{22}$$

and at the end of the neck the coefficient $\lambda_\tau(x)$ has the following numerical values:

$$\lambda_\tau^5(\ell_0) = 41,542 \cdot 10^{-6}$$
$$\lambda_\tau^{10}(\ell_0) = 48,503 \cdot 10^{-6}$$
$$\lambda_\tau^5(\ell_0) = 49,383 \cdot 10^{-6} \tag{23}$$

The coefficient $\lambda_\tau(x)$ at the sections of neck zone, depending on x $(0 \le x \le \ell_0)$ changes in the following form:

$$\lambda_\tau^5(x) = \frac{41,542}{1 + 0,2213(1 - \frac{x}{\ell_0})} \cdot 10^{-6},$$
$$\lambda_\tau^{10}(x) = \frac{48,503}{1 + 0,4260(1 - \frac{x}{\ell_0})} \cdot 10^{-6}$$
$$\lambda_\tau^{15}(x) = \frac{49,383}{1 + 0,4519(1 - \frac{x}{\ell_0})} \cdot 10^{-6} \tag{24}$$

On the base of these experimental data, we suggest the following models of mechanical deformation of a glass bundle (filament) polymerized into a rubber at the neck zone depending on x $(0 \leq x \leq \ell_0)$: for the mark $БC6 \times 34 \times 1 \times 3 \times 5$:

$$\vec{\sigma}_F^5(x) = 0,655 \cdot 10^6 \cdot [1 + 0,2213(1 - \frac{x}{\ell_0})] \cdot \{\vec{\varepsilon}_x + 0,684 \frac{x}{\ell_0} \cdot \vec{\varepsilon}_\nu + \frac{41,542 \cdot 10^{-6}}{1 + 0,2213(1 - \frac{x}{\ell_0})} \cdot \vec{\varepsilon}_\tau\}$$

for the mark $БC6 \times 34 \times 1 \times 3 \times 10$:

$$\vec{\sigma}_F^{10}(x) = 0,561 \cdot 10^6 \cdot [1 + 0,4260(1 - \frac{x}{\ell_0})] \cdot \{\vec{\varepsilon}_x + 0,585 \frac{x}{\ell_0} \cdot \vec{\varepsilon}_\nu + \frac{48,503 \cdot 10^{-6}}{1 + 0,4260(1 - \frac{x}{\ell_0})} \cdot \vec{\varepsilon}_\tau\}$$

for the mark $БC6 \times 34 \times 1 \times 3 \times 15$:

$$\vec{\sigma}_F^{15}(x) = 0,551 \cdot 10^6 \cdot [1 + 0,4519(1 - \frac{x}{\ell_0})] \cdot \cdot\{\vec{\varepsilon}_x + 0,512 \frac{x}{\ell_0} \cdot \vec{\varepsilon}_\nu + \frac{49,383 \cdot 10^{-6}}{1 + 0,4519(1 - \frac{x}{\ell_0})} \cdot \vec{\varepsilon}_\tau\} \quad (25)$$

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